Analytic beam shaping for flattened output irradiance profile

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ABSTRACT

A flattened Lorentzian irradiance profile, which was introduced by Brenner in 2003, offers prospects for analytic design of beam shaping optics when transforming a Gaussian beam into a soft-flat-top profile. It is shown that for the flattened Lorentzian profile the output power integral and the ray mapping function are analytic functions. This paper presents a systematic derivation of the flattened Lorentzian profile from the super-Lorentzian profile and exploration of properties of the flattened Lorentzian profile and compares its behavior to the super-Gaussian, flattened Gaussian, and Fermi-Dirac profiles. Details are presented for evaluation of the FL profile normalization, profile matching conditions, ray mapping function, and M^2.

1. INTRODUCTION

Laser beam shaping has become an important component of laser-based application, such as, isotope separation, materials processing, laser printing, lithography, and laboratory research. Aspheric optics were proposed by Kreuzer[1,2] in 1964 for use in lossless laser beam shaping with a pair of plano-aspheric lenses, whose surface contours are determined by imposing the geometrical optics law of intensity and the constant optical path length condition between the input and output beams. Since lossless beam shaping requires use of aspheric optics, there was an interlude of thirty years from the first proposal to use a pair of plano-aspheric lenses for shaping a Gaussian beam into a uniform irradiance beam to the first publication of experimental results by Jiang, Shealy, and Martin,[3,4] using a pair of plano-aspheric lenses in the Galilean configuration for beam shaping of a Gaussian beam into a top-hat profile. Hoffnagle and Jefferson[5,6] described an optical system that transforms a laser beam with an axially-symmetric irradiance profile into a beam with a different axially-symmetric profile with continuous, sigmoidal irradiance distribution, such as a Fermi-Dirac distribution. The design method of Kreuzer was used to determine sag of aspheric surfaces in a Keplerian configuration.

To avoid diffraction effects in regions of sharp changes in the irradiance, several analytic functions with a uniform central region and a continuous variation from the uniform region to the almost null region have been studied and reported in the literature. There are four specific flattened beam profiles that have been widely used in beam shaping applications: the super-Gaussian (SG),[7] the flattened-Gaussian (FG),[8] the Fermi-Dirac (FD),[5] and the super-Lorentzian (SL)[9] distribution. Shealy and Hoffnagle[10] have shown that when the shape and width parameters of these four distributions are chosen such that each profile has the same radius and slope of the irradiance at its half-height point, then beam shaping optics designed to produce either of these four flattened output irradiance profiles will produce similar irradiance distributions as the beam propagates. Therefore, using a profile which simplifies the design, fabrication and testing of beam shapers is preferred.

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Brenner[11,12] proposed in 2003 a new functional form for a soft-flat-top irradiance profile, which is a variant of the SL profile and which satisfies two properties that are required for analytic design of beam shaping optics. Namely, the output power integral of the irradiance profile can be solved analytically, and the ray mapping function can be solved and inverted analytically to express the output ray height as an analytic function of the input ray height. In the paper, we shall refer to the irradiance profile introduced by Brenner as a flattened Lorentzian profile (FL). We will show that the functional form of the FL profile can be obtained in a systematic way from the SL profile. Also, we investigate the properties of the FL profile: the normalization constant, the ray mapping function, the M^2 quantity, and application of the profile matching conditions to compare the FL profile with the SG, FD, and FG profiles.

2. FLATTENED LORENTZIAN IRRADIANCE PROFILE

Brenner[12] proposed the following expression for a soft-flat-top profile:

$$I_{FL}(\rho; N) = \left(\frac{1}{1+\rho^N}\right)^{\frac{2}{N}+1}, \tag{1}$$

where ρ is a dimensionaless variable equal to the radial coordinate divided by the profile width parameter. He has shown that the output power integral of the FL profile over a spherical area of radius ρ_{out} on the output aperture is given by

$$P_{\text{out}}(\rho_{\text{out}}) = \int_0^{2\pi} \int_0^{\rho_{\text{out}}} I_{\text{FL}}(\rho', N)\rho' d\rho' d\varphi, \tag{2a}$$

$$= \pi \left(\frac{\rho_{\text{out}}^N}{1+\rho_{\text{out}}^N}\right)^{\frac{2}{N}}. \tag{2b}$$

Brenner inverts Eq. (2b) and uses conservation of energy to justify equating $P_{\text{in}}(\rho_{\text{in}})$ and $P_{\text{out}}(\rho_{\text{out}})$ to obtain

$$\rho_{\text{out}} = \left[\left(\frac{P_{\text{in}}(\rho_{\text{in}})}{\pi}\right)^{-\frac{N}{2}} - 1\right]^{\frac{1}{N}}, \tag{3}$$

where the input ray passing through the radial point, ρ_{in}, is assumed to also pass through the output radial point, ρ_{out}. Brenner also evaluates Eq. (3) for an input Gaussian beam to obtain the ray mapping function for a Gaussian to FL profile transform. Equations (1) - (3) are interesting results which will be examined in more details in this paper.

