PH 221-3A Fall 2007

Force and Motion

Lecture 6-7

Chapter 5
(Halliday/Resnick/Walker, Fundamentals of Physics 8th edition)
Chapter 5

Force and Motion

In chapters 2 and 4 we have studied “kinematics” i.e. described the motion of objects using parameters such as the position vector, velocity and acceleration without any insights as to what caused the motion. This is the task of chapters 5 and 6 in which the part of mechanics known as “dynamics” will be developed. In this chapter we will introduce Newton’s three laws of motion which is at the heart of classical mechanics. We must note that Newton’s laws describe physical phenomena of a vast range. For example Newton’s laws explain the motion of stars and planets. We must also note that Newton’s laws fail in the following two circumstances:

1. When the speed of objects approaches (1% or more) the speed of light in vacuum ($c = 8\times10^8$ m/s). In this case we must use Einstein’s special theory of relativity (1905)
2. When the objects under study become very small (e.g. electrons, atoms etc) In this case we must use quantum mechanics (1926)
Dynamics – the study of forces and their effects on the motion of the bodies

1. The concepts of force, mass, and inertia

**Force** – in common use is any push or pull exerted on a body

**Inertia** – natural tendency of an object to remain at rest or in a motion at a constant speed along a straight line.

**Mass** – is a quantitative measure of inertia. Unit – kilogram (kg)

2. Newton’s first law of motion

In the absence of external forces a body at rest remains at rest, a body in motion continues to move at constant velocity
An inertial reference frame is one in which Newton’s law of inertia is valid.

- The acceleration of an inertial reference frame is zero.
- Earth is a good approximation of an inertial reference frame.

3. Newton’s Second Law of Motion

An external force acting on a body gives it an acceleration that is in the direction of the force and has a magnitude inversely proportional to the mass of the body.

\[ \text{F} \]

\[ \text{a} = \frac{\text{F}}{\text{m}} \text{ or } \text{ma} = \text{F}, \text{ valid only in inertial reference frame} \]

Superposition of forces – law of nature

If several forces \( F_1, F_2, F_3, \ldots F_n \) act simultaneously on a body, then the acceleration is the same as that produced by a single force \( F_{\text{net}} \) given by a vector sum of individual forces.

\[ \text{a} = \frac{F_{\text{net}}}{m} = \frac{F_1}{m} + \frac{F_2}{m} + \ldots \frac{F_n}{m} = \text{a}_1 + \text{a}_2 \]

+ \ldots \text{a}_n

Each force produces an acceleration independently of the other forces.
The Vector Nature of the Newton’s 2\textsuperscript{nd} laws

\[ \sum F = ma \]
\[ \sum F_x = ma_x \]
\[ \sum F_y = ma_y \]

Components are positive or negative numbers.

Free body diagrams and the second law

It is a diagram that represents the object and the forces that act on it. Only forces that act on the object appear in a free body diagram.

5. Newton’s Third Law of Motion

Every action (force) has an equal, but opposite reaction.

All forces occur in pairs, action-reaction forces
Newton’s First Law

Scientists before Newton thought that a force (the word “influence” was used) was required in order to keep an object moving at constant velocity. An object was thought to be in its “natural state” when it was at rest. This mistake was made before friction was recognized to be a force. For example, if we slide an object on a floor with an initial speed \( v_o \) very soon the object will come to rest. If on the other hand we slide the same object on a very slippery surface such as ice, the object will travel a much larger distance before it stops. Newton checked his ideas on the motion of the moon and the planets. In space there is no friction, therefore he was able to determine the correct form of what is since known as: “Newton’s first law”

If no force acts on a body, the body’s velocity cannot change; that is the body cannot accelerate

Note: If several forces act on a body (say \( F_A \), \( F_B \), and \( F_C \)) the net force \( F_{\text{net}} \) is defined as: \( F_{\text{net}} = F_A + F_B + F_C \) i.e. \( F_{\text{net}} \) is the vector sum of \( F_A, F_B, \) and \( F_C \).
**Force:** The concept of force was tentatively defined as a push or pull exerted on an object. We can define a force exerted on an object quantitatively by measuring the acceleration it causes using the following procedure.

