ALL QUESTIONS ARE WORTH 20 POINTS

NOTE: Clearly write out solutions and answers (circle the answers) by section for each part (a., b., c., etc.)

Important Formulas:

**Ch. 33. ELECTROMAGNETIC WAVES**

1. Electromagnetic waves $E = E_m \sin(kx - \omega t)$
   $B = B_m \sin(kx - \omega t)$
   \[ c = \frac{E}{B} = \sqrt{\frac{1}{\mu_0 \varepsilon_0}} \]

2. Poynting vector
   \[ \vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} \] - the rate per unit area at which energy is transported
   \[ S_{\text{avg}} = I = \frac{1}{c \mu_0} E_m^2 \] - the time averaged rate per unit area or intensity
   \[ I = \frac{P}{4 \pi r^2} \] - the intensity of the waves at distance $r$ from a point source of power $P$

3. Radiation pressure
   \[ F = \frac{IA}{c} \] (force in case of total absorption)
   \[ F = \frac{2IA}{c} \] 9 force in case of total reflection back along path
   \[ p_r = \frac{I}{c} \] (radiation pressure in case of total absorption)
   \[ p_r = \frac{2I}{c} \] (radiation pressure in case of total reflection back along path)

4. Polarizing sheets
   \[ I = \frac{1}{2} I_0 \] transmitted intensity of initially unpolarized light of intensity $I_0$
   \[ I = I_0 \cos^2 \theta \] transmitted intensity of initially polarized light of intensity $I_0$

5. Reflection and refraction
   **Law of reflection:** The angle of incidence equals the angle of reflection;
   Index of refraction: \[ n = \frac{c}{v} = \frac{\lambda}{\lambda_m} \]
Law of refraction (Snell’s law): \[ \frac{\sin \theta_1}{\sin \theta_2} = \frac{v_1}{v_2} \text{ or } n_1 \sin \theta_1 = n_2 \sin \theta_2; \]

1. **Total Internal Reflection**
   Critical angle for total internal reflection for boundary between two materials (where \( n_1 > n_2 \)):
   \[ \sin \theta_c = \frac{n_2}{n_1}; \]
   Critical angle for material-air boundary (where \( n \) is the index of refraction of the material):
   \[ \sin \theta_c = \frac{1}{n}; \]

2. **Polarization by Reflection**
   A reflected wave will be fully polarized, with its electric vector perpendicular to the plane of incidence, if it strikes a boundary at Brewster angle \( \theta_B \), where
   \[ \theta_B = \tan^{-1} \frac{n_2}{n_1}; \]

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**Ch. 34. IMAGES**

1. **Focal length of spherical mirror:** \( f = \pm \frac{1}{2} r \) (\( f \) is positive for concave mirror, negative for convex);
2. **Spherical Mirror equation:** \( \frac{1}{p} + \frac{1}{i} = \frac{1}{f} = \frac{2}{r} \) (\( p \) and \( i \) are positive if object or image is in front of the mirror, negative if behind; \( f > 0 \), concave; \( f < 0 \), convex);
3. **Spherical Refracting Surface:**
   \[ \frac{n_1}{p} + \frac{n_2}{i} = \frac{n_2 - n_1}{i} \] (single surface)
   Where \( n_1 \) is the index of refraction of the material where the object is located, \( n_2 \) is the index of refraction of the material on the other side of refraction surface, and \( r \) is the radius of curvature of the surface.
4. **Thin lens**
   \[ \frac{1}{p} + \frac{1}{i} = \frac{1}{f} = \frac{1}{r} - \frac{1}{r_1}; \]
   \( p \) is positive if object is on near side of lens, negative if on far side; \( i \) is positive if image is on far side of the lens, negative if on near side; \( f > 0 \), convex; \( f < 0 \), concave, a convex lens surface that faces the object has a positive radius of curvature, a concave lens surface that faces the object has a negative radius of curvature.
5. **Lateral Magnification produced by a spherical mirror or a thin lens**
   \[ m = - \frac{i}{p} \] (when \( m \) is positive image is upright; when \( m \) is negative image is inverted);
6. **Angular magnification of magnifier**
   \[ m_\theta = \frac{2.5 \, cm}{f} \]
12. Overall magnification of compound microscope:

\[ M = m \times \frac{25 \text{ cm}}{f_e} \times \left(1 - \frac{s}{f_{ob}}\right), \text{ s is the tube length;} \]