Now, we show how to obtain in a systematic way the FL profile from the SL profile. Assume that the FL irradiance profile can be written as

$$I_{\text{FL}}(u) = \frac{I_0}{[1+u^q]^Q} \tag{4}$$

where $u = R/R_{\text{FL}}$, R specifies the radial height of an output ray, R_{FL} specifies the beam width, (q, Q) are beam shape parameters, and I_0 is the profile normalization constant which is determined by setting the total power contained within the beam profile over all space equal to unity. Q is an unknown shape parameter at this point, which will be determined by requiring the output power integral to be an analytic function:

$$P_{\text{out}}(\rho) = 2\pi I_0 \int_0^{\rho} \frac{u\, du}{[1+u^q]^Q}. \tag{5}$$

To facilitate evaluation of Eq. (5), it is convenient to consider the integral representation of the hypergeometric function [13, Eq. 15.3.1]

$$F(a, b; c; z) = \frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)} \int_0^1 \frac{t^{b-1}(1-t)^{(c-b-1)}\, dt}{(1-tz)^a} \tag{6}$$

with the following parameters and change of variables:

$$a = Q;\ b = \frac{2}{q};\ c = 1 + \frac{2}{q};\ z = -\rho^q;\ t = \left(\frac{u}{\rho}\right)^q. \tag{7}$$

Then, Eq. (6) can be expressed as follows when using Eqs. (7)

$$F\left(Q, \frac{2}{q}; 1 + \frac{2}{q}; -\rho^q\right) = \frac{2}{\rho^2}\int_0^\rho \frac{u\, du}{[1+u^q]^Q}, \tag{8}$$

where the properties of the Gamma functions [13, Eq. 6.1.15] were used to simplify the ratio of the Gamma functions appearing in Eq. (6). Then, Eq. (5) with Eq. (8) becomes after reversing the order of the first two parameters of the hypergeometric function, as permitted by their definition. [13, Eq. 15.1.1]

$$P_{\text{out}}(\rho) = \pi I_0 \rho^2 F\left(\frac{2}{q}, Q; 1 + \frac{2}{q}; -\rho^q\right), \tag{9a}$$

$$= \frac{\pi\, I_0\, \rho^2}{[1+\rho^q]^{\frac{2}{q}}} \tag{9b}$$

where Eq. (9b) follows, if we assume that

$$Q = 1 + \frac{2}{q}, \tag{10}$$

and if we use the following property of the hypergeometric function when the second and third parameters are equal: [13, Eq. 15.1.8]

$$F(N, b; b; -z) = \frac{1}{(1+z)^N}. \tag{11}$$

Equation (9b) implies that the FL profile as defined in Eq. (4) with Eq. (10) can be written as

$$I_{\text{FL}}(u) = \frac{I_0}{[1+u^q]^{1+\frac{2}{q}}}, \tag{12}$$

which has the same functional form as defined in Eq. (1). As we shall show, it is conventional to define the normalization of an irradiance profile such that its power over all space is unity. Now, we compute the normalization constant for the FL irradiance profile defined in Eq. (12):

$$1 = 2\pi \int_0^\infty \frac{I_0}{[1+(R/R_{\text{FL}})^q]^{1+\frac{2}{q}}} R dR, \tag{13a}$$

$$= \pi R_{\text{FL}}^2 I_0. \tag{13b}$$

Solving Eq. (13b) for I_0 gives

$$I_0 = \frac{1}{\pi R_{\text{FL}}^2}. \tag{14}$$

It is interesting to note that the normalization constant of the FL profile is independent of the beam shape parameter q and only depends on the beam width parameter R_{FL} with the same form as the top-hat profile. Therefore, specifying a value for R_{FL} fixes I_0, according to Eq. (14), and vice-versa.

Since it is conventional in beam propagation, profile matching, and M^2 analysis to normalize the input and output power of the beam over all space to unity, we shall use the normalization given by Eq. (14), and we shall refer to Eq. (12) as defining the FL profile instead of the form of Eq. (1). Therefore, the FL profile can be written as:

$$I_{\text{FL}}(R/R_{\text{FL}}) = \frac{1}{\pi R_{\text{FL}}^2 [1+(R/R_{\text{FL}})^q]^{1+\frac{2}{q}}}, \tag{15}$$