We place an object of mass \( m = 1 \) kg on a frictionless surface and measure the acceleration \( a \) that results from the application of a force \( F \). The force is adjusted so that \( a = 1 \) m/s\(^2\). We then say that \( F = 1 \) newton (symbol: N).

**Note:** If several forces act on a body (say \( \vec{F}_A, \vec{F}_B, \) and \( \vec{F}_C \)) the net force \( \vec{F}_{net} \) is defined as: \( \vec{F}_{net} = \vec{F}_A + \vec{F}_B + \vec{F}_C \). i.e. \( \vec{F}_{net} \) is the vector sum of \( \vec{F}_A, \vec{F}_B, \) and \( \vec{F}_C \).
Mass: Mass is an *intrinsic* characteristic of a body that automatically comes with the existence of the body. But what is it exactly? It turns out that mass of a body is the characteristic that relates a force $F$ applied on the body and the resulting *acceleration* $a$.

Consider that we have a body of mass $m_o = 1 \text{ kg}$ on which we apply a force $F = 1 \text{ N}$. According to the definition of the newton, $F$ causes an acceleration $a_o = 1 \text{ m/s}^2$. We now apply $F$ on a second body of unknown mass $m_X$ which results in an acceleration $a_X$. The ratio of the accelerations is inversely proportional to the ratio of the masses

$$\frac{m_X}{m_o} = \frac{a_o}{a_X} \rightarrow m_X = m_o \frac{a_o}{a_X}$$

Thus by measuring $a_X$ we are able to determine the mass $m_X$ of any object.
**Newton’s Second Law**

The results of the discussions on the relations between the net force $F_{\text{net}}$ applied on an object of mass $m$ and the resulting acceleration $a$ can be summarized in the following statement known as: “**Newton’s second law**”

![Diagram of Newton's Second Law](image)

The net force on a body is equal to the product of the body’s mass and its acceleration

In equation form Newton’s second law can be written as:

$$\vec{F}_{\text{net}} = m\vec{a}$$

The above equation is a compact way of summarizing three separate equations, one for each coordinate axis:

$$F_{\text{net},x} = ma_x \quad F_{\text{net},y} = ma_y \quad F_{\text{net},z} = ma_z$$
Newton’s Third Law:

When two bodies interact by exerting forces on each other, the forces are equal in magnitude and opposite in direction.

For example consider a book leaning against a bookcase. We label $\vec{F}_{BC}$ the force exerted on the book by the case. Using the same convention we label $\vec{F}_{CB}$ the force exerted on the case by the book. Newton's third law can be written as:

$\vec{F}_{BC} = -\vec{F}_{CB}$

The book together with the bookcase are known as a "third-law force pair".

A second example is shown in the picture to the left. The third-law pair consists of the earth and a cantaloupe. Using the same convention as above we can express Newton's third law as:

$\vec{F}_{CE} = -\vec{F}_{EC}$
Inertial Reference Frames:

We define a reference frame as “inertial” if Newton’s three laws of motion hold. In contrast, reference frames in which Newton’s law are not obeyed are labeled “non-inertial”.

Newton believed that such at least one inertial reference frame \( R \) exists. Any other inertial frame \( R' \) that moves with \textbf{constant velocity} with respect to \( R \) is also an inertial reference frame. In contrast, a reference frame \( R'' \) which \textit{accelerates} with respect to \( R \) is a non-inertial reference frame.

The earth rotates about its axis once every 24 hours and thus it is accelerating with respect to an inertial reference frame. Thus we are making an approximation when we consider the earth to be an inertial reference frame. This approximation is excellent for most small scale phenomena. Nevertheless for large scale phenomena such as global wind systems, this is not the case and corrections to Newton’s laws must be used.
Applying Newton’s Laws / Free body Diagrams

Part of the procedure of solving a mechanics problem using Newton’s laws is drawing a free body diagram. This means that among the many parts of a given problem we choose one which we call the “system”. Then we choose axes and enter all the forces that are acting on the system and omitting those acting on objects that were not included in the system.

