Kinematics in One Dimension

Lectures 3,4

Chapter 2
(Cutnell & Johnon, Physics 7th edition)
Motion along a straight line

Studies the motion of bodies

Deals with force as the cause of changes in motion

Key Concepts

1. Ideal particle – a point-like mass of infinitesimal size
2. Rectangular coordinates and origin point
3. Reference frame – coordinate grid with suitably adjusted clocks
Ch. 2 Problem 2
One afternoon, a couple walks three-fourths of the way around a circular lake, the radius of which is 1.50 km. They start at the west side of the lake and head due south at the beginning of their walk.

a) What is the distance they travel?
b) What are the magnitude and direction (relative to due east) of the couple’s displacement?

\[ \text{Distance} = \frac{3}{4} \times \text{Circumference of Lake} = \frac{3}{4} \times 2\pi R = \frac{3}{4} \times 2\pi (1.50\, \text{km}) = 7.07\, \text{km} \]

\[ \text{Displacement} = \sqrt{r^2 + r^2} = \sqrt{1.50^2 + 1.50^2} = 2.12\, \text{km} \]

\[ \theta = \tan^{-1} \left( \frac{1.50\, \text{km}}{1.50\, \text{km}} \right) = \tan^{-1}(1) = 45^\circ \text{ North of East} \]
Average Speed

\[ \text{average speed} = \frac{\text{distance traveled}}{\text{time taken}} \]  
Unit: m/s

<table>
<thead>
<tr>
<th>Speed</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light</td>
<td>$3.0 \times 10^8$ m/s</td>
</tr>
<tr>
<td>Recession of fastest known quasar</td>
<td>$2.8 \times 10^8$ m/s</td>
</tr>
<tr>
<td>Electron around nucleus (hydrogen)</td>
<td>$2.2 \times 10^6$ m/s</td>
</tr>
<tr>
<td>Earth around Sun</td>
<td>$3.0 \times 10^4$ m/s</td>
</tr>
<tr>
<td>Rifle bullet (muzzle velocity) (a)</td>
<td>$7 \times 10^2$ m/s</td>
</tr>
<tr>
<td>Random motion of molecules in air (average)</td>
<td>$4.5 \times 10^2$ m/s</td>
</tr>
<tr>
<td>Sound</td>
<td>$3.3 \times 10^2$ m/s</td>
</tr>
<tr>
<td>Jet airliner (Boeing 747, maximum airspeed)</td>
<td>$2.7 \times 10^2$ m/s</td>
</tr>
<tr>
<td>Cheetah (maximum) (b)</td>
<td>28 m/s</td>
</tr>
<tr>
<td>Federal highway speed limit (55 mi/h)</td>
<td>25 m/s</td>
</tr>
<tr>
<td>Man (maximum)</td>
<td>12 m/s</td>
</tr>
<tr>
<td>Man (walking briskly)</td>
<td>1.3 m/s</td>
</tr>
<tr>
<td>Snail</td>
<td>$\approx 10^{-3}$ m/s</td>
</tr>
<tr>
<td>Glacier (c)</td>
<td>$\approx 10^{-6}$ m/s</td>
</tr>
<tr>
<td>Rate of growth of hair (human)</td>
<td>$3 \times 10^{-9}$ m/s</td>
</tr>
<tr>
<td>Continental drift</td>
<td>$\approx 10^{-9}$ m/s</td>
</tr>
</tbody>
</table>

Speed is independent of the direction of motion
Average velocity for motion along a straight line

Graphical representation of motion by means of a plot of the position vs. the time coordinate

The X axis coincides with the straight line
If the displacement points in the + direction the average velocity is positive

Average velocity is a vector that points in the same direction as the displacement.

Velocity is positive or negative

Average velocity for the interval from $t_1$ to $t_2$ is equal to the ratio $\frac{x_2 - x_1}{t_2 - t_1}$, where $x_2 - x_1 = \Delta x$.
Instantaneous Velocity ($V_{\text{inst}}$)

Uniform motion \[ V_{\text{inst}} = V_{\text{av}} = V_{\text{const}} \]

With $\Delta t$ extremely small

The inst. Velocity at a given time = the slope of the tangent that touches the plot at that time
To find the $V_{\text{inst}}$ at different times we draw tangents to the plot at these times and measure their slopes.
Problem 60
A person who walks for exercise produces the position-time graph
a) Without doing any calculations, decide which segments of the graph (A, B, C or D) indicate positive, negative and zero average velocities.

b) Calculate the average velocity for each segment to verify your answers to part A