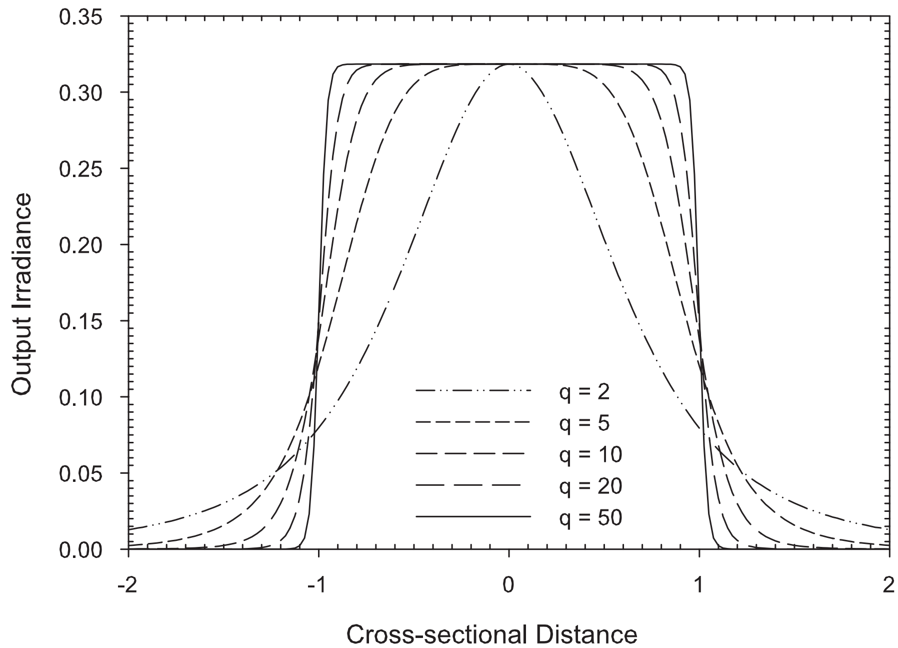

Figure 1. Cross-sectional plot of the irradiance for the fattened Lorentzian profile with the shape parameters $q = 2, 5, 10, 20, 50$ and the width parameter $R_{\mathrm{FL}} = 1$. All profiles are normalized such that the total energy contained within each irradiance distribution over all space is equal to unity.

where R specifies the radial coordinate on the output aperture. Figure 1 presents a plot of the irradiance versus R/R_{FL} over an output plane for $q = 2, 5, 10, 50$. It can be noted in Fig. 1 that the beam width parameter R_{FL} of the FL profile is no longer equal to the radius when the irradiance is equal to half of its axial value, as was the case for the SL profile. It is interesting to list and discuss several features of the FL irradiance profile defined by Eq. (15):

1. The axial value of the irradiance $I_{\mathrm{FL}}(R = 0) = I_0 = 1/\pi R_{\mathrm{FL}}^2$ is independent of the shape parameter q.

2. When $R = R_{\mathrm{FL}}$, then

$$I_{\mathrm{FL}}\left(R = R_{\mathrm{FL}}\right) = \frac{I_0}{2^{1+\frac{2}{q}}}, \tag{16}$$

which implies that for large q the irradiance at $R = R_{\mathrm{FL}}$ approaches one-half of its axial value.

3. The radius for which the irradiance is equal to half of its axial value is computed by solving the following equation for $R_{\frac{1}{2},\mathrm{FL}}$

$$\frac{I_{\mathrm{FL}}\left(R_{\frac{1}{2},\mathrm{FL}}/R_{\mathrm{FL}}\right)}{I_0} \equiv \frac{1}{2} = \left[1 + \left(\frac{R_{\frac{1}{2},\mathrm{FL}}}{R_{\mathrm{FL}}}\right)^q\right]^{-\left(1+\frac{2}{q}\right)}. \tag{17}$$

Raising both sides of Eq. (17) to the power of $q/(q+2)$ and solving for $R_{\frac{1}{2},\mathrm{FL}}$ gives

$$R_{\frac{1}{2},\mathrm{FL}} = R_{\mathrm{FL}} \sqrt[q]{2^{\frac{q}{2+q}} - 1}. \tag{18}$$

Figure 2 presents plots of the ratio of the radius of half-height to the beam width parameter of the FL, SG, FG, and FD profiles as a function of the corresponding shape parameters. It follows that radius of half-height approaches the corresponding width parameter as the shape parameters approaches infinity. It is also interesting to note from Fig. 2 that each flattened profile considered has a very different functional dependence on its shape parameter of the ratio of the radius of half-height to its beam width parameter. Shealy and Hoffnagle[10] describe the different meanings of the beam width parameters for the FD, SG, and FG profiles, where R_{FD} is defined to be the radius at which its irradiance is equal to half of its normalization constant; R_{SG} is the radius of beam for which its irradiance is equal to e^{-2} of its axial value; and R_{FG} is radius of the beam for which the irradiance is equal to $\Gamma(N+1, N+1)^2/\Gamma(N+1)^2$ of its axial value, where $\Gamma(a, z)$ is the incomplete gamma function. However, the beam width parameter, R_{FL}, of the FL profile is defined by Eq. (14) and has the same meaning as the beam width parameter of the top-hat profile. Namely, R_{FL} is equal to $[\pi I_0]^{-1/2}$.