An example is given in the figure below. This is a problem that involves two blocks labeled "A" and "B" on which an external force $\vec{F}_{app}$ is exerted.

We have the following "system" choices:

a. System = block A + block B. The only horizontal force is $\vec{F}_{app}$

b. System = block A. There are now two horizontal forces: $\vec{F}_{app}$ and $\vec{F}_{AB}$

c. System = block B. The only horizontal force is $\vec{F}_{BA}$
Concepts at a glance

External Forces
1. Gravitational Force
2. Normal Force
3. Frictional Forces
4. Tension Force

Problem Solving Strategy:
1) Select an object
2) Show Free body diagram
3) Choose a set of x an y axis and resolve all forces into components
4) Apply $\Sigma F_x = ma_x$; $\Sigma F_y = ma_y$;
5) Solve equations of step 4 for the unknown quantities
Newton’s Law of Universal Gravitation

Every particle attracts every other particle with a force directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

\[ F = \frac{GMm}{r^2} \]

- Gravitational force does not require any contact between the interacting particles
- The gravitational force between two particles is unaffected by the presence of intervening masses.
Magnitude of the gravitational force exerted by a particle of mass 1kg on another particle of mass 1kg.

\[ F = \frac{G m_1 m_2}{r^2} \]
Newton’s Theorem

The net gravitational force between two spherical bodies acts just as though each body were concentrated at the center of its respective sphere.

Acceleration of free fall
Calculate the gravitational force that the Earth exerts on an astronaut of mass 75kg in a space capsule at a height of 1000km above the surface of the Earth. Compare with the gravitational force that this astronaut would experience if on the surface of the Earth.

Given:
\[ M_a = 75 \text{ kg} \]
\[ M_E = 6 \times 10^{24} \text{ kg} \]
\[ h = 1000 \text{ km} = 10^6 \text{ m} \]
\[ r_E = 6.4 \times 10^6 \text{ m} \]

Find \( F_R = ? \); \( F_{E_s} = ? \)

1) \[ F_R = \frac{G m_a M_E}{R^2} = \frac{G m_a M_E}{(r_E + h)^2} = \frac{(6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2)(75 \text{ kg})(6 \times 10^{24} \text{ kg})}{(6.4 \times 10^6 \text{ m} + 10^6 \text{ m})^2} = 5.5 \times 10^2 \text{ N} \]

at 1000km

2) \[ F_{E_s} = \frac{G m_a M_E}{r_E^2} = \frac{(6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2)(75 \text{ kg})(6 \times 10^{24} \text{ kg})}{(6.4 \times 10^6 \text{ m})^2} = 7.3 \times 10^2 \text{ N} \]
on the Surface of the Earth
Somewhere between the Earth and the Moon there is a point where the gravitational pull of the Earth on a particle exactly balances that of the Moon. At what distance from the Earth is this point?

\[
\sum F = m \ddot{x} = 0
\]

\[
\sum F_x = -\frac{G M_E m}{R^2} + \frac{G M_m m}{(d-R)^2} = 0
\]

\[
\frac{G M_m m}{(d-R)^2} = \frac{G M_E m}{R^2}
\]

\[
(d-R)^2 = \frac{M_m M_e}{M_e}
\]

\[
d^2 - 2dR + R^2 - \frac{M_m}{M_e} R^2 = 0
\]

\[
R^2 (1 - \frac{M_m}{M_e}) - 2dR + d^2 = 0
\]

\[
R^2 (1 - \frac{7.35 \times 10^{22}}{5.98 \times 10^{24}}) - 2 (3.84 \times 10^8)
\]

\[
= 0.988 R^2 - 7.68 \times 10^8 R + 147 \times 10^8 = 0
\]