---

a) The sign of the average velocity during a segment corresponds to the sign of the slope of the segment. The slope, and hence the average velocity, is positive for segments A and D, and negative for C, zero for segment B

b) 
\[
V_A = \frac{1.00 \text{ km} - 0 \text{ km}}{0.20 \text{ h} - 0 \text{ h}} = 5.0 \text{ km/h}
\]
\[
V_B = \frac{1.00 \text{ km} - 1.00 \text{ km}}{0.40 \text{ h} - 0.20 \text{ h}} = 0.0 \text{ km/h}
\]
\[
V_C = \frac{0.25 \text{ km} - 1.00 \text{ km}}{0.60 \text{ h} - 0.40 \text{ h}} = -3.8 \text{ km/h}
\]
\[
V_D = \frac{1.25 \text{ km} - 0.25 \text{ km}}{1.00 \text{ h} - 0.60 \text{ h}} = 2.5 \text{ km/h}
\]
1. [1pt] The following graphs show the position (x) of a car as a function of time t. The east is chosen as +x direction. Which graphs are consistent with the following descriptions? If more than one graphs match to the description, enter two or more letters without space. For example, AC.

(a) The car is initially at rest and travels due east.

(b) The car is traveling due west and increasing its speed.

(c) The car is initially traveling due west and stops.

(d) The car is moving with a constant velocity.

2. [1pt] A commuter airplane, starting from rest on an airport runway, accelerates for 22.5 s before taking off. Its speed at takeoff is 51.8 m/s (116 mi/hr). (a) Calculate the acceleration of the plane, assuming it remains constant.

(b) How far did the plane move while accelerating for 22.5 s?

3. [1pt] A Boeing 767 jet taking off from an airport accelerates from rest for 31.0 s before leaving the ground. Its acceleration is 2.16 m/s². (a) Assuming that the acceleration is constant, calculate the plane's speed at takeoff in m/s.

What is the takeoff speed in km/hr?

4. [1pt] An object is moving in a straight line with a constant
A runner runs 100m in 10s, rests 60s and returns at a walk in 80s. What is the average speed for the complete motion? What is the average velocity?

- The runner moved a total distance $d = 100 + 0 + 100 = 200$ m
- The round trip took $t = 10 + 60 + 80 = 150$ s
  \[ V_{av} = \frac{d}{t} = \frac{200\text{m}}{150\text{s}} = 1.3\ \text{m/s} \]
- After the motion, the runner is precisely located at the starting point
  \[ \Rightarrow \text{His position didn’t change} \]
  \[ \Rightarrow V_{av} = \frac{\Delta x}{t} = \frac{0}{t} = 0 \]
The average acceleration is defined to provide a measure of how much the velocity changes per unit of elapsed time.

- $a_{av}$ is a vector that points in the same direction as $\Delta V$
- $+ \text{ and } -$ indicate two possible directions for the acceleration vector

$\Rightarrow a_{av}$ directed to the left
$\Rightarrow$ it’s component on the positive x direction is negative

**Instantaneous acceleration**

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{V}}{\Delta t}$$
Equations of kinematics for constant acceleration

- Assume \( x_0 = 0 \) when \( t_0 = 0 \) => \( \Delta x = x - x_0 = x \)
- Since motion is along a straight line all the vectors of displacement, velocity and acceleration are along this line and we will substitute them with their magnitudes having plus or minus signs conveying the direction of these vectors
- Assume \( v = v_0 \) at \( t_0 = 0 \) and \( v \) at \( t \)
- Assume \( a = \) constant
  => \( a_{av} = a = v-v_0/t \) or \( v = v_0 + at \)
- From the definition of the \( v_{av} \)
  \( v_{av} = x-x_0/t-t_0 = x/t \) or \( x = v_{av}t \)
- Because \( a = \) constant, \( v \) increases at a constant rate.
  => \( v_{av} \) is midway between \( v_0 \) and \( v_{\text{final}} \)
  \( v_{av} = \frac{1}{2}(v_0+v) \) => \( x = \frac{1}{2}(v_0+v)t \)
  \( x = v_{av}t = \frac{1}{2}(v_0+v)t = \frac{1}{2}(v_0+v_0+at)t = v_0t + at^2/2 \)
  \( x = v_0t + at^2/2 \)

\( x = \frac{1}{2}(v_0+v)t = \frac{1}{2}(v_0+v)(v-v_0/a) = v^2-v_0^2/2a \) => \( v^2 = v_0^2 + 2ax \)
Ch. 2 Problem 22

a) What is the magnitude of the average acceleration of a skier who, starting from rest, reaches a speed of 8.0 m/s when going down a slope for 5.0s?

b) How far does the skier travel in this time?

