MS23: Nonlinear PDEs of Conservation Laws and Fluid Mechanics III

Non-Strictly Hyperbolic Systems, Singularities and Bifurcation

Katarzyna Saxton, *Loyola University*

We examine a 2x2 genuinely nonlinear hyperbolic system in (e, p) variables which loses the property of strict hyperbolicity wherever $e = 0$. In hyperbolically degenerate cases this may occur on sets $x =$ constant, along which singularities can form, even in the presence of damping in the system. Furthermore, these sets may bifurcate at some time to create secondary curves prior to blow up. We discuss the existence and properties of these additional curves. Numerical examples are considered for Riemann and smooth data.

Further Results on Irregular Weak Reflection

Allen Tesdall, *CUNY Staten Island*

Recent numerical solutions and shock tube experiments have shown the existence of a complex reflection pattern known as GMR which provides a resolution of the triple point paradox. This pattern is characterized by a discontinuous transition from supersonic to subsonic flow at the rear of each patch in a sequence of tiny supersonic patches. We study numerically the possibility of an alternate structure in which the transition from supersonic to subsonic flow is smooth.

On the Properties of Weak Solutions Describing Dynamic Cavitation in Nonlinear Elasticity

Alexey Miroshnikov, *University of Massachusetts*

In this work we study the problem of dynamic cavity formation in isotropic compressible nonlinear elastic media. Cavitating solutions were introduced by J.M. Ball [1982, Phil. Trans. R. Soc. Lond. A] in elastostatics and by K.A. Pericak-Spector and S. Spector [1988, Arch. Rational Mech. Anal.] in elastodynamics. They turn out to decrease the total mechanical energy and provide a striking example of non-uniqueness of entropy weak solutions (in the sense of hyperbolic conservation laws) for polyconvex energies. In our work we established various further properties of cavitating solutions. For the equations of radial elasticity we construct self-similar weak solutions that describe a cavity emanating from a state of uniform deformation. For dimensions d = 2,3 we show that cavity formation is necessarily associated with a unique precursor shock. We also study the bifurcation diagram and do a detailed analysis of the singular asymptotics associated to cavity initiation as a function of the cavity speed of the self-similar profiles. We show that for stress-free cavities the critical stretching associated with dynamically cavitating solutions coincides with the critical stretching in the bifurcation diagram of equilibrium elasticity.

Spectral and Nonlinear Stability of Viscous Detonation Waves

Greg Lyng, *University of Wyoming*

In this talk, we outline a program, combining analytical and numerical Evans-function techniques, for evaluating the spectral and nonlinear stability of viscous detonation waves. In the relatively simple case of Majda's qualitative combustion model, this program has been completely carried out, and we describe how to obtain nonlinear stability results for both strong (Lax-type) and weak (under compressive) detonation waves. Finally, we discuss the extension of this program to the physically relevant case of the Navier-Stokes equations modeling a compressible mixture of reacting gases. The results in this case are interesting.