4. Finally, it will be necessary to evaluate the slope of the irradiance at the radius of half-height of the irradiance as part of using the profile matching conditions. Taking the derivative of the irradiance from Eq. (15) with respect to R gives

$$\frac{dI_{\mathrm{out}}(R/R_{\mathrm{FL}})}{dR} = \frac{-I_0(q+2)(R/R_{\mathrm{FL}})^q}{R[1+(R/R_{\mathrm{FL}})^q]^{\frac{2(q+1)}{q}}}, \tag{19}$$

which will be used in the next section when applying the profile matching conditions to determine values of q and R_{FL} so that a FL profile will match another flattened profile by having the same width at half-height and the same slope of the radius at the radius of half-height.

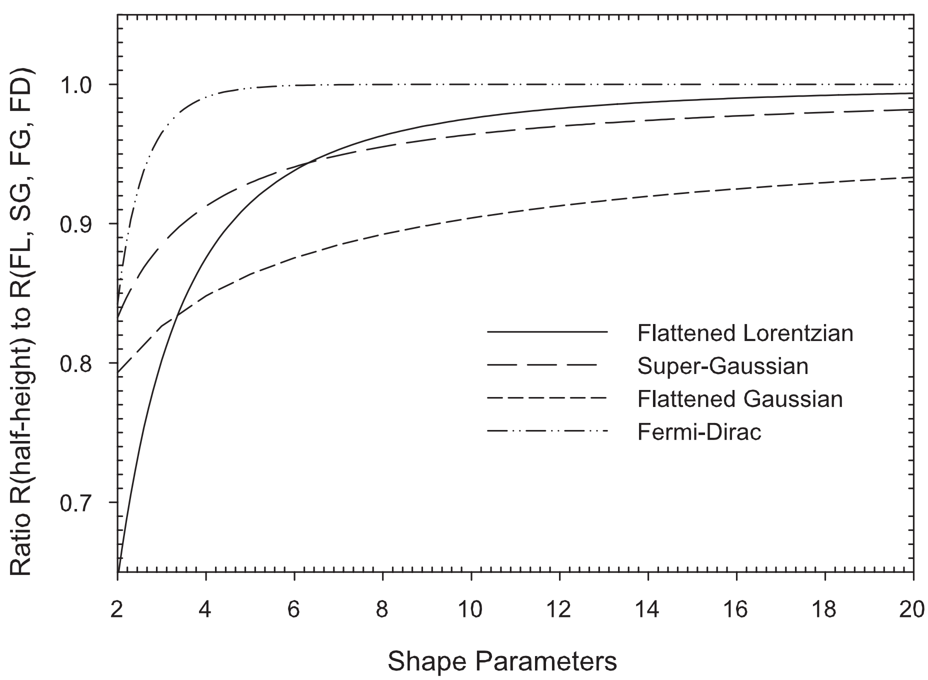

Figure 2. Plot of ratio of the radius of half-height to the beam width parameters as function of the shape parameters q, p, N, β of the FL, SG, FG, and FD profiles.

3. MATCHING PROFILES

In this section, we will evaluate the shape and beam width parameters of the FL profile so that it will have the same shape as a FG profile, using the profile matching conditions as defined by Shealy and Hoffnagle.[10] Explicitly, we defined two or more irradiance distributions as matching profiles when the following conditions are satisfied:

1. Equal radius when the irradiance is equal to half of its axial value. (20a)
2. Equal slope of the irradiance at the radius of the half height point. (20b)

Following a similar procedure as presented by Shealy and Hoffnagle,[10] we assume that the shape and width parameters of a FG irradiance profile are given by

$$N = \text{ non-negative integer,} \tag{21a}$$
$$R_{\mathrm{FG}} = 1, \tag{21b}$$

and we will use the profile matching conditions Eqs. (20) to determine the shape and width parameters of a FL irradiance profile. We will also compare the matched FL profile with the known FG profile as well as the matched profiles for the SG, and FD profiles.[10]

From Eqs. (12) and (13) of the work by Shealy and Hoffnagle,[10] we can compute explicit values of the radius at half-height, $R_{\frac{1}{2},\mathrm{FG}}$, and the slope of the FG irradiance profile at the radius of half-height, $m_{\frac{1}{2},\mathrm{FG}}$ from known values of N and R_{FG}. From Eq. (20), we know

$$R_{\frac{1}{2},\mathrm{FL}} = R_{\frac{1}{2},\mathrm{FG}} \tag{22a}$$
$$m_{\frac{1}{2},\mathrm{FL}} = m_{\frac{1}{2},\mathrm{FG}}. \tag{22b}$$