5. Solve the equation with respect to the unknown kinematic variable

\[ \text{average acceleration} = \frac{v_f - v_i}{t} \]

\[ a_{avg} = \frac{8.0 \text{ m/s} - 0}{5.0 \text{ s}} = 1.6 \text{ m/s}^2 \]

\[ x = \frac{1}{2} (v_i + v_f) t = \frac{1}{2} (8.0 \text{ m/s} + 0) (5.0 \text{ s}) = 20 \times 10 \text{ m} \]

3. Fill up the table of given kinematic variables with appropriate + or – signs.

4. Verify that given information contains at least three of the five kinematics variables and choose one kinematic equation relating three given kinematic variables with the unknown one.

1. Make a drawing

2. Show positive direction of motion and choose a convenient origin point

\[ v_t = v_i + at \]

\[ x = \frac{1}{2} (v_i + v_f) t \]

\[ x = v_i t + \frac{1}{2} a t^2 \]

\[ v^2 = v_i^2 + 2ax \]
The driver of an automobile traveling at 95 km/h perceives an obstacle on the road and slams on the brakes. Calculate the total stopping distance in meters. Assume that the reaction time of the driver is 0.75s (so there is a time interval of 0.75s during which the automobile continues at constant speed while the driver gets ready to apply the brakes and that the deceleration of the automobile is 7.8 m/s² when the brakes are applied.

\[
\text{Given: } v_0 = v = 95 \text{ m/s}, \quad t_0 = 0, \quad t_1 = 0.75 \text{s}, \quad a = -7.8 \text{ m/s}^2, \quad v_2 = 0
\]

\[
x_2 - x_0 = x_1 - x_0 + x_2 - x_1 = v_1 t_1 + \frac{1}{2} a (v_1^2 - v_0^2)
\]

\[
= \left(95 \text{ m/s}, 0.75 \text{s}, \frac{1}{2} (-7.8 \text{ m/s}^2) (95^2 - 95^2)
\]

\[
= 20 + 44.6 = 64.6 \text{ m} = 65 \text{ m}
\]

Often the motion of an object is divided into segments, each with a different acceleration. When solving each problem, it is important to realize that the final velocity for one segment is the initial velocity for the next segment.
Equations of Kinematics

The operation manual of a passenger automobile states that the stopping distance is 50m when the brakes are fully applied at 96 km/h. What is the deceleration?

\[ a = \frac{1}{2} \left( \frac{v_f^2 - v_i^2}{x} \right) \]

Given:
- \( v_i = 96 \text{ km/h} \)
- \( v_f = 0 \)
- \( x = 50 \text{ m} \)

Find: \( a = ? \)

\[ v_f^2 = v_i^2 + 2ax \]

\[ a = \frac{1}{2} \left( \frac{27^2 - 96^2}{50} \right) = \frac{729 - 9216}{100} = \frac{-8487}{100} = -84.87 \text{ m/s}^2 \]

2) \[ x = x_0 + v_i t + \frac{1}{2} a t^2 \]

\[ a = \frac{v_f - v_i}{t} \]

Substitute \( t = \frac{v_f}{a} \) in (1)

\[ x = v_i \left( \frac{v_f}{a} \right) + \frac{a(v_f)^2}{2(a)} = -\frac{v_i^2}{a} + \frac{v_f^2}{2a} = -\frac{v_i^2}{a} \]

\[ a = \frac{v_i^2}{2x} = \frac{27^2}{2 \times 50} = \frac{729}{100} = 7.3 \text{ m/s}^2 \]

Direction is opposite to
Equations of Kinematics

Ch. 2 Problem 30
A speedboat starts from rest and accelerates at $+2.01\text{m/s}^2$ for 7.00s. At the end of this time the boat continues for an additional 6.00s with an acceleration of $+0.518\text{m/s}^2$. Following this, the boat accelerates at $-1.49\text{m/s}^2$ for 8.00s.

a) What is the velocity of the boat at $t = 21.0\text{s}$?

b) Find the total displacement of the boat.

---

a) The velocity at the end of the first (7.00s) period is

$$v_1 = v_0 + a_1t_1 = (2.01\text{m/s}^2)(7.00\text{s})$$

- at the end of the second period:

$$v_2 = v_1 + a_2t_2 = v_1 + (0.518\text{m/s}^2)(6.00\text{s})$$

- at the end of the third period:

$$v_3 = v_2 + a_3t_3 = v_2 + (-1.49\text{m/s}^2)(8.00\text{s}) = 5.26\text{m/s}$$

b) The displacement for the first time period is found from

$$x_1 = v_0t_1 + \frac{1}{2}a_1t_1^2$$

$$x_1 = \frac{1}{2}(2.01\text{m/s}^2)(7.00\text{s})^2 = 49.2\text{m}$$

Similarly, $x_2 = 93.7\text{m}$; $x_3 = 89.7\text{m}$

Total displacement:

$$x = x_1 + x_2 + x_3 = 233\text{m}$$

!!!. When the motion of an object is divided into segments, remember that the final velocity of one segment is the initial velocity for the next segment.
The motion of 2 objects may be interrelated so they share a common variable. The fact that the motions are interrelated is an important piece of information. In such cases, data for only two variables need be specified for each object.