13. Angular magnification of telescope:

\[ m = -\frac{f_{ob}}{f_e} \]

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**Ch. 35 INTERFERENCE**

1. Two-slit interference pattern (Young experiment):
   - **maxima:** \( dsin \theta = 0, \lambda, 2\lambda, \ldots \)
   - **minima:** \( dsin \theta = 1/2\lambda, 3/2\lambda, 5/2\lambda, \ldots \)

2. Intensity in Two-Slit Interference

\[ I = 4 I_o \cos^2 \frac{1}{2} \phi; \text{ where } \phi = \frac{2\pi}{\lambda} dsin \theta \]

3. Thin Film Interference

\[ 2t + \Delta = mL_f, \text{ constructive interference} \]

\[ 2t + \Delta = (m + \frac{1}{2})L_f, \text{ destructive interference} \]

where \( m = 0,1,2,3,\ldots \); \( \lambda = \frac{\lambda}{n_f} \); \( \Delta = 0 \text{ or } \frac{\lambda_f}{2} \)

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**Ch. 36. DIFFRACTION**

1. Single-Slit Diffraction

\[ a \sin \theta = mL, \text{ for } m = 1,2,3,\ldots \text{ (minima)} \]

\[ I(\theta) = I_o \left(\frac{\sin \alpha}{\alpha}\right)^2, \text{ where } \alpha = \frac{\pi a}{\lambda} \sin \theta \]

2. Circular Aperture Diffraction

\[ \sin \theta = 1.22 \frac{\lambda}{d}, \text{ (first minimum - circular aperture)} \]

3. Rayleigh’s Criterion (resolution for a circular aperture):

\[ \theta_R = 1.22 \frac{\lambda}{d} \text{ (Rayleigh’s criterion)} \]

4. Double-Slit Diffraction

\[ I(\theta) = I_o \left(\cos^2 \beta \right) \left(\frac{\sin \alpha}{\alpha}\right)^2 \text{ (double slit)} \]

where \( \beta = \left(\frac{\pi d}{\lambda}\right) \sin \theta \), \( \alpha = \left(\frac{\pi a}{\lambda}\right) \sin \theta \)

5. Diffraction Grating
\[ d \sin \theta = m\lambda, \text{ for } m=0,1,2,... \text{ (maxima)} \]

with half-widths of the lines given by

\[ \Delta \theta_{hw} = \frac{\lambda}{Nd \cos \theta} \text{ (half-widths)}, \]

The dispersion is given by

\[ D = \frac{\Delta \theta}{\Delta \lambda} = \frac{m}{d \cos \theta} \]

The resolving power \( R \) is given by

\[ R = \frac{\lambda_{avg}}{\Delta \lambda} = Nm \]

1. X-Ray Diffraction
   \[ 2d \sin \theta = m\lambda, \text{ for } m=1,2,3,... \text{ (Bragg's law)} \]
1. A glass block with an index of refraction of 1.7 is immersed in an unknown liquid. A ray of light inside the block undergoes total internal reflection as shown in the figure. Which one of the following relations best indicates what may be concluded concerning the index of refraction of the liquid, $n_L$?

(a) $n_L < 1.0$  
(b) $n_L \geq 1.1$  
(c) $n_L \geq 1.3$  
(d) $n_L \leq 1.1$  
(e) $n_L \leq 1.3$

Since at angle of incidence $\theta = 50^\circ$ we already have total internal reflection, it means that