\[
0.988 x^2 - 7.68 \times 10^8 x + 147 \times 10^8 = 0
\]

\[
x = \frac{7.68 \times 10^8 \pm \sqrt{(7.68 \times 10^8)^2 - 4 (0.988)(147 \times 10^8)}}{2 (0.988)}
\]

\[
R = \frac{7.68 \times 10^8 \pm \sqrt{(7.68 \times 10^8)^2 - 4 (0.988)(147 \times 10^8)}}{2 (0.988)}
\]

\[
= 7.68 \times 10^8 \pm 9.42 \times 10^7
\]

\[
= 3.41 \times 10^8
\]

\[
\text{take the root that is less than d}
\]
The Gravitational Force: It is the force that the earth exerts on any object (in the picture a cantaloupe). It is directed towards the center of the earth. Its magnitude is given by Newton’s second law.

\[
\vec{F}_g = m\vec{a} = -mg\hat{j}
\]

\[
|\vec{F}_g| = mg
\]

Weight: The weight of a body is defined as the magnitude of the force required to prevent the body from falling freely.

\[
F_{net,y} = ma_y = W - mg = 0 \rightarrow W = mg
\]

Note: The weight of an object is not its mass. If the object is moved to a location where the acceleration of gravity is different (e.g. the moon where \(g_m = 1.7 \text{ m/s}^2\)), the mass does not change but the weight does.
True Weight

The true weight of an object on the Earth is the gravitational force that the Earth exerts on the object. The true weight always acts downward toward the center of the Earth.

SI Unit of Weight (N)

Mass is a quantitative measure of inertia. It is an intrinsic property of matter and doesn’t change as an object is moved from one location to another.

Apparent Weight

The apparent weight is the force that the object exerts on the scale with which it is in contact.

The apparent weight and true weight are not always equal.
Apparent Weight

1. No acceleration ($v$ = constant)
   \[ A_W = W \]
   \[ A_W = W \]

2. Upward acceleration
   \[ N - W = ma \]
   \[ N = W + ma \]

3. Downward acceleration
   \[ N - W = -ma \]
   \[ N = W - ma \]

4. Free-fall
   \[ N - W = -mg \]
   \[ N = W - mg \]

Conclusion:
- \[ A_W = W - ma \]
- \[ A_W < W \]
- \[ A_W = 0 \]
A person of mass $m$ is standing on scales while riding in an elevator. When the elevator accelerates upward uniformly at a rate “$a$” the scales read 700 N. When the elevator accelerates downward at the same rate, the scales read 400 N.

a) Calculate the uniform acceleration “$a$”
b) Calculate the mass of the person
**Contact Forces:** As the name implies these forces act between two objects that are in contact. The contact forces have two components. One that is acting along the normal to the contact surface (normal force) and a second component that is acting parallel to the contact surface (frictional force).

**Normal Force:** When a body presses against a surface, the surface deforms and pushes on the body with a normal force perpendicular to the contact surface. An example is shown in the picture to the left. A block of mass $m$ rests on a table.

\[ F_{net,y} = ma_y = F_N - mg = 0 \rightarrow F_N = mg \]

**Note:** In this case $F_N = mg$. This is not always the case.

**Friction:** If we slide or attempt to slide an object over a surface, the motion is resisted by a bonding between the object and the surface. This force is known as “friction”. More on friction in chapter 6.
The Normal Force

The normal force $F_N$ is one component of the force that a surface exerts on an object with which it is in contact, namely, the component that is perpendicular to the surface.

Free body diagram
**Tension:** This is the force exerted by a rope or a cable attached to an object. Tension has the following characteristics:
1. It is always directed along the rope.
2. It is always pulling the object.
3. It has the same value along the rope. (For example, between points A and B)

The following assumptions are made:

a. The rope has negligible mass compared to the mass of the object it pulls.

b. The rope does not stretch.

If a pulley is used as in fig. (b) and fig. (c), we assume that the pulley is massless and frictionless.
The Tension Force

Each particle in the rope applies a force to its neighbor. As a result the force $T$ is transmitted to the box.

