CS4: Contributed Session IV

Error Estimates for a Regularization of a Formulation of the Porous Medium Equation

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We consider the equation

$$\frac{\partial(\phi S)}{\partial t} - \nabla \cdot (k(S)\nabla S) = Q(S) \qquad (1)$$

which is a more general form of the classical Porous Medium Equation (PME)

$$\frac{\partial S}{\partial t} - \triangle(Sm) = Q(S) \qquad (2)$$

In Equation (1), which is obtained through a mathematical modeling of an immiscible and incompressible two-phase flow through a porous medium (assuming here there is no transport), one often assumes that the porosity ϕ is either independent of the time variable or changes little with time. In this case one obtains the equation

$$\frac{\partial(S)}{\partial t} - \nabla \cdot (k(S)\nabla S) = Q(S) \qquad (3)$$

This can bring in some loss of information even when the product rule is used and the term $\frac{\partial \phi}{\partial t}S$ is buried in the righthand side. In this presentation, we consider the case where the porosity $\phi = \phi(x,t)$ is a function of both the spatial variable x and the temporal variable t. Because of the degeneracies $(k(0) = k(1) = 0)$, we regularize the problem (1) in the usual way and derive error estimates for the regularization problem.

Two-phase Generalized Forchheimer Flows in Porous Media

Thinh Kieu, *University of North Georgia*

We derive a non-linear system of parabolic equations to describe the onedimensional two-phase generalized Forchheimer flows of incompressible, immiscible fuids in porous media, with the presence of capillary forces. We prove the existence of non-constant steady state solutions under relevant constraints on relative permeabilities and capillary pressure. A weighted maximum principle is proved with the weight function depending on the steady state. Utilizing this crucial property, we establish the corresponding weighted stability for the perturbation and derive long time estimates for its weighted L_1-norm. Moreover, the stability for velocities (on bounded intervals) is obtained by using Bernstein's estimate technique.

On the Physics of Incompressible Fluids

Jonas Holdeman

The Navier-Stokes equation would seem to be 'settled law' governing incompressible fluid flow. But is it? Does this differential-algebraic equation hide a more complex structure? An integro-differential equation perhaps? Is there a physical basis for this IDE, or is it a 'mathematical trick'? If a 'physical' basis, what would first-principles derivation entail? How might this affect computation? Would this clarify or confuse physical and mathematical understanding? These intriguing questions and more will be addressed.

Effect of Round Cavities on Flow and Heat Transfer Characteristics in Converging Pipes: A Numerical Study

Khalid Alammar, *King Saud University*

Using the standard k-e turbulence model, a developing, steady two-dimensional turbulent flow inside straight and converging pipes was simulated with and without round cavities. Effect of cavities on flow and heat transfer characteristics was investigated. Uncertainty was approximated through validation and grid independence. The simulation revealed circulation within the cavities. Cavity boundaries were shown to contribute significantly towards turbulence production. Cavity presence was shown to enhance overall heat transfer while increasing pressure drop significantly across the pipe.