$\theta_c \leq 50^\circ$ or $\sin \theta_c \leq \sin 50^\circ$

We also know that

$N_g \sin \theta_c = N_L \sin \theta_c$

$\Rightarrow$ $N_g \sin 50^\circ \geq N_L$

$\Rightarrow$ $(N_L \leq 1.3)$
2. An object of 2.0 cm height is placed 30 cm from a converging lens with a focal length 20 cm. A concave mirror with focal length 15 cm is located 80 cm to the right of the lens as shown in figure. (Note: the figure is not drawn to scale)
(a) determine the location of the final image by construction
(b) characterize whether image is real or virtual, upright or inverted, reduced in size or enlarged
(c) determine the location of the final image by use of lens and mirror equations
(d) what is the height of the image

\[
\frac{1}{S_o} + \frac{1}{S_i'} = \frac{1}{f} \Rightarrow \frac{1}{S_i'} = \frac{1}{f} - \frac{1}{S_o} = \frac{S_o - f}{S_o f}
\]

\[
S_i' = \frac{S_o f}{S_o - f} = \frac{30 \cdot 20}{30 - 20} = \frac{600}{10} = 60 \text{ cm}
\]

\[
\frac{1}{S_m} + \frac{1}{S_m'} = \frac{1}{f_m} ; \quad S_m' = \frac{S_m f_m}{S_m - f_m} = \frac{20 \cdot 15}{20 - 15} = \frac{300}{5} = 60 \text{ cm}
\]

\[
S_m = 80 \text{ cm} - S_o = 80 - 60 = 20 \text{ cm}
\]

\[
\frac{h'}{h} = M = \left( -\frac{S_m'}{S_m} \right) \left( -\frac{S_o}{S_i'} \right) = \left( -\frac{60}{30} \right) \times \left( -\frac{60}{20} \right) = \frac{1}{2} \Rightarrow \frac{h'}{h} = \frac{-1}{2}
\]

\[
h' = h \cdot M = 12 \text{ cm}
\]

\( h' = h \cdot M = 12 \text{ cm} \)
3. A diverging lens has a focal length of \(-10\) cm. A 3-cm object is placed 25 cm from the lens.
(a) Determine the location of the final image by construction
(b) Determine the approximate distance between the object and the image.
(c) Describe the image.
(d) What is the magnification of the image?

\[
\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{5-f}{5s} \Rightarrow s' = \frac{fs}{s+f} = \frac{(-10)(25)}{25-(-10)} = -\frac{10 \cdot 25}{35} = -\frac{10}{35} = -\frac{2}{7}, 14\text{cm}
\]

\[
D = |s| - |s'| = 25 - 7 = (18\text{cm})
\]

\(\text{c) upright, reduced in size, virtual}\)

\(\text{d) } m = -\frac{s'}{s} = -\frac{7}{25} = +0.28\)
4. Mrs. York has been prescribed eyeglasses with lenses that have a refractive power of \(+3.2\) diopters. The glasses are worn 2.0 cm from her eyes. With the lenses, she can read a magazine held 25 cm from her eyes. What is the near point of her eye without glasses?

\[
\frac{1}{s} + \frac{1}{s'} = f \quad \frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = 3.2 - \frac{1}{0.23} = 3.2 - 4.35 = -1.148
\]

\[s' = -0.87 \text{ m}\]

\[N = |s'| + 2.0 \text{ cm} = 87 + 2.0 = 89 \text{ cm}\]
5. An astronomical telescope is designed with an overall magnification of 35.
   a) If the objective has a focal length of 100 cm, what should be the focal length of the eyepiece?
   b) How far should the eyepiece be from the objective?

\[
\text{(a) } M_{\text{tel}} = \frac{f_{\text{obj}}}{f_{\text{eye}}} \implies f_{\text{eye}} = \frac{f_{\text{obj}}}{M_{\text{tel}}} = \frac{100}{35} = 2.9 \text{ cm}
\]

\[
\text{(b) for telescope} \\
L = f_{\text{obj}} + f_{\text{eye}} = 100 + 2.9 \text{ cm} = 1.03 \times 10^2 \text{ cm}
\]
6. Light is incident on two slits that are separated by 0.2 mm. The figure shows the resulting interference pattern observed on a screen 1.0 m from the slits. Determine the wavelength of light used in this experiment.

Given: $m = 3$

$y_{m=3} = 7.5$ mm.

