Geometrical Methods for Design of Laser Beam Shaping Systems


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OUTLINE

• Overview of current practices and history
• Geometrical methods for design
• Applications:
  ▪ Rotationally symmetric systems
    ➢ 1 mirror, 1 lens, 2 plano-aspheric lenses
  ▪ Non-rotationally symmetric systems
    ➢ 2 mirror with no central obscuration
  ▪ Genetic algorithm optimization methods
    ➢ 3-element spherical surface GRIN system
Current Practices of Beam Shaping*

• Process of redistributing the irradiance and phase

• Two functional categories:
  • Beam Integrators
  • Field Mapping

Beam Integrators

[Diagram showing a beam integrator with labels for D, d, f, F, and S]
Field Mapping Beam Shaper
Physical versus Geometrical Optics

\[ \beta = \frac{2\sqrt{2\pi} r_0 Y_0}{f \lambda} \]

- \( \lambda \) = wavelength,
- \( r_0 \) = waist or radius of input beam,
- \( Y_0 \) = half-width of the desired output dimension,
- \( f \) = focal length of the focusing optic, or the working distance from the optical system to the target plane.

Beam Shaping Guidelines:

- \( \beta < 4 \), Beam shaping will not produce acceptable results
- \( 4 < \beta < 32 \), Diffraction effects are significant
- \( \beta > 32 \), Geometrical optics methods should be adequate
Historical Background

• **Frieden, Appl. Opt. 4.11, 1400-1403, 1965:** “Lossless conversion of a plane wave to a plane wave of uniform irradiance.”

• **Kreuzer, US patent 3,476,463, 1969:** “Coherent light optical system yielding an output beam of desired intensity distribution at a desired equi-phase surface.”

• **Rhodes & Shealy, Appl. Opt. 19, 3545-3553, 1980:** “Refractive optical systems for irradiance redistribution of collimated radiation – their design and analysis.”

Frieden, Appl. Opt. 4.11, 1400-1403, 1965:
“Lossless conversion of a plane wave to a plane wave of uniform irradiance.”

Conservation of Energy:

\[
R(r) = \pm R_{\text{max}} \left[ \frac{1 - \exp\left(-r^2/2\alpha^2\right)}{1 - \exp\left(-r_{\text{max}}^2/2\alpha^2\right)} \right]^{1/2}
\]

\[
z(r) = \int f(r) \, dr + C
\]

- Intensity shaping leads to OPL variation of 20\(\lambda\)
- Need to shape of output wavefront when phase is important
- Frieden requires rays to be parallel Z-axis
- Leads to OPL variation of \(\lambda/20\)
• Kreuzer, US patent 3,476,463, 1969:

“Coherent light optical system yielding an output beam of desired intensity distribution at a desired equi-phase surface.”

• Conservation of Energy & Ray Trace Equations:

\[ r + \mathcal{K}(s, S) \sin \theta - R_{\text{max}} \left[ \frac{1 - e^{-2(r/r_0)^2}}{1 - e^{-2(r_{\text{max}}/r_0)^2}} \right]^{1/2} = 0 \]

• Constant OPL:

\[ d(n - 1) + \mathcal{K} \left( 1 - n \cos \theta \right) = 0 \]

Mirror Surface Equations:

\[
\begin{align*}
Z(r) &= \int_{0}^{r} \frac{dr}{\sqrt{(n^2 - 1) + \left( \frac{(n-1)d}{R-r} \right)^2}} \\
Z(R) &= \int_{0}^{R} \frac{dR}{\sqrt{(n^2 - 1) + \left( \frac{(n-1)d}{R-R} \right)^2}}
\end{align*}
\]
Energy Balance:

\[ R = \frac{r_0^2}{2I_{\text{out}}} \left[ 1 - \exp\left(-\frac{2r^2}{r_0^2}\right) \right] \]

\[ I_{\text{out}} = \frac{r_0^2}{2R_{\text{max}}^2} \left[ 1 - \exp\left(-\frac{2r_{\text{max}}^2}{r_0^2}\right) \right] \]

Ray Trace Equation:

\[ z(r) = \int f(r) \, dr + C \]

Constant OPL:

\[ Z(r) = z(r) + g(r) \]
Cornwell, “Non-projective transformations in optics,”
Ph.D. Dissertation, University of Miami, Coral Gables, 1980

- Differential Power
  \[ I_{in}(x, y)dx\,dy = I_{out}(X, Y)\,dX\,dY \]
- Conservation of Energy: \( E_{in} = E_{out} \)
- Magnifications of ray coordinates
  \[ m_x(x) = \frac{1}{x} \left[ C_1 \int_0^x \frac{a_x(u)\,du}{A_X(um_x(u))} + C_2 \right] \]

- OPL condition
- Determine sag \( z(r) \) of first surface
- Determine inverse magnification
- Determine sag \( Z(R) \) of second surface
Overview of Geometrical Methods

• Conservation of energy within a bundle of rays – geometrical optics intensity law.

