MA207 TEST 2 28 OCTOBER 15

> Please show **all** your work! Answers without supporting work will not be given full credit. Write answers in spaces provided. You have 1 hour and 20 minutes to complete this exam. You may use a personal calculator, any tables provided, and any reference sheets provided. By placing your name on the line below, you agree to uphold the Honor Code during this test.

Name: Key

1. An insurance company determines that the claims (in dollars) on a certain insurance product are normally distributed with mean \$50000 and standard deviation \$9968. In 2007, 49 claims were made, and the average of these claims was \$52848. In 2008, 64 claims were made, and the average of these claims was \$47196.50. In which year (2007 or 2008) was the average claim more unusual? (Show all work.) (10pts)

I'm going to do this problem by calculating the z-scores for $\bar{X}_{2007}$ and $\bar{X}_{2008}$. Well, $\bar{X}_{2007} \sim N(50000, 1424)$ & $\bar{X}_{2008} \sim N(50000, 1246)$.

So $z_{2007} = \frac{52848-50000}{1424} = 2$ and $z_{2008} = \frac{47196.5-50000}{1246} = -2.25$

Because $|z_{2007}| < |z_{2008}|$, the avg. claims in 2008 were more unusual.

Answer: 2008

2. Two samples are taken from the same large population. Sample A has 100 things in it. Sample B has 120 things in it. Which sample's mean has the smaller variation? (5pts)

Answer: Sample B

3. $X \sim N(10, 2)$ and $Y \sim N(10, 2)$ and X and Y are independent. What's the distribution of $X - Y$? (5pts)

$\mu_{X-Y} = \mu_X - \mu_Y = 10 - 10 = 0$. $\sigma^2_{X-Y} = \sigma^2_X + \sigma^2_Y = 2^2 + 2^2 = 8$.

And the sum or difference of normal r.v.s is normal. Thus :)

Answer: $N(0, \sqrt{8})$

4. If $P(A \text{ and } B) = 0.5$ and $P(A) = 0.75$, find $P(B|A)$. (5pts)

$$P(B|A) = \frac{P(A \& B)}{P(A)} = \frac{0.5}{0.75} = \frac{2}{3}$$

Answer: $\frac{2}{3}$

5. A sample is taken of 50 different students from BSC's 1200 students, and in this sample, we count the number of students who are from Georgia. 15% of BSC's students are from Georgia. Which of the following distributions would be the sampling distribution of this count? Select only one. (5pts)

- $B(1200, 15)$
- $B(50, 15)$
- $N(7.5, 2.525)$ ← this answer would also receive a lot of credit because this is the normal approximation of the correct answer.
- $N(50, 0.15)$
- $N(50, 0.85)$
- $B(50, 0.15)$
- $N(50, 15)$

6. A 95% confidence interval is constructed to test the claim that the mean of a population is 15. What's the significance of this hypothesis test? (5pts)

Claim: $\mu = 15$

$*H_0: \mu = 15$

$H_a: \mu \neq 15$

the test is two-tailed which matches the null & alt. hyp.

So $\alpha = 1 - 0.95 = 0.05$

Answer: 0.05 or 5%.

7. Which of the following pairs of null and alternative hypothesis would be used to test the claim that the population mean was no more than (but possibly equal to) 2? (5pts)

- $H_0: \mu = 2$ and $H_a: \mu \neq 2$
- $H_0: \mu = 2$ and $H_a: \mu < 2$
- $H_0: \mu = 2$ and $H_a: \mu > 2$
- $H_0: \mu = 2$ and $H_a: \mu \leq 2$
- $H_0: \mu = 2$ and $H_a: \mu \geq 2$
- $H_0: \mu \leq 2$ and $H_a: \mu > 2$
- $H_0: \mu \geq 2$ and $H_a: \mu < 2$

Claim: $\mu \leq 2$

$*H_0: \mu = 2$

$H_a: \mu > 2$

8. In what type of significance test do you compare the level of significance with the probability that the test statistic would take a value as extreme or more extreme than that actually observed in the sample? (5pts)

- confidence interval significance test
- P-value significance test
- critical value significance test
- none of these