Then, Eq. (18) can be rewritten as

$$R_{\mathrm{FL}} = \frac{R_{\frac{1}{2},\mathrm{FG}}}{\sqrt[q]{2^{\frac{q}{2+q}} - 1}} \tag{23}$$

where R_{FL} is the width parameter and q is the shape parameter of the FL profile. Equation (23) can be used to compute R_{FL} when q and the matching profile half-width $R_{\frac{1}{2},\mathrm{FG}}$ are known.

Now, we use the slope at radius of half-height matching condition to compute q. We evaluate $m_{\frac{1}{2},\mathrm{FL}}$ from Eq. (19) for $R = R_{\frac{1}{2},\mathrm{FL}}$, and then use Eqs. (22a) and (22b) to obtain:

$$m_{\frac{1}{2},\mathrm{FG}} + \frac{(q+2)\left[2^{\frac{q}{(2+q)}} - 1\right]^{1+\frac{2}{q}}}{\pi 2^{\frac{2(q+1)}{(2+q)}} R^3_{\frac{1}{2},\mathrm{FG}}} = 0, \tag{24}$$

which can be solved for the FL shape parameter q when $m_{\frac{1}{2},\mathrm{FG}}$ and $R_{\frac{1}{2},\mathrm{FG}}$ are known. Then, R_{FL} can be evaluated from Eq. (23).

Table 1 compares the shape and width parameters using the matched profile procedure for the FG, SG, FD, and FL profiles. From the data given in Table 1, it would be interesting to plot several cross-sectional irradiance plots to evaluate trends. Figure 3a presents a comparison between the SG, FD and FL shape parameters as a function of N for matched profiles. q and β have very similar functional dependencies on N. Figure 3b presents a comparison between the width parameters of the SG, FD, and FL width parameters as a function of N for profiles with matched parameters, where R_{FL} and R_{FD} have very similar dependence on N.

Table 1. Comparison of beam shape and width parameters of matched profiles.

N	$\mathbf{R}_{\frac{1}{2},\mathrm{FG}}$	**p**	$\mathbf{R}_{\mathrm{SG}}$	β	**q**	R_{FL}
4	0.848116	5.27137	1.03695	7.81989	7.55728	0.884187
9	0.898685	7.81645	1.02916	11.4961	11.13822	0.91690
18	0.929474	11.1402	1.02223	16.2003	15.77475	0.939103
24	0.939402	12.9283	1.01964	18.7111	18.26193	0.946728
36	0.951073	15.9539	1.01639	22.9436	22.46562	0.956028
49	0.958405	18.7193	1.01422	26.8016	26.30481	0.962072
64	0.963846	21.4950	1.01255	30.6677	30.15660	0.966667
81	0.968040	24.2775	1.01123	34.5392	34.01690	0.970277
100	0.971368	27.0649	1.01015	38.4145	37.88319	0.973185