$d = 0.2$ mm

$D = 1.0$ m

\[ d \sin \theta = m \lambda \]

Constructive interference

\[ \lambda = \frac{d \sin \theta}{m} \]

\[ \theta = \sin \theta \approx \tan \theta = \frac{y}{D} \]

\[ \Rightarrow \lambda = \frac{d \cdot y}{m \cdot D} = \frac{(0.2 \times 10^{-3}) \cdot (7.5 \times 10^{-3})}{3 \cdot 1.0} \approx \frac{5 \times 10^{-7}}{m} \]

\[ = 500 \text{ nm} \]
7. What is the least thickness of a soap film \((d=?)\), placed on the polished surface of material with \(n=1.3\), which will appear bright (constructive interference) when viewed with sodium light \((\lambda=589.3 \text{ nm})\) reflected practically perpendicular to the film? The refractive index for soap solution is \(n=1.38\).

\[ \lambda=589.3 \text{ nm} \]

\[ \text{Air} \]

\[ \text{a} \]

\[ \text{b} \]

\[ \text{d} \]

\[ n=1.3 \]

1) Analysis of \( \Delta \)
- \( a \) is \( \frac{1}{2} \) shifted with respect to \( i \)
- \( b \) is not shifted with respect to \( i \)

\[ \Rightarrow a \text{ is } \frac{1}{2} \text{ shifted with respect to } b \]

\[ \Rightarrow \Delta = \frac{1}{2} \]

2) Condition for constructive interference

\[ 2d + \frac{\lambda}{2} = \lambda f ; \quad 2 \frac{\lambda}{2} = 3 \frac{\lambda}{2} \]

\[ \Rightarrow 2d = \frac{\lambda f}{2} ; \quad 3 \frac{\lambda f}{2} = 5 \frac{\lambda f}{2} \]

\[ \Rightarrow 2d_{\text{min}} = \frac{\lambda f}{2} ; \quad \lambda f = \frac{\lambda}{n} = \frac{589.3 \text{ nm}}{1.38} = 427 \text{ nm} \]

\[ \Rightarrow d_{\text{min}} = \frac{\lambda f}{4} = \frac{427 \text{ nm}}{4} = 107 \times 10^{-2} = 1.1 \times 10^{-2} \text{ nm} \]
8. The wavelength of light emitted from two distant objects is 715 nm. The minimum angle at which these objects are resolved when using binoculars is $10^{-3}$ degrees. What is the smallest diameter of the objective lens of the binoculars?

In the limiting case due to Raleigh criterion

$$\theta_c = \frac{1.22\lambda}{d},$$

where $d$ - diameter of the binocular lens

From the figure

$$\theta_c = 10^{-3} \text{ degree} = \frac{10^{-3} \times 2\pi}{360} = 1.745 \times 10^{-5} \text{ rad}$$

$$\lambda = 715 \text{ nm}$$

$$\Rightarrow d = \frac{1.22\lambda}{\theta_c} = \frac{1.22 \times 715 \times 10^{-9}}{1.745 \times 10^{-5} \text{ rad}} = 5.0 \times 10^{-2} \text{ m} = 5.0 \text{ cm}$$
9. Light from a laser ($\lambda = 640 \text{ nm}$) passes through a diffraction grating and spreads out into three beams as shown in the figure. Determine the spacing between the slits of the grating.

\[
d \sin \theta_0 = 0.1 \quad \Rightarrow \quad \sin \theta_0 = 0
\]

\[
d \sin \theta_1 = 1.1 \quad \Rightarrow \quad d = \frac{1}{\sin \theta_1} = \tan \theta_1 = \frac{y}{2}
\]

\[
= \frac{\lambda}{\sin (\arctan \frac{y}{2})} = \frac{640 \times 10^{-9} \text{ m}}{\sin (\arctan \frac{4}{3})} = \frac{640 \times 10^{-9} \text{ m}}{\sin 53.12^\circ} = \frac{640 \times 10^{-9} \text{ m}}{0.8} = 800 \times 10^{-9} \text{ m} = 8.00 \times 10^2 \text{ nm}
\]