Problem 33

Two soccer players start from rest 48m apart. They run directly toward each other, both players accelerating. The first player has an acceleration whose magnitude is 0.50m/s². The second player’s acceleration has a magnitude of 0.30m/s².

a) How much time passes before they collide?
b) At the instant they collide, how far has the first player run?
Sometimes there are 2 possible answers to a kinematics problem, each answers corresponding to a different situation.

**Example**
The spacecraft is traveling with a velocity of +3250m/s. Suddenly the retrorockets are fired and the spacecraft begins to slow down with an acceleration whose magnitude is 10.0m/s². What is the velocity of the spacecraft when the displacement of the craft is +215km relative to the point where the retrorockets began firing?

**Reasoning:**
Since the spacecraft is slowing down, \( \bar{a} \) has a direction opposite to the velocity

\[ \Rightarrow a = -10.0 \text{m/s}^2 \]

**Solution:**

\[ v^2 = v_0^2 + 2ax = (3250 \text{m/s})^2 + 2(-10.0 \text{m/s}^2)(215000 \text{m}) = 6.3 \times 10^6 \text{ m}^2/\text{s}^2 \]

\[ v = \pm\sqrt{6.3 \times 10^6 \text{ m}^2/\text{s}^2} = \pm2500 \text{ m/s} \]

Both of these answers correspond to the same displacement \( x = +215 \text{km} \), but each arises in a different part of the motion.
Galileo Galilei was an Italian scientist who formulated the basic law of falling bodies, which he verified by careful measurements.

In the absence of air resistance, he found that all bodies at the same location above the earth fall vertically with the same acceleration.
Freely Falling Bodies

• The effect of gravity causes objects to fall downward
• In the absence of air resistance, it is found that all bodies at the same location above the earth fall vertically with the same acceleration
• If the distance of the fall is small compared to the radius of the earth, the acceleration remains constant throughout the fall

![Diagram](image)

• Idealized motion, in which air resistance is neglected is known as free fall
• Since the acceleration is nearly constant in free fall, the equations of kinematics can be used
• Since the motion occurs in vertical or “y” direction we simply replace “x” with “y” in kinematics equations

\[
\begin{align*}
v &= v_o + at \\
y &= \frac{1}{2}(v_o + v)t \\
y &= v_o t + \frac{1}{2}at^2 \\
v^2 &= v_o^2 + 2ay \\
v &= v_o - gt \\
y &= \frac{1}{2}(v_o + v)t \\
y &= v_o t - \frac{1}{2}gt^2 \\
v^2 &= v_o^2 - 2gy
\end{align*}
\]
A Falling Stone. The Velocity of a Falling Stone

A stone is dropped from rest from the top of a tall building. After 3.00s of free fall

a) What is the displacement of the stone?

b) What is the velocity of the stone?

- Because of the acceleration due to gravity, the magnitude of the stone’s downward velocity increases by 9.80 m/s during each second of free fall \( v = v_0 + at \)
- Since the stone is moving downward in the negative direction the value for \( v \) should be negative \( v = v_0 + at = (-9.80 \text{ m/s}^2)(3.00 \text{ s}) = -29.4 \text{ m/s} \)
How High Does It Go?

A golf ball rebounds from the floor and travels straight upward with a speed of 5.0 m/s. To what max height does the ball rise?

\[ v = v_0 + at \]
\[ y = \frac{v^2 - v_0^2}{2a} = \frac{0 - (5.0 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = 1.3 \text{ m}. \]
How Long Is It in the Air?

An arrow is fired from ground level straight upward with an initial speed of 15 m/s. How long is the arrow in the air before it strikes the ground?

- During the time the arrow travels upward, gravity causes its speed to decrease to 0
- On the way down, gravity causes the arrow to regain the lost speed
- => the time for the arrow to go up is equal to the time to go down

\[
\begin{align*}
\text{up:} & \quad \theta = 0, \quad y(t) = y_0 + v_0 t - \frac{1}{2} g t^2 \\
\text{down:} & \quad v = v_0 + g t, \quad t = \frac{v - v_0}{g}
\end{align*}
\]

\[
\begin{align*}
\text{up:} & \quad t_{\text{up}} = \frac{v_0 - v_f}{g} = \frac{0 - (15 \text{ m/s})}{-9.8 \text{ m/s}^2} = 1.55 \text{ s} \\
\text{down:} & \quad t_{\text{down}} = \frac{(15 \text{ m/s}) - (0)}{-9.8 \text{ m/s}^2} = 1.55 \text{ s} \Rightarrow t_{\text{up}} + t_{\text{down}} = 3.10 \text{ s}
\end{align*}
\]