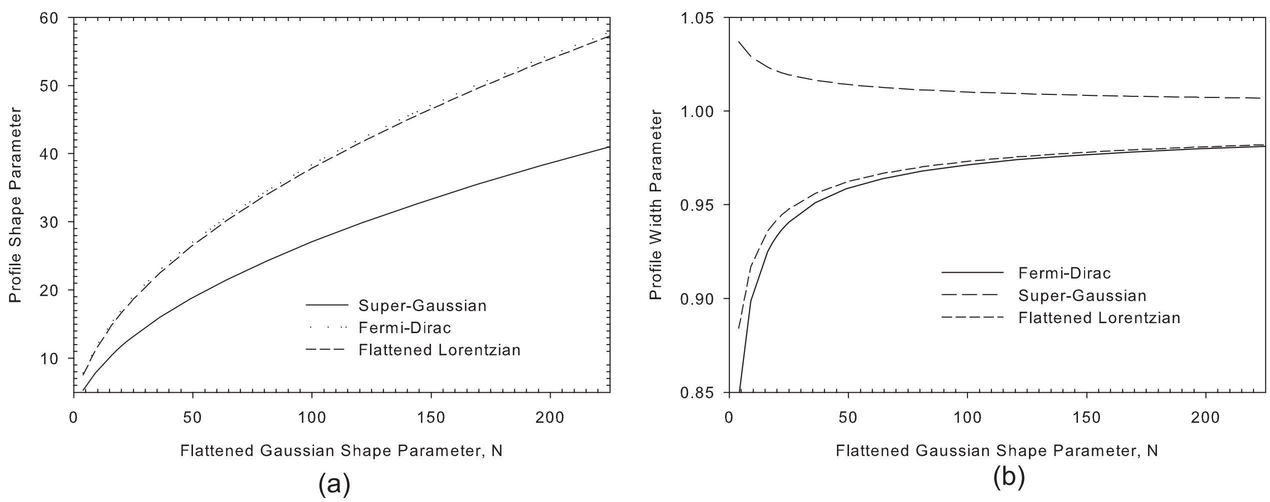

Figure 3. (a) Comparison between shape parameters of the SG, FD, and FL shape parameters as a function of the FG shape parameters for profiles with matched parameters. (b) Comparison between width parameters of the SG, FD, and FL shape parameters as a function of the FG shape parameters for profiles with matched parameters.

4. RAY-MAPPING FUNCTION

Now, we write an analytic expression for the ray mapping function $h(r) \equiv \epsilon R$ which relates an input ray at a radial distance r from the optical axis to an output ray at a radial height R from the optical axis, where ϵ is a parity variable that is used to allow for rays to cross the optical axis between the input and output aperture while keeping the radial coordinates positive. The geometrical optics law of intensity determines the functional structure of $h(r)$ as well as the input and output irradiance profiles. Assume that the input beam profile is Gaussian

$$I_{\mathrm{in}}(r/w_0) = \frac{2}{\pi w_0^2} \exp\left[-2\left(\frac{r}{w_0}\right)^2\right] \tag{25}$$

where w_0 is the input beam waist. Also, assume that the output beam profile is a FL profile whose irradiance is given by Eq. (15). Applying conservation of energy between the input and output planes for an arbitrary ray gives

$$\int_0^r 2\pi I_{in}(r'/w_0)\, r' dr' = \int_0^R 2\pi I_{\mathrm{out}}(R'/R_{\mathrm{FL}})\, R' dR'. \tag{26}$$

Integrating Eq. (26) gives

$$1-\exp\left[-2\left(\frac{r}{w_0}\right)^2\right]=\left[1+\left(R/R_{\mathrm{FL}}\right)^{-q}\right]^{-(2/q)} \tag{27}$$

Solving Eq. (27) for $r\,(R)$ gives

$$h^{-1}\,(R)\equiv\epsilon r=\epsilon w_0\sqrt{-\frac{1}{2}\ln\left\{1-\left[1+\left(R/R_{\mathrm{FL}}\right)^{-q}\right]^{-(2/q)}\right\}} \tag{28}$$

where

$$\epsilon=\begin{cases}+1 & \text{Galilean configuration}\\ -1 & \text{Keplerian configuration.}\end{cases} \tag{29}$$

Now, solve for R as a function of r by first taking the square root of Eq. (27). Next, raise both sides of the intermediate results to the q^{th} power and solve for the quantity $(R/R_0)^q$. Finally, the ray mapping function for beam transformation of a Gaussian beam into a FL profile is obtained by taking the q^{th} root to obtain:

$$h\,(r)\equiv\epsilon R=\frac{\epsilon R_{\mathrm{FL}}\sqrt{1-\exp\left[-2\left(\frac{r}{w_0}\right)^2\right]}}{\sqrt[q]{1-\left\{1-\exp\left[-2\left(\frac{r}{w_0}\right)^2\right]\right\}^{\frac{q}{2}}}}. \tag{30}$$

It is interesting to note that the numerator of Eq. (30) is the ray mapping function for tranforming the input Gaussian beam into a top-hat output profile, while the denominator of Eq. (30) modifies the ray mapping function of a top-hat output profile into the flattened Lorentzian profile. Figure 4 presents a comparison between the normalized output ray mapping function R/R_{FL} as a function of the input ray coordinates r/w_0 for shape parameters $q=10,15,20,30$ of the FL profile and a top-hat profile.

Following the design method of Kreuzer,[2] which uses conservation of energy and the constant optical path length conditions between the input and output beams, one obtains expressions for the slope as a function of the radial distance from the symmetry axis of two aspheric surfaces of a beam shaper. Numerical methods have been used to compute the sag and slope of the aspheric surfaces of refractive beam shapers in the Galilean configuration[3,4] and in the Kaplerian configuration.[5,6] The analytic expression of the ray mapping function given by Eq. (30) may be used to specify tolerances in the sag figure errors which would be required to obtained a flattened output beam profile that would be able to propagate a specified distance while retaining a desired uniformity.