• Ray trace equations.

• Constant optical path length condition.
Geometrical Optics Intensity Law

- Conservation of energy within geometrical optics – intensity law – follows from scalar wave equation:

\[
\left( \nabla^2 + n^2 k_0^2 \right) e(\mathbf{r}) = 0 \quad \text{Assume } e(\mathbf{r}) = e_0(\mathbf{r}) \exp[ik_0 S(\mathbf{r})]
\]

\[
(\nabla S)^2 = n^2
\]

\[
2e_0 \nabla S \cdot \nabla e_0 + e_0^2 \nabla^2 S = 0
\]

\[
\nabla \cdot \left( I \frac{\nabla S(\mathbf{r})}{n(\mathbf{r})} \right) = \nabla \cdot \left( I(\mathbf{r}) \hat{a}(\mathbf{r}) \right) = 0 \quad I_1 dA_1 = I_2 dA_2
\]
Ray Trace Equations

• Ray Trace Equations:

\[ \frac{d}{ds} \left( n(r) \frac{dr}{ds} \right) = \nabla n(r) \quad r(s) = as + b \]

• Law Reflection & Refraction:

\[ A = a - 2\hat{n}(a \cdot \hat{n}) \]

\[ n'A = na + \left[ n'\cos i' - n\cos i \right] \hat{n} \]
Constant Optical Path Length Condition

• Impose the constant optical path length condition for all rays:

\[ \text{OPL}(C) = \int_C n(x, y, z) \, ds \]

\[ (OPL)_0 = (OPL)_r. \]
Summary of Geometrical Methods for Laser Beam Shaping

• Geometrical optics intensity law:

\[ \nabla \cdot (I \mathbf{a}) = 0 \]

\[ I_1 \, dA_1 = I_2 \, dA_2 \]

• Constant optical path length condition:

\[ (OPL)_0 = (OPL)_r \]
Optical Design of Laser Beam Shaping Systems: Solution of Differential Equations versus Global Optimization

- Geometrical methods leads to equations of the optical surfaces:
  \[ z(r) = \int f(r) dr + C \]
  \[ Z(r) = z(r) + g(r) \]

- Global Optimization with discrete & continuous variables:
  - Beam shaping merit function

\[ M = \frac{M_{\text{Diameter}} M_{\text{Collimation}} M_{\text{Uniformity}}}{4} \]
One-mirror Profile Shaping System

- Ray trace equation
  \[-(R - r)z'' + 2(Z - z)z' + (R - r) = 0\]

- Energy balance
  \[I_{\text{in}}(r)2\pi rdz = I_{\text{out}}(R)2\pi R[dr^2 + dZ^2]^{1/2} = I_{\text{out}}(R)2\pi R\left[1 + \left(\frac{dZ}{dR}\right)^2\right]dR\]

- Differential equation of mirror
  \[\frac{z''}{z'} = \frac{1}{(R - r)} \left\{ \frac{I_{\text{in}}(r)}{I_{\text{out}}(R)} \left(\frac{r}{R}\right) \left[\frac{1 - z'^2}{1 + z'^2} + \frac{2z'}{(1 + z'^2)} \left(\frac{dZ}{dR}\right)\right] \right\} - 1 \]
Collimated Gaussian input beam uniformly illuminating Cylindrical receiver (McDermit, p. 84).
Two Element Beam Shaping Systems

• Conservation of Energy: 
  \[ R(r) = \pm \sqrt{\frac{2}{I_{\text{out}}}} \int_0^r I_{\text{in}}(u) u \, du \]

• Constant OPL:
  \[(OPL)_0 = (OPL)_r\]

• Equations of the optical surfaces:
  \[ z(r) = \int f(r) \, dr + C \]
  \[ Z(r) = z(r) + g(r) \]
Two Lens Beam Shaping System

