Stable PCA for Detection Anomaly in Video Surveillance

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Problem
Given a sequence of video frames, how does one identify whether anomalous activities stand out in the current frame when compared to the previous frames. There are several methods proposed for this problem, namely in subspace tracking methods, such as Dynamic Sparse Coding [2], and Robust Tensor Subspace Learning [3]. Our work is motivated by [4] and [1].

Our Approach
Our approach is to utilize the stable PCA technique [1] to decompose the data matrix into its background and foreground. The columns of the data matrix is the vectorized video frames. Then the foreground frames are then measured for anomaly.

Principal Component Pursuit
Principal component pursuit is a convex programming method for separating a data matrix into a sum of a low rank and sparse matrices. The following optimization,

\[ \min_{L, S} \|L\|_* + \lambda \|S\|_1 \quad \text{subject to} \quad L + S = M \]

is solved for the optimal low rank \(L^*\) and sparse \(S^*\) matrices from a given data matrix \(M\). Here we denote \(\|L\|_* = \sum_i \sigma_i(L)\) as the trace class norm of \(L\) and \(\|S\|_1 = \sum_{ij} |S_{ij}|\) as the \(\ell_1\) norm of \(S\).

Future Work
- Develop principal component pursuit for tensors.
- Develop local with global anomaly detections.
- Implement subspace detection.

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Algorithm
The algorithm for is based on augmented lagrange multiplier (ALM) ([33,51] in Candès) with the augmented Lagrangian

\[ l(L, S, Y) = \|L\|_* + \lambda \|S\|_1 + \langle Y, M - L - S \rangle + \frac{\mu}{2} \|M - L - S\|_F^2 \]

with multiplier \(Y\).

Algorithm 1 (Principal Component Pursuit by Alternating Directions)
1: initialize: \(S_0 = Y_0 = 0, \mu > 0\).
2: while not converged do
3: compute \(L_{k+1} = D_{\mu^{-1}}(M - S_k + \mu^{-1}Y_k)\);
4: compute \(S_{k+1} = S_{\lambda^{-1}}(M - L_{k+1} + \mu^{-1}Y_k)\);
5: compute \(Y_{k+1} = Y_k + \mu(M - L_{k+1} - S_{k+1})\);
6: end while
7: output: \(L, S\).

Let \(S_{\lambda} : R \rightarrow R\) is the shrinkage operator defined as \(S_{\lambda} = \text{sgn}(x) \max(|x| - \frac{\lambda}{\mu}, 0)\) and \(D_{\lambda^{-1}}(X) = U\Sigma_{\lambda} - 1(V)^*\) with \(X = U\Sigma V^*\) is the singular value thresholding operator.

Spare Matrix (Foreground) Modeling From Video
An Example of Data Separation: \(M\) is a frame in the video, \(L\) is the low rank matrix (background), and \(S\) is the sparse matrix (foreground)

Anomaly Classification: Let the error for frame \(t\) be the Frobenius 2-norm

\[ e_t = \|S_t\|_F^2. \]

Classify frame \(t\) as abnormal when

\[ e_t \geq \text{mean}(e_i) + 2 \cdot \text{std}(e_i) \text{ for } i = 1 \ldots t \]

The figures with red boundary are classified as abnormal.

References