Optical Design of Laser Beam Shaping Systems

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Outline of Presentation

- Overview of history and current practices
- Geometrical methods for design
- Applications:
  - Two-plano-aspheric lens system
  - Two-mirror system with no central obscuration
  - Three-element GRIN system
Historical Background

- **Frieden, Appl. Opt. 4.11, 1400-1403, 1965:** “Lossless conversion of a plane wave to a plane wave of uniform irradiance.”

- **Kreuzer, US patent 3,476,463, 1969:** “Coherent light optical system yielding an output beam of desired intensity distribution at a desired equi-phase surface.”

Contemporary Beam Shaping*

- Process of redistributing the irradiance and phase

- Two functional categories:
  - Field Mapping
  - Beam Integrators

Field Mapping Beam Shaper
Beam Integrators
Physical versus Geometrical Optics

\[ \beta = \frac{2\sqrt{2\pi} r_0 Y_0}{f \lambda} \]

\( \lambda \) = wavelength, \( r_0 \) = waist or radius of input beam,
\( Y_0 \) = half-width of the desired output dimension
\( f \) = focal length of the focusing optic, or the working distance from the optical system to the target plane

Beam Shaping Guidelines:

\( \beta < 4 \), Beam shaping will not produce acceptable results
\( 4 < \beta < 32 \), Diffraction effects are significant
\( \beta > 32 \), Geometrical optics methods should be adequate
Selected Chapter Titles:


• “Geometrical Methods,” D.L. Shealy


• “Beam Shaping with Diffractive Diffusers,” D.R. Brown.


• “Current Technology of Beam Profile Measurements,” C.B. Roundy.
Overview of Geometrical Methods

• Geometrical optics intensity law:

\[ \nabla \cdot (I \mathbf{a}) = 0 \]

\[ I_1 \, dA_1 = I_2 \, dA_2 \]

• Constant optical path length condition:

\[ (OPL)_0 = (OPL)_r \]

- Geometrical methods leads to equations of the optical surfaces:
  \[ z(r) = \int f(r) dr + C \]
  \[ Z(r) = z(r) + g(r) \]

- Global Optimization with discrete & continuous variables:
  - Beam shaping merit function

\[ M = M_{\text{Diameter}} M_{\text{Collimation}} M_{\text{Uniformity}} \]
Applications of Geometrical Methods

- Two plano-aspheric lens system for shaping rotationally symmetric Gaussian beam.

- Two mirror system with no central obscuration for shaping elliptical Gaussian beam.

- Three-element GRIN system for shaping rotationally symmetric Gaussian beam.
Two Lens Beam Shaping System
Jiang, Ph.D. Dissertation, UAB, 1993

Energy Balance:

\[ R = \sqrt{\frac{r_0^2}{2I_{out}}} \left[ 1 - \exp\left( -2r^2/r_0^2 \right) \right] \]

\[ I_{out} = \frac{r_0^2}{2R_{\text{max}}^2} \left[ 1 - \exp\left( -2r_{\text{max}}^2/r_0^2 \right) \right] \]

Ray Trace Equation:

\[ z' = \frac{-(R-r)(Z-z) \pm (n/n_0)(R-r)\sqrt{(Z-z)^2 + (R-r)^2}}{(1-(n/n_0)^2)(Z-z)^2 - (n/n_0)^2(R-r)^2} \]

Constant OPL:

\[ (Z-z) = \frac{n(n-n_0)d + \left[ n_0^2(n-1)^2d^2 + (n^2-n_0^2)(R-r)^2 \right]^{1/2}}{n^2-n_0^2} \]
Input and Output Beam Profiles
Beam Shaping Applications

In a holographic projection processing system featured on 10 January 1999 issue of *Applied Optics*, a two-lens beam shaping optic increased the quality of micro-optical arrays.
Two Mirror Beam Shaping System
Shealy & Chao, Proc SPIE 4443-03, 2001