Jiang, Ph.D. Dissertation, UAB, 1993

Energy Balance:

\[
R = \frac{\frac{r_0^2}{2I_{\text{out}}}}{1 - \exp\left(-\frac{2r^2}{r_0^2}\right)}
\]

\[
I_{\text{out}} = \frac{r_0^2}{2R_{\text{max}}^2} \left[1 - \exp\left(-\frac{2r_{\text{max}}^2}{r_0^2}\right)\right]
\]

Ray Trace Equation:

\[
z' = \frac{-(R - r)(Z - z) \pm (n/n_0)(R - r)\sqrt{(Z - z)^2 + (R - r)^2}}{(1 - (n/n_0)^2)(Z - z)^2 - (n/n_0)^2(R - r)^2}
\]

\[
z(r) = \int f(r)dr + C
\]

Constant OPL:

\[
(Z - z) = \frac{n(n - n_0)d + \left[n_0^2(n - 1)^2d^2 + (n^2 - n_0^2)(R - r)^2\right]^{1/2}}{n^2 - n_0^2}
\]

\[
Z(r) = z(r) + g(r)
\]
Surface Parameters of HeCd (441.57nm) Laser Beam Shaping system

\[ z(h) = \frac{c h^2}{1 + \sqrt{1 - (1 + k) c^2 h^2}} + \sum_{j=2}^{5} A_{2j} h^{2j} \]

<table>
<thead>
<tr>
<th>Lens Surface Parameters</th>
<th>Primary</th>
<th>Secondary</th>
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<tbody>
<tr>
<td>Diameter (mm)</td>
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<td>30.0</td>
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<tr>
<td>Vertex Radius (mm)</td>
<td>47.861445</td>
<td>113.64905</td>
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<tr>
<td>Index of Refraction</td>
<td>(1.43916 (CaF_2))</td>
<td>(1.43916 (CaF_2))</td>
</tr>
<tr>
<td>Thickness (mm)</td>
<td>10.0</td>
<td>10.0</td>
</tr>
<tr>
<td>Conic Constant, (\kappa)</td>
<td>–11143607</td>
<td>–1.4877144</td>
</tr>
<tr>
<td>(A_4 \text{ (mm}^{-3})</td>
<td>(-7.1532887 \times 10^{-5})</td>
<td>(-2.6322455 \times 10^{-6})</td>
</tr>
<tr>
<td>(A_6 \text{ (mm}^{-5})</td>
<td>(3.3729843 \times 10^{-7})</td>
<td>(9.4058758 \times 10^{-9})</td>
</tr>
<tr>
<td>(A_8 \text{ (mm}^{-7})</td>
<td>(-1.4916816 \times 10^{-9})</td>
<td>(-2.3096843 \times 10^{-10})</td>
</tr>
<tr>
<td>(A_{10} \text{ (mm}^{-9})</td>
<td>(5.9836543 \times 10^{-12})</td>
<td>(1.5839557 \times 10^{-12})</td>
</tr>
<tr>
<td>(A_{12} \text{ (mm}^{-11})</td>
<td>(-1.5166511 \times 10^{-14})</td>
<td>(-4.8438745 \times 10^{-15})</td>
</tr>
</tbody>
</table>
Input and Output Beam Profiles
Beam Shaping Applications

In a holographic projection processing system featured on 10 January 1999 issue of *Applied Optics*, a two-lens beam shaping optic increased the quality of micro-optical arrays.
Relationship between $\lambda$ and Lens Spacing

\[ n^2 = 1 + \sum_{i=1}^{3} \frac{A_i \lambda^2}{\lambda^2 - \lambda_i^2} \]

From the Optical Design:

\[ Z - z = \frac{(R-r)\left[1+n\sqrt{1+(1-n^2)z'^2}\right]}{(n^2-1)z'} \]

Solve for $d$ so that rays conserve energy as they leave second lens:

\[ d = \frac{n(Z-z) - \sqrt{(Z-z)^2 + (R-r)^2}}{n-1} \]
Newport - Refractive Beam Shaper*

*Based on New Product Concept literature distributed at SPIE 2002, Seattle.
Gaussian to Flat Top

- High Efficiency – Accepts 99.7% of the input beam while minimizing diffraction by using a Fermi-Dirac output beam profile

- High Uniformity - 78% incident power is within region with 5% rms power variation
Collimated Output Beam
Large Bandwidth – 257 to 1550nm