Free body diagram of the rope

\[ \Sigma F_x = ma_x \]
\[ T - T_1 = ma \]
only when $m = 0$ (massless rope)
\[ T = T_1 \]

The ability of a massless rope to transmit tension without changes from one end to the other is not affected when the rope passes around objects such as the pulley (provided the pulley itself is massless and frictionless).
3. A massless cord passing over an easily turned pulley has an unknown mass "m_1" hanging from one end and mass m_2 = 9 kg hanging from the other. (This arrangement is called Atwood's machine.) The acceleration of the masses a = 1.23 m/s^2. Find:
(a) mass "m_1";
(b) the tension in the cord.

\[ T_1 = T_2 = T \]
\[ a_1 = a_2 = a \]

\[ \frac{m_2 g - T}{m_1} = m_2 a_2 \]
\[ \frac{T_1 - m_1 g}{m_2} = m_1 a_1 \]
\[ m_2 (g - a) = m_1 (g + a) \]

\[ a = \left( \frac{m_2 - m_1}{m_2 + m_1} \right) g \]

\[ m_1 = 7 \text{ kg} \]

\[ T = \left( \frac{m_1 m_2 g}{m_1 + m_2} \right) \]

\[ T = 77 \text{ N} \]
Weighting the Earth

\[ F = \frac{G M_e m}{R_e^2} = m g \]

\[ g = \frac{G M_e}{R_e^2} \]

Atwood machine:

\[ a = \frac{g (m_2 - m_1)}{m_2 + m_1} \]

\[ g = \frac{a (m_2 + m_1)}{m_2 - m_1} = 9.81 \text{ m/s}^2 \]

\[ R_e = 6.38 \times 10^6 \text{ m} \]

\[ M_e = \frac{R_e^2 g}{G} = \frac{(6.38 \times 10^6 \text{ m})^2 \times 9.81 \text{ m/s}^2}{6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2} = 5.98 \times 10^24 \text{ kg} \]
1. One mass, \( m_1 = 155 \text{ g} \), of an ideal Atwood machine rests on the floor 1.25 m below the other mass, \( m_2 = 175 \text{ g} \). If the masses are released from rest, how long does it take \( m_2 \) to reach the floor?

Given:

\[
\begin{align*}
\text{Given:} \\
\quad m_1 &= 0.155 \text{ kg} \\
\quad m_2 &= 0.175 \text{ kg} \\
\quad y &= 1.25 \text{ m} \\
\end{align*}
\]

Find \( t = ? \)

(1) \[
\begin{align*}
T - m_1 g &= m_1 a \\
m_2 g - T &= m_2 a \\
\frac{m_2 g - m_1 g}{m_2 - m_1} &= a
\end{align*}
\]

\[
\Rightarrow a = \frac{g (m_2 - m_1)}{m_1 + m_2} = 0.594 \text{ m/s}^2
\]

(2) \[
\begin{array}{|c|c|c|c|}
\hline
y & a & v_0 & v & t \\
\hline
-1.25 & \frac{g (m_2 - m_1)}{m_1 + m_2} & 0 & ? & ? \\
\hline
\end{array}
\]

\[
y = v_0 t + \frac{1}{2} a t^2
\]

\[
t = \sqrt{\frac{2 y}{a}} = \sqrt{\frac{+2 (-1.25)}{-0.175 - 0.155) y} = 2.05 \text{ s}
\]
Non-equilibrium applications of Newton’s Laws of Motion

(a) Diagram showing a truck and a trailer connected by a drawbar. The mass of the trailer is $m_2 = 27000$ kg and the mass of the truck is $m_1 = 8500$ kg. The acceleration is $a = 0.78$ m/s$^2$.