5. M^2 EVALUATION

One figure of merit for beam propagation which has found widespread acceptance is Siegman's M^2 parameter.[14] It has been expressed analytically for the FD,[10] SG,[15] and FG[16] profiles and shown that M^2 increases monotonically with the shape parameters of these profiles. This means that the propagation range of a flattened beam profile decreases. The more steeply the edges of the beam roll off, the larger M^2 becomes, and in the limit of infinite beam shape parameter, for which the profile approaches a top-hat, M^2 diverges. Studies of the M^2 dependence and of the Fresnel-Kirchhoff integral both illustrate the same basic point: that in the design of beam shaping optics there is a trade-off to be made between achieving a beam with simultaneously large uniformity and efficiency, and one that propagates well. A gradual roll-off at the edge of the beam is necessary for long propagation range, and this implies that the beam shape parameter cannot be chosen to be arbitrarily large. In addition, we have seen that for a practical design it is important to be aware of the effect of finite aperture on the propagation of the shaped beam.

Following the same methods used by Shealy and Hoffnagle,[10] we evaluate M^2 for the FL profile by first evaluating σ_r^2 and σ_p^2 and then by computing M^2 from

$$M^2=2\pi\sqrt{\sigma_r^2\sigma_p^2}. \tag{31}$$

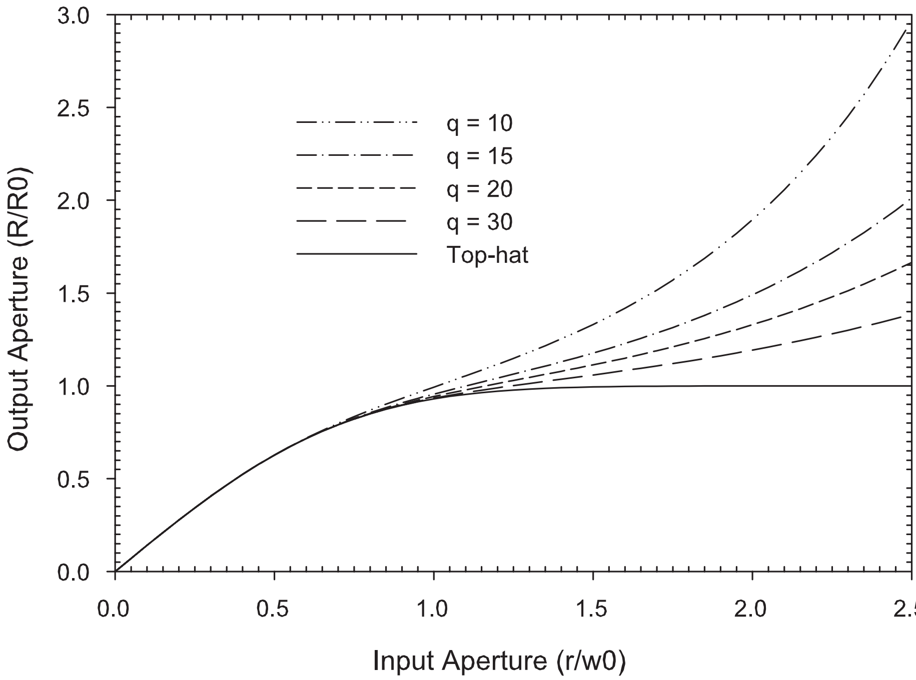

Figure 4. Comparison between the normalized output ray mapping function R/R_{FL} as a function of the input ray coordinates r/w_0 for shape parameters $Q = 10, 15, 20, 30$ of the FL profile and for a top-hat output profile.

where σ_r^2 and σ_p^2 are the second moments of the transverse irradiance distribution in real and Fourier space with $\rho = R/R_{FL}$:

$$\sigma_r^2 = 2\pi R_{FL}^4 \int_0^\infty I(\rho)\,\rho^3 d\rho \tag{32a}$$

$$\sigma_p^2 = 2\pi \int_0^\infty \hat{I}(k) k^3\, dk. \tag{32b}$$

where $I(r)$ is a normalized radial profile and $\hat{I}(k)$ is its Fourier transform. Evaluating σ_r^2 for the FL profile from Eq. (32a) gives

$$\sigma_r^2 = 2R_{\mathrm{FL}}^2 \int_0^\infty \frac{\rho^3 d\rho}{\left[1+\rho^q\right]^{1+\frac{2}{q}}} \tag{33a}$$

$$= \frac{qR_{\mathrm{FL}}^2\, 2^{\frac{4}{q}}\Gamma\left(1-\frac{2}{q}\right)\Gamma\left(\frac{3}{2}+\frac{2}{q}\right)}{\sqrt{\pi}\,(4+q)}. \tag{33b}$$