[Graph showing a broad range of wavelengths from 257 to 1550nm, with the beam shaper design minimizing variation of output intensity at wavelengths from 257 to 1550nm.]
Non-Rotationally Symmetric System

- Differential Power
  \[ I_{in}(x, y)dx\,dy = I_{out}(X, Y)dXdY \]

- Conservation of Energy: Ein=Eout

- Magnifications of ray coordinates
  \[ m_x(x) = \frac{1}{x} \left[ C_1 \int_0^x \frac{a_x(u)\,du}{A_x(u)\,m_x(u)} + C_2 \right] \]

- OPL condition

- Determine sag \( z(r) \) of first surface

- Determine inverse magnification

- Determine sag \( Z(R) \) of second surface
Laser Beam Shaping System for Cymer ELS-5000A KrF Excimer Laser

See http://www.cymer.com
Two Mirror Beam Shaping System
Shealy & Chao, Proc SPIE 4443-03, 2001

Input beam has 1-to-3 beam waist in x-to-y directions:

\[ I_{in}(x,y) = \exp \left[ -2 \left( \frac{x}{x_0} \right)^2 \right] \exp \left[ -2 \left( \frac{y}{y_0} \right)^2 \right] \]

\[ I_{out} = A_X A_Y = \text{const.} \]
Sag of First Mirror

Parameters:

\[ L = 10; \ h = 5 \]
\[ l_0 = \sqrt{L^2 + h^2} \]
\[ r_0 = 1; \ x_0 = \frac{r_0}{2}; \ x_{\text{max}} = r_0 \]
\[ y_0 = \frac{3r_0}{2}; \ y_{\text{max}} = 3r_0 \]
\[ X_{\text{max}} = 2r_0 = Y_{\text{max}} \]

\[
z(x, y)(L + l_0) = \frac{1}{2}(x^2 + y^2) + \frac{r_0^2}{\sqrt{2\pi} \ \text{erf}(2\sqrt{2})} \times \left\{ 4 - \exp\left[-2\left(\frac{2x}{r_0}\right)^2\right] - 3\exp\left[-2\left(\frac{2y}{3r_0}\right)^2\right] - \frac{2\sqrt{2\pi}}{r_0} \left[ x\text{erf}\left(\frac{2\sqrt{2}x}{r_0}\right) + y\text{erf}\left(\frac{2\sqrt{2}y}{3r_0}\right) \right] \right\} \]

Tucson, 21 Nov 2002
Optical Sciences Colloquium
Optical Performance Analysis*

- ZEMAX was used.

- Elliptical Gaussian input beam profile was modeled as user-defined surface.

- Two mirror surfaces were also modeled as user-defined surfaces.

Relative Illumination

Input Beam

Output Beam
First Mirror Surface Analysis

![Graphs showing mirror surface analysis](image)
Second Mirror Surface Analysis

![Graph showing mirror surface analysis](image)
Tolerance Analysis

- In A, the first mirror was decentered 10% of its diameter about x-axis.
- In B, both mirrors were decentered by 2.5% of their diameter about x- and y-axes and tilted about the three axes by 0.25 degrees.
Conclusions for 2 Mirror System

• Designed a two-mirror system with no central obscuration for shaping a 3:1 elliptical Gaussian beam into a uniform output beam.

• ZEMAX used to do optical performance analysis:
  – First mirror has strong aspherical component along direction of smaller waist (120µm for 6mm diameter mirror).
  – Output beam profile is stable for decentering of less than 2.5% of mirror diameter and 0.25 degrees about coordinate axis.
Optical Design of Laser Beam Shaping Systems: Solution of Differential Equations versus Global Optimization

- Geometrical methods leads to equations of the optical surfaces:
  \[ z(r) = \int f(r)dr + C \quad Z(r) = z(r) + g(r) \]

- Global Optimization with discrete & continuous variables:
  - Beam shaping merit function
  \[ M = M_{\text{Diameter}} M_{\text{Collimation}} M_{\text{Uniformity}} \]
Genetic Algorithms (GAs)

• Useful Tool in Geometrical Optics?

• Advantages
  – Ease of implementation and broad applicability
  – Relative immunity to local extrema
  – Easily made parallel

• Drawbacks
  – Random starting designs
  – “This planet does not lack for life forms, but there is a paucity of intelligent ones.”