Input beam has 1-to-3 beam waist in x-to-y directions:
Parameters:

\[ I_{out} = A_{X_0} A_{Y_0} = \text{const.} \]

\[ I_{in}(x, y) = \exp \left[ -2 \left( \frac{x}{x_0} \right)^2 \right] \exp \left[ -2 \left( \frac{y}{y_0} \right)^2 \right] \]
Sag of First Mirror

Parameters:

\[ L = 10; \quad h = 5 \]
\[ l_0 = \sqrt{L^2 + h^2} \]
\[ r_0 = 1; \quad x_0 = \frac{r_0}{2}; \quad x_{\text{max}} = r_0 \]
\[ y_0 = \frac{3r_0}{2}; \quad y_{\text{max}} = 3r_0 \]
\[ X_{\text{max}} = 2r_0 = Y_{\text{max}} \]

\[
z(x, y)(L + l_0) = \frac{1}{2}(x^2 + y^2) + \frac{r_0^2}{\sqrt{2\pi} \operatorname{erf}(2\sqrt{2})}
\times \left\{ 4 - \exp\left[-2\left(\frac{2x}{r_0}\right)^2\right] - 3\exp\left[-2\left(\frac{2y}{3r_0}\right)^2\right] - \frac{2\sqrt{2\pi}}{r_0} \left[ x \operatorname{erf}\left(\frac{2\sqrt{2}x}{r_0}\right) + y \operatorname{erf}\left(\frac{2\sqrt{2}y}{3r_0}\right) \right] \right\}
\]
Optical Performance Analysis*

- ZEMAX was used.

- Elliptical Gaussian input beam profile was modeled as user-defined surface.

- Two mirror surfaces were also modeled as user-defined surfaces.

Relative Illumination

Input Beam

Output Beam
First Mirror Surface Analysis
Second Mirror Surface Analysis
Tolerance Analysis

- In A, the first mirror was decentered 10% of its diameter about x-axis.
- In B, both mirrors were decentered by 2.5% of their diameter about x- and y-axes and tilted about the three axes by 0.25 degrees.
Conclusions for 2 Mirror System

• Designed a two-mirror system with no central obscuration for shaping a 3:1 elliptical Gaussian beam into a uniform output beam.

• ZEMAX used to do optical performance analysis:
  – First mirror has strong aspherical component along direction of smaller waist (120µm for 6mm diameter mirror).
  – Output beam profile is stable for decentering of less than 2.5% of mirror diameter and 0.25 degrees about coordinate axis.
3-Element GRIN Beam Shaping System


- Can a spherical-surface GRIN shaping system be designed using catalog GRIN materials?
- System would have practical applications.
- Problem is well suited for Genetic Algorithms:
  - discrete parameters
    - Number of lens elements
    - GRIN catalog number
  - Continuous parameters: radii, thickness
Initialize GA by randomly picking new individuals

Evaluate Merit Function for each individual in generation

Perform genetic operations (reproduction, mutations, cross-overs); produce new generation

Is the population stagnant? (Micro-GA check)

Keep best individual and replace the remainder with randomly-selected individuals

Termination criterion reached?

End
Merit Function Used in GA Optimization

\[ M = \frac{M_{\text{Diameter}} M_{\text{Collimation}}}{M_{\text{Uniformity}}} = \exp\left[ -s \left( \frac{R_{\text{Target}} - R_N}{2} \right)^2 \right] \exp\left[ - \left( 1 - \prod_{i=1}^{N} \cos^Q (\gamma_i) \right)^2 \right] \]

\[ \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left( I_{\text{out}} (R_i) - \left[ \frac{1}{N} \sum_{k=1}^{N} I_{\text{out}} (R_k) \right] \right)} \]

