

Please show **all** your work! Answers without supporting work will not be given full credit. Write answers in spaces provided. You have 1 hour and 20 minutes to complete this exam. You may use a personal calculator, any tables provided, and any reference sheets provided. By placing your name on the line below, you agree to uphold the Honor Code during this test.

Name: Key

1. A die is rolled many times to see if it is loaded. It lands on each side in the following way.

Side	1	2	3	4	5	6
Rolls	5	6	6	10	8	7

Create the expected counts table which would be used to test the claim that the die is not loaded, i.e. that the distribution of rolls should be uniform. (10pts)

Side	1	2	3	4	5	6
Rolls	7	7	7	7	7	7

← There are 42 observations and  $P(1) = P(2) = \dots = P(6) = \frac{1}{6}$ .

2. True or false: A one-way ANOVA tests that the population ~~variances~~ means are the same for three or more populations. (5pts)

Answer: False

3. A two-way table test is conducted on a  $3 \times 4$  table, and the test statistic is found to be 11. What can we conclude about the row and column variables? (5pts)

5%<sub>0</sub>,  $df = 6$ ,  $\chi^2$ -dist: critical value = 12.592 | If we use  $\alpha = 0.05$ , they would be indep.  
 10%<sub>0</sub>,  $df = 6$ ,  $\chi^2$ -dist: critical value = 10.645 | Answer: If we use  $\alpha = 0.10$ , they would not be indep.

4. We have the following two-way table. Does the table meet the requirements for a two-way table test? Why or why not? (5pts)

	Red	Green	Yellow	Total
Car	10	4	4	18
Truck	1	1	4	6
SUV	1	1	0	2
Total	12	6	8	26

Answer: No! The average of the expected counts would be  $2.8\bar{8}$ , which is less than 5!!

5. The line of best fit for 25  $(x, y)$  pairs of data is given by  $\hat{y} = 10 + 2x$ . We also know that  $\bar{x} = 12$  and  $\bar{y} = 8$ . We perform a 5% test of significance for  $\rho$  and get a  $P$ -value of 0.045. Our value of  $r$  was 0.97. What proportion of the variance in  $y$  can be explained by the variance in the linear regression model? (5pts)

Answer: It's  $r^2$ ! So it's 94.09%!

6. We have the following table for a test for significance for  $\beta_1$  where the alternative hypothesis is that  $\beta_1 \neq 0$ . The critical  $F$  value for this test is 11.26. Fill in the missing parts of the table. Does this test support the claim that  $\beta_1 = 0$ ? (5pts)

Source	DF	SS	MS	F
Model	1	77.092	77.092	0.391625
Error	8	3.133		
Total	9	80.225		

← Thus, we fail to reject  $H_0: \beta_1 = 0$  and support our claim

Answer: Yes!

7. Is a prediction interval for a future observation or a confidence interval for the mean response wider? (5pts)

Answer: prediction interval is wider

8. Which two tests are numerically equivalent? Circle only one. (5pts)

- a goodness-of-fit test and a two-way table test
- a test for population linear correlation and a test for slope of a population regression line
- a two-way ANOVA and two one-way ANOVAs
- all of these
- none of these

9. For the data below, we have that  $r = 0.6728$ . Does the data support the claim that  $x$  and  $y$  are linearly correlated? Use  $\alpha = 0.01$  and a critical value test. (15pts)

$x$	1	2	3	4	5	6	7	8	9	10
$y$	2	4	5	12	6	3	8	10	11	10

Test statistic:  $t = \frac{0.6728\sqrt{10-2}}{\sqrt{1-0.6728^2}} = 3.477$

Critical value: (from  $t$  distribution with 8 degrees of freedom)

3.355.

thus! We reject  $H_0: \rho = 0$  and support  $H_a: \rho \neq 0$ . ← this lines up w/ our claim

Answer: Yes!!

10. A medical researcher wants to tell if a test for diabetes is effective. She collects the following data from a sample of 100 patients.

	Diabetic	Not Diabetic
Test Positive	35	15
Test Negative	15	35

← observed

Use  $\alpha = 0.0005$  to test the claim that the result of the test is independent of whether or not the patient has diabetes. Use a critical value test. (15pts)

expected  $\rightarrow$

	D	ND
TP	25	25
TN	25	25

$$\Rightarrow \chi^2 = \frac{(35-25)^2}{25} + \frac{(15-25)^2}{25} + \frac{(35-25)^2}{25} + \frac{(15-25)^2}{25} = 16$$

The critical value (from  $\chi^2$  table with 1 degree of freedom) is 12.12. Thus, reject  $H_0$ : the factors are independent. Support  $H_a$ : the factors are dependent.

Answer: The data does not support the claim. It looks like the result of the test for diabetes does depend on the status of the patient.

11. If  $\hat{y} = 10 + x$  and  $SE_{\hat{y}} = 0.5$ , produce a 90% prediction interval for  $x = 11$ , assuming that 11 is in the range of observed  $x$  values and that the data is actually linearly correlated. (10pts) *Assume  $n=12$ .*

$$\hat{y} = 21, t^* = 2.228, SE_{\hat{y}} = 0.5, \text{ so}$$

$$PI = (21 - 2.228(0.5), 21 + 2.228(0.5)) \\ = (19.886, 22.114)$$



Answer: \_\_\_\_\_

12. Fill in the missing parts of the two-way ANOVA test. At a 5% significance level, does it appear that there is an effect due to the interaction of the two factors? (The critical value for this test is 2.15 at the 5% significance level.) Also, tell what type of distribution the test statistic has — include degrees of freedom. (15pts)

Source	DF	SS	MS	F
A	2	53.733333	26.86666	15.3038
B	4	146.83333	36.70833	<del>75.3038</del> 20.9098
AB	8	135.766	16.970833	9.6669
Error	45	79	1.75555	
Total	59	415.33333		

The test statistic has an  $F(8, 45)$  distribution.

So we reject  $H_0$ : there is no effect due to the interaction of factors A & B.

Answer: \_\_\_\_\_

*Yes, it does appear that there is an effect due to interaction!*

13. Which is larger  $SE_{\hat{\mu}}$  or  $SE_{\hat{y}}$ ? (1pt)