2. Shafer, 192.
GA Concepts

- Generating solutions
- Populations and individuals
- Parents and children: producing new populations

Initialize GA by randomly picking new individuals

Evaluate Merit Function for each individual in generation

Perform genetic operations (reproduction, mutations, cross-overs); produce new generation

Is the population stagnant? (Micro-GA check)

Step eligible for parallelization

Keep best individual and replace the remainder with randomly-selected individuals

Is the population stagnant? (Micro-GA check)

Termination criterion reached?

End

End

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GA Concepts

- Biological Paradigm and Nomenclature
- Expressing solutions in terms of binary strings: genetic code
Irradiance Profile Calculations

• A fast, efficient method is needed to calculate the irradiance profile on a specified Output Surface.

• The Geometrical Optics form of the Energy Conservation Law is harnessed for these calculations:

\[
\left( \nabla^2 + n^2 k_0^2 \right) e(r) = 0
\]

\[
e(r) = e_0(r) \exp[i k_0 S(r)]
\]

\[
(\nabla S)^2 = n^2 \quad 2e_0 \nabla S \cdot \nabla e_0 + e_0^2 \nabla^2 S = 0
\]

\[
\nabla \cdot (f \mathbf{v}) = f \nabla \cdot \mathbf{v} + \mathbf{v} \cdot \nabla f
\]

\[
\nabla \cdot \left( I_1 \mathbf{a} \right) = 0
\]

\[
I_1 dA_1 = I_2 dA_2
\]
\[ E = \int_{\Omega} \sigma(\rho) \left( \hat{n}^{in}(\rho) \cdot v^{in}(\rho) \right) da = \int_{\Omega} u(P) \left( \hat{n}^{out}(P) \cdot v^{out}(P) \right) dA, \]

\[ I_1 \, dA_1 = I_2 \, dA_2 \quad \text{and} \quad u(P_i) = \sigma(\rho_i) \left( \frac{\cos(i^{in}_i) \rho_i (\rho_i - \rho_{i-1}) \cos(x^{out}_i)}{\cos(i^{out}_i) P_i (P_i - P_{i-1})} \right). \]

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Merit Function Topography for Beam Shaper/Projector

\[ M = \frac{1}{\mu} \exp \left[ -0.01 \left\{ 50 - P \right\}^2 \right], \]

\[ \mu = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \{ u(P_i) - \bar{u} \}^2} \]

\[ \bar{u} = \frac{1}{N} \sum_{i=1}^{N} u(P_i). \]
One Lens Beam Shaping System

Evans & Shealy in Applied Optics vol. 37, 5216-5221, 1998

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Beam Profile on Input Plane

\[ \sigma(\rho_i) = \exp\left(-\alpha \rho_i^2\right) \]
Beam Profile on Output Surface

\[ u(P_i) = \sigma(\rho_i) \left( \frac{\cos(i^\text{in}_i) \rho_i (\rho_i - \rho_{i-1}) \cos(\chi^\text{out}_i)}{\cos(i^\text{out}_i) P_i (P_i - P_{i-1})} \right). \]
Results for GA-optimized Beam Shaper/Projector System

- CODE V used to evaluate merit function
- Simultaneous execution on 4 Sun Sparcs
- Manually Tweaking the Merit Function Landscape
- Total search time of 7 hours
- “Best” solution found
Genetic Algorithm Optimization Method


- Can a spherical-surface GRIN beam shaping system be designed using catalog GRIN materials?
- Problem is well suited for Genetic Algorithms:
  - discrete parameters
    - Number of lens elements
    - GRIN catalog number
  - Continuous parameters: radii, thickness
Gradient-Index (GRIN) Shaper Problem*

\[
\begin{align*}
n_1 &= a_1 + b_1 z + c_1 z^2 \\
n_2 &= a_2 + b_2 z + c_2 z^2
\end{align*}
\]

Beam Shaping Merit Function 
Used in GA Optimization

\[ M = \frac{M_{\text{Diameter}} M_{\text{Collimation}}}{M_{\text{Uniformity}}} = \exp \left[ -s \left( R_{\text{Target}} - R_N \right)^2 \right] \exp \left[ - \left( 1 - \prod_{i=1}^{N} \cos^Q (\gamma_i) \right)^2 \right] \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left\{ I_{\text{out}} (R_i) - \left[ \frac{1}{N} \sum_{k=1}^{N} I_{\text{out}} (R_k) \right] \right\}} \]