For analytic irradiance profiles, it is convenient to evaluate σ_p^2 using an identity from Fourier-transform theory, whose proof has recently appeared in the optics literature,[10]

$$\sigma_p^2 = \frac{1}{2\pi}\int_0^\infty \left|\frac{du(\rho)}{d\rho}\right|^2 \rho\, d\rho, \tag{34}$$

where $u(\rho)$ is the optical field amplitude and is equal to $\sqrt{I(\rho)}$ with $I(\rho)$ being the irradiance given by Eq. (15)

for the FL profile. Evaluating Eq. (34) with Eqs. (15) and (19) follow with intermediate results

$$\sigma_p^2 = \frac{I_0}{2\pi}\int_0^{\infty}\left[\frac{(q+2)\,\rho^{q-1}}{[1+\rho^q]^{\frac{3q+2}{2q}}}\right]^2 \rho d\rho; \tag{35a}$$

$$= \frac{I_0}{2\pi}\left[\frac{(2+q)^2\,\rho^{2q}}{8q}F\left(3+\frac{2}{q},2;3;-\rho^q\right)\right|_{\rho=0}^{\infty}; \tag{35b}$$

$$= \frac{q\,(2+q)}{16\pi^2 R_{\mathrm{FL}}^2\,(q+1)}. \tag{35c}$$

Therefore, M^2 for the FL profile is given by

$$M^2 = \frac{q}{2}\left[\frac{2^{\frac{4}{q}}\,(q+2)}{\sqrt{\pi}\,(q+1)\,(q+4)}\Gamma\left(1-\frac{2}{q}\right)\Gamma\left(\frac{3}{2}+\frac{2}{q}\right)\right]^{1/2}. \tag{36}$$

Figure 5 presents a plot of M^2 for the SG, FL, FD, and FG irradiance profiles as a function of the corresponding shape parameters, without using the matching conditions.

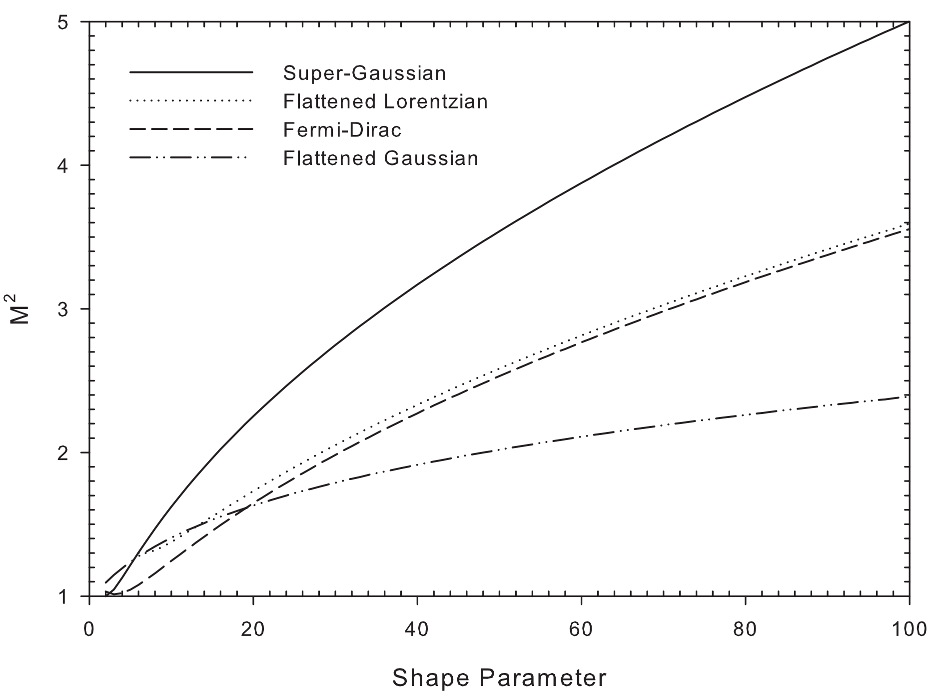

Figure 5. Plot of M^2 as function of the corresponding shape parameters for the SG, FD, and FG irradiance profiles.

6. CONCLUSION

A flattened Lorentzian (FL) irradiance profile, which was introduced by Brenner in 2003, offers prospects for analytic design of beam shaping optics when transforming a Gaussian beam into a soft-flat-top profile. We have presented a systematic derivation of the FL profile from the super-Lorentzian profile and exploration of properties of the flattened Lorentzian profile. Its normalization constant was shown to have a simple from of $[\pi R_{\rm FL}^2]^{-1}$, which does not depend of the shape parameter q of the FL profile. The profile matching conditions of Shealy and Hoffnagle[10] have been used to show that for appropriate choices of shape and width parameters, the FL profile has similar shape as either flattened Gaussian, super-Gaussian, or Fermi-Dirac profiles. An analytic express for the ray mapping function and M^2 for the FL profile have been obtained. It remains to be determined if these interesting properties of the FL profile will enable one to better understand the dependence of the aspheric surface sag of input and output profiles or if analytic expressions for aspheric surface sag functions can be obtained for the Gaussian-to-FL beam transform.

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