9. A beverage bottling company is trying a new plastic bottle. They want the new plastic bottles to weigh the same as the old plastic bottles. A SRS of 25 old bottles is taken, and its mean and standard deviation are 25g and 0.5g respectively. A SRS of 30 new bottles is taken, and its mean and standard deviation are 24.5 and 0.75 respectively. Create a 90% confidence interval for $\mu_{\text{old}} - \mu_{\text{new}}$. With 0.1 significance, does it seem like the bottles weigh the same or not? (25pts)

not useful for this problem, but wanted you to see it calculated

$$t = \frac{(25-24.5)-(\mu_1-\mu_2)}{\sqrt{\frac{0.25}{25}+\frac{0.5625}{30}}} = 2.94884.$$

this is 0 because we're creating a confidence interval for $\mu_1-\mu_2$ under the assumption that $\mu_1=\mu_2$.

we won't use this because we're constructing a confidence interval.

$$df = \frac{\left(\frac{0.5^2}{25}+\frac{0.75^2}{30}\right)^2}{\frac{1}{24}\left(\frac{0.5^2}{25}\right)^2+\frac{1}{29}\left(\frac{0.75^2}{30}\right)^2} = 50.74.$$

so use 50 as your df in the table.

from the table, your t critical value will be 1.676.

Thus, the 90% confidence interval will be

$$(\bar{X}_{old}-\bar{X}_{new}) \pm (1.676)\left(\sqrt{\frac{0.25}{25}+\frac{0.5625}{30}}\right) = 0.5 \pm 0.2842$$

$$= (0.2158, 0.7842)$$

because 0 is not in this interval, it does not look like the bottles have the same weight.

Answer: They don't have same weight.

Cont.

10. A county elections official says that voter turnout in the latest election was greater than 65%. A SRS of 200 voters was taken, and 132 of them had voted in the latest election. Give the null and alternative hypothesis necessary to test the official's claim, tell which of the two hypotheses supports the claim, and conduct a 5% significance P-value test. State whether or not the official's claim is supported by this SRS. (20pts)

Claim: $p > 0.65$

$H_0: p = 0.65$

$\circledast H_a: p > 0.65$

z-Test statistic: $\frac{\hat{p} - p}{\sigma_p}$

$\hat{p} \sim N\left(p, \frac{\sqrt{npq}}{n}\right) = N(0.65, 0.03373)$.

$\hat{p} = \frac{132}{200} = 0.66$.

$\Longrightarrow$ z-Test statistic $= \frac{0.66 - 0.65}{0.03373} = 0.296$.

use table or calculator.

Calculate P-value: $P(z > 0.296) = 1 - P(z < 0.296) = 0.3859$.

Because $0.3859 > \alpha = 0.05$, we do not reject our null hypothesis. The official's claim is not supported by this SRS.

Note: In this case, rejecting H_0 is supporting the official's claim.

Answer: Official's claim not supported.

The End.

11. Say you want a sample mean to have standard deviation no more than than 2 and that you are sampling a population with standard deviation 25. How large should your sample be to guarantee that the standard deviation of the sample mean is no more than 2? (5pts)

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \Longrightarrow 2 \cancel{\leq} \geq \frac{25}{\sqrt{n}} \Longrightarrow$$

$\sqrt{n} \cancel{\leq} \geq 12.5 \Longrightarrow n \geq 156.25$, so your sample should be at least 157.

Answer: 157 or larger

12. Say you're testing to see if Population A has a larger proportion than Population B with significance $\alpha = 0.05$. If you get a P value of 0.09, do you support or not support the claim that Population A has a larger proportion than Population B? (5pts)

Claim: $p_A > p_B \Longrightarrow p_A - p_B > 0$

H_0: $p_A - p_B = 0$

(*) H_a: $p_A - p_B > 0$

Because $0.09 > \alpha = 0.05$, we do not reject H_0 and thus do not support our claim.

Answer: No.

13. (Bonus) Would you use a t or a z test statistic if you're conducting a significance test for a population mean when you know s? (1pt) Use t. We only use z when σ is known.

Cont.