- \( R_{\text{target}} \) = Output Beam Radius
- \( R_N \) = Marginal Ray Height on Output Plane
- \( \gamma_i \) = Angle \( i^{\text{th}} \) Ray Make with the Optical Axis
- \( Q \) and \( s \) = Convergence Constants
GA Solution to GRIN: Part 1

Lens 1: LightPath G1SF, positive gradient

Lens 2: LightPath G1SF, negative gradient

Connector
## Parameters Optimized for Free-form GA-designed GRIN Problem

<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Type</th>
<th>Limits</th>
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</thead>
<tbody>
<tr>
<td>1 Number of Elements</td>
<td>Discrete</td>
<td>1-4 (integer)</td>
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<tr>
<td>2 Radius of curvature of left surface of Element 1</td>
<td>Continuous</td>
<td>-100 to 100</td>
</tr>
<tr>
<td>3 Radius of curvature of right surface of Element 1</td>
<td>Continuous</td>
<td>-100 to 100</td>
</tr>
<tr>
<td>4 Thickness of Element 1</td>
<td>Continuous</td>
<td>1 to 10</td>
</tr>
<tr>
<td>5 Distance between Element 1 and Element 2</td>
<td>Continuous</td>
<td>1 to 10</td>
</tr>
<tr>
<td>6 GRIN glass type for Element 1</td>
<td>Discrete</td>
<td>1-6 (integer)</td>
</tr>
<tr>
<td>7 Positive or Negative GRIN for Element 1</td>
<td>Discrete</td>
<td>0 or 1</td>
</tr>
<tr>
<td>8 Radius of curvature of left surface of Element 2</td>
<td>Continuous</td>
<td>-100 to 100</td>
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<td>9 Radius of curvature of right surface of Element 2</td>
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<td>-100 to 100</td>
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<td>12 GRIN glass type for Element 2</td>
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<td>1-6 (integer)</td>
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<td>13 Positive or Negative GRIN for Element 2</td>
<td>Discrete</td>
<td>0 or 1</td>
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<td>14 Radius of curvature of left surface of Element 3</td>
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<td>15 Radius of curvature of right surface of Element 3</td>
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<td>-100 to 100</td>
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<td>1 to 10</td>
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<td>17 Distance between Element 3 and Element 4</td>
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<td>1 to 10</td>
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<td>0 or 1</td>
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<td>20 Radius of curvature of left surface of Element 4</td>
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<td>21 Radius of curvature of right surface of Element 4</td>
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<td>-100 to 100</td>
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<td>22 Thickness of Element 4</td>
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<td>23 Distance between Element 4 and Surface 10 (a dummy surface)</td>
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<td>1 to 10</td>
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<tr>
<td>24 GRIN glass type for Element 4</td>
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<td>1-6 (integer)</td>
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<td>25 Positive or Negative GRIN for Element 4</td>
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<td>26 Distance from Surface 10 (a dummy surface) to the Output Plane</td>
<td>Continuous</td>
<td>1 to 10</td>
</tr>
</tbody>
</table>
Determining when a Solution is Found

\[ M_{\text{best}} \]

Generation
3-Element GRIN Shaping System
3-Element GRIN Shaping System

- Average evaluation time for a generation: 7.80s
- Total execution time: 26.8 hrs
- Integrating Output Profile over Output Surface yields 21.9 units; integrating Input Profile over Input Surface yields 21.7 units

<table>
<thead>
<tr>
<th>Parameter</th>
<th>First Element</th>
<th>Second Element</th>
<th>Third Element</th>
</tr>
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<tr>
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<td>Left Surface</td>
<td>Right Surface</td>
<td>Left Surface</td>
</tr>
<tr>
<td>Vertex radius ((1/c), \text{mm})</td>
<td>-61.6</td>
<td>80.4</td>
<td>-12.5</td>
</tr>
<tr>
<td>Surface type</td>
<td>spherical</td>
<td>spherical</td>
<td>spherical</td>
</tr>
<tr>
<td>Glass Type ((\text{UDG C1})^2)</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>GRIN Direction</td>
<td>negative</td>
<td>positive</td>
<td>negative</td>
</tr>
</tbody>
</table>
Summary and Conclusions

• Geometrical methods for design of laser beam shaping systems uses:
  – Conservation of energy within a bundle of rays,
  – Constant optical path length condition.

• Numerical and analytical techniques have been used to design a 1 and 2-element beam shaping systems.

• Laser beam shaping merit function used with genetic algorithms to design a 3-element GRIN system with spherical surface lenses.