\( R_{\text{target}} = \text{Output Beam Radius} \)
\( R_N = \text{Marginal Ray Height on Output Plane} \)
\( \gamma_i = \text{Angle } i^{th} \text{ Ray Make with the Optical Axis} \)
\( Q \text{ and } s = \text{Convergence Constants} \)
<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Type</th>
<th>Limits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Elements</td>
<td>Discrete</td>
<td>1-4 (integer)</td>
</tr>
<tr>
<td>Radius of curvature of left surface of Element 1</td>
<td>Continuous</td>
<td>-100 to 100</td>
</tr>
<tr>
<td>Radius of curvature of right surface of Element 1</td>
<td>Continuous</td>
<td>-100 to 100</td>
</tr>
<tr>
<td>Thickness of Element 1</td>
<td>Continuous</td>
<td>1 to 10</td>
</tr>
<tr>
<td>Distance between Element 1 and Element 2</td>
<td>Continuous</td>
<td>1 to 10</td>
</tr>
<tr>
<td>GRIN glass type for Element 1</td>
<td>Discrete</td>
<td>1-6 (integer)</td>
</tr>
<tr>
<td>Positive or Negative GRIN for Element 1</td>
<td>Discrete</td>
<td>0 or 1</td>
</tr>
<tr>
<td>Radius of curvature of left surface of Element 2</td>
<td>Continuous</td>
<td>-100 to 100</td>
</tr>
<tr>
<td>Radius of curvature of right surface of Element 2</td>
<td>Continuous</td>
<td>-100 to 100</td>
</tr>
<tr>
<td>Thickness of Element 2</td>
<td>Continuous</td>
<td>1 to 10</td>
</tr>
<tr>
<td>Distance between Element 2 and Element 3</td>
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<td>1 to 10</td>
</tr>
<tr>
<td>GRIN glass type for Element 2</td>
<td>Discrete</td>
<td>1-6 (integer)</td>
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<td>Positive or Negative GRIN for Element 2</td>
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<td>0 or 1</td>
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<tr>
<td>Radius of curvature of left surface of Element 3</td>
<td>Continuous</td>
<td>-100 to 100</td>
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<tr>
<td>Radius of curvature of right surface of Element 3</td>
<td>Continuous</td>
<td>-100 to 100</td>
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<td>Thickness of Element 3</td>
<td>Continuous</td>
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<td>Distance between Element 3 and Element 4</td>
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<td>1 to 10</td>
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<td>GRIN glass type for Element 3</td>
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<td>Positive or Negative GRIN for Element 3</td>
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<td>0 or 1</td>
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<td>Radius of curvature of left surface of Element 4</td>
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<td>-100 to 100</td>
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<tr>
<td>Radius of curvature of right surface of Element 4</td>
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<tr>
<td>Thickness of Element 4</td>
<td>Continuous</td>
<td>1 to 10</td>
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<tr>
<td>Distance between Element 4 and Surface 10 (a dummy surface)</td>
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<td>1 to 10</td>
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<td>GRIN glass type for Element 4</td>
<td>Discrete</td>
<td>1-6 (integer)</td>
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<td>Positive or Negative GRIN for Element 4</td>
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<td>0 or 1</td>
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<tr>
<td>Distance from Surface 10 (a dummy surface) to the Output Plane</td>
<td>Continuous</td>
<td>1 to 100</td>
</tr>
</tbody>
</table>
Determining when a Solution is Found
3-Element GRIN Shaping System
3-Element GRIN Shaping System

- Average evaluation time for a generation: 7.80s
- Total execution time: 26.8 hrs
- Integrating Output Profile over Output Surface yields 21.9 units; integrating Input Profile over Input Surface yields 21.7 units

<table>
<thead>
<tr>
<th>Parameter</th>
<th>First Element</th>
<th>Second Element</th>
<th>Third Element</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Left Surface</td>
<td>Right Surface</td>
<td>Left Surface</td>
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<tr>
<td>Thickness, mm</td>
<td>9.99</td>
<td>10.0</td>
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<td>Vertex radius (1/ε), mm</td>
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<td>2</td>
<td>1</td>
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<tr>
<td>GRIN Direction</td>
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<td>positive</td>
<td>negative</td>
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</table>
Summary and Conclusions

• Geometrical methods for design of laser beam shaping systems uses:
  – Conservation of energy within a bundle of rays,
  – Constant optical path length condition.

• Numerical and analytical techniques used to design a 2-plano-aspherical lens system and a 2-mirror system with no central obscuration.

• Laser beam shaping merit function used with genetic algorithms to design a 3-element GRIN system with spherical surface lenses.