(b) Free-body diagrams

1. $\sum F_x = T = m_2 a_x = 21000$ N
2. Since the drawbar is massless, $T = T'$
3. $\sum F_x = D - T = m_1 a_x = 28000$ N
4. Three blocks \((m_1=1 \text{ kg}, m_2=2.0 \text{ kg}, m_3=3.0 \text{ kg})\) are pulled along a frictionless surface by a horizontal force \(F=18 \text{ N}\).
(a) What is the acceleration of the system?
(b) What are the tension forces on the light strings?

\[
\begin{align*}
E_F &= ma \\
F &= (m_1 + m_2 + m_3) a \\
a &= \frac{F}{m_1 + m_2 + m_3} = \frac{18.0 \text{ N}}{(1+2+3) \text{ kg}} = 3.0 \text{ m/s}^2
\end{align*}
\]

1) \[
\begin{align*}
\sum F &= 0 \\
x: \sum F_x &= m_3 a \\
F - T_2 &= m_3 a \\
\Rightarrow T_2 &= F - m_3 a = 18 - 3.0 \times 3.0 = 9.0 \text{ N}
\end{align*}
\]

2) \[
\begin{align*}
\sum F &= 0 \\
x: \sum F_x &= m_2 a \\
T_2 - T_1 &= m_2 a \\
T_1 &= T_2 - m_2 a = 9.0 - 2 \times 3 = 3.0 \text{ N}
\end{align*}
\]
Accelerating Blocks

Given:
\( m_1 = 8.0 \text{ kg} \)
\( m_2 = 22.0 \text{ kg} \)

Surface frictionless
Pulley is massless and frictionless

Find: \( \tau = ? \)

\[ W_1 \sin 30.0^\circ \]

Block 1:
\[ \Sigma F_x = -W_1 \sin 30^\circ + F_T = m_1 a \]

Block 2:
\[ \Sigma F_y = T - W_2 = m_2 (\tau) \]

\[ a = \frac{W_2 - W_1 \sin 30^\circ}{m_1 + m_2} = 5.8 \text{ m/s}^2 \]

\[ T = m_2 (\tau - g) = m_2 \left[ g - g \left( \frac{m_2 - m_1 \sin 30^\circ}{m_1 + m_2} \right) \right] = m_2 \left[ \frac{m_1 g + m_2 g - m_1 g \sin 30^\circ}{m_1 + m_2} \right] = m_2 \frac{m_1 g (1 - \sin 30^\circ)}{m_1 + m_2} = 86.3 \text{ N} \]
2. Two blocks having masses \(m_1=3.0\) kg and \(m_2=5.0\) kg are connected by a massless string that passes over a massless, frictionless pulley. The blocks are in contact with frictionless planes of angles 37° and 53°. Determine the acceleration of the blocks.

\[
\begin{align*}
(1) & \quad N_1 - m_1g \cos \Theta = 0 \\
(2) & \quad N_2 - m_2g \cos \phi = 0 \\
(3) & \quad T - m_1g \sin \Theta = m_1a \\
(4) & \quad T + m_2g \sin \phi = m_2a \\
\end{align*}
\]

\[
\begin{align*}
(4) + (2) & \quad T + m_2g \sin \phi = m_2a \\
\frac{T - m_1g \sin \Theta}{m_1} & = \frac{-m_1g \sin \Theta + m_2g \sin \phi}{m_2 + m_1} \\
& \quad \Rightarrow a = \frac{g (-m_1 \sin \Theta + m_2 \sin \phi)}{m_2 + m_1} = \frac{g (m_2 \sin \phi - m_1 \sin \Theta)}{m_2 + m_1} \\
& \quad \Rightarrow a = \frac{g (m_2 \sin \phi - m_1 \sin \Theta)}{m_2 + m_1} \\
& \quad = \frac{9.8 \, \text{m/s}^2 \left(5.0 \, \text{kg}\right) \sin 53° - (3.0 \, \text{kg}) \sin 37°}{(3.0 \, \text{kg}) + (5.0 \, \text{kg})} = \frac{9.8 \times 3.5938 - 1.85255}{8.0} \\
& \quad = 2.6799 = 2.7 \, \text{m/s}^2
\end{align*}
\]