PH 202-1E Fall 2006

Electric Circuits

Lectures 8-11

Chapter 20
(Cutnell & Johnson, Physics 6th edition)
Currents and Ohm's Law

Under static conditions, there can exist no electric field inside a conductor.

- A charged capacitor
- A separate uncharged piece of copper wire

Both the capacitor and the wire are in electrostatic equilibrium.

We suddenly deposit opposite amounts of electric charge on the opposite ends of a long metallic conductor. The charges will generate electric fields along and inside the conductor. This $E$ directs the charges toward each other. When the charges meet, they cancel and $E = 0$, the conductor reaches equilibrium.

The approach to equilibrium is very rapid $\tau \approx 45 \mu s$.

The wire and the plate now form a single capacitor, which is not in electrostatic equilibrium.

We can continually supply more electric charge to the ends and keep the conductor in a permanent state of disequilibrium when we connect a copper wire to the terminals of a battery or an electric generator.

The electric charges will continually flow from one terminal to the other, forcing an electric current.
Electric current

A wire is connected between the two terminals of a generator. The electric charges are propelled from one end of the wire to the other due to $E$ existing along and within the wire.

If the conductor is a straight wire of constant thickness, then $E = \text{const}$.

The electric field causes the flow of charge or electric current, from one end of the wire to the other.

The electric current is defined as the charge $q$ flowing through some given place of the wire divided by the time $\Delta t$ it takes to pass the given place.

$$I = \frac{q}{\Delta t}$$

The current is the rate at which charge enters the wire at one end or the rate at which charge leaves at the other end.

The SI unit of current is the ampere (A), a flow of charge of one coulomb per second.

1 ampere = 1 A = 1 C/s
In metallic conductors, the charge carriers are electrons - a current is a flow of electrons. In electrolytes, the charge carriers are positive ions, negative ions, or both.

Convention
For the sake of mathematical uniformity, whenever we need to indicate the direction of the current along a conductor - we consider that the current has the direction of an assumed positive flow of charge. We pretend that the moving charges are always positive charges.

The transport of negative charge in one direction is equivalent to the transfer of positive charge in the opposite direction.
Problem. The electric current in the light bulb of a flashlight is 0.5 A. How much electric charge flows through the light bulb in one hour? How many electrons pass through the light bulb?

\[ I = 0.5 \, \text{A} = 0.5 \, \text{C/s} \]
\[ Q = It = (0.5 \, \text{C/s})(3600 \, \text{s}) = 1.8 \times 10^3 \, \text{C} \]
\[ n = \frac{Q}{e} = \frac{1.8 \times 10^3}{1.6 \times 10^{-19}} = 1.1 \times 10^{22} \, \text{electrons} \]

Problem. In a typical lightning stroke, the electric current is about 20,000 A and it lasts about 10^{-4} s. The direction of the current is from the ground to the cloud. What is the charge (magnitude and sign) that this stroke deposits on the ground?

\[ Q = It = (2 \times 10^4 \, \text{C/s})(10^{-4} \, \text{s}) = 2 \, \text{C} \]

Current is flow of "-" from ground to cloud, leaving behind a negative charge of 2 C.
Direct current (DC) circuits

The electric circuits installed in your automobile and the electric circuits installed in your home carry two different kinds of currents: direct currents (DC) and alternating currents (AC).

DC: - flows steadily along the wires
- remains constant except when it is switched on or off.

AC: - periodically reverses its direction of flow along the wire
- like a pendulum swinging back and forth, it oscillates sinusoidally from one direction (positive) to the opposite direction (negative).

To keep any current flowing in the wires of the circuit we must connect the ends of the wire to a "pump of electricity", a device that continuously supplies electric charges to one end of the wire and removes them from the other.

A steady pump: batteries, dry cells, fuel cells, solar cells, electric generators.
Electromotive Force

A steady, time-independent current flow through a simple circuit consisting of a single wire, connected to the terminals of a battery. The battery must do work on the charges in order to keep them moving around the circuit.

A positive charge at point P is pushed along the wire by E. The KE of the charge is constant as it is dissipated by friction within the wire. Its potential energy decreases change. The charge reaches the other end of the wire. Because the electric field does work on the charge, the electric potential steadily decreases.

To keep the current flowing, the battery must "pump" the charge from low potential terminals to the high potential terminals. This produces chemical energy. Electromotive force (EMF) characterizes the device as a source of electric potential energy.

The EMF of the battery delivers a charge as charge is often called the voltage of the source.

The EMF is often called the voltage of the source.
Mechanical analog of the battery-wire circuit

A channel, by means the water runs down the hill, returning to the pump is analogous to the wire connected between the terminals of the battery. The hydraulic pump is the role analogous to the role of the battery. It increases the gravitational potential energy of the water. The hydraulic pump is a source of gravitational potential energy. It produces this energy from an external supply of chemical mechanism or electrical energy.
Positive (+) terminal

Negative (−) terminal

Positive (+) terminal (raised button)

Negative (−) terminal (metallic bottom surface)
Problem

The electric starter motor in an automobile equipped with a 12-V battery draws a current of 80 A when in operation.

(a) Suppose it takes the starter motor 3.0 s to start the engine. What amount of electric energy has been withdrawn from the battery?

(b) The automobile is equipped with a generator that delivers 5.0 A to the battery when the engine is running. How long must the engine run so that the generator can restore the energy in the battery to its original level?

(a) The battery does 12.9 J of work on each coulomb that passes through. It is equal to the amount of electric energy that has been withdrawn.

\[ Q = I \cdot t = 80 \, \text{A} \times 3 \, \text{s} = 240 \, \text{C} \]

The total energy is 12.9 J x 240 C = \(2.9 \times 10^3\) J

(b) \[ U = V \cdot Q = V \cdot I_2 \cdot t_2 \]

\[ t_2 = \frac{U}{V \cdot I_2} = \frac{2.9 \times 10^3 \, \text{J}}{12 \, \text{V} \times 5 \, \text{A}} = 48 \, \text{s} \]
Resistance and Ohm's law

Consider the behavior of a current in a metallic conductor.

- The negative charge of the free electrons balances the positive charge of the Cu ions that make up the crystal lattice.
- The ions remain at rest and a current is simply a flow of the gas of electrons.
- When we apply the electric field - it pushes the gas of electrons along the wire.

The gas of electrons does not accelerate, but the individual electrons have a much higher speed 10^6 m/s - speed of random motion. The electron gradually drifts from left to right 10^14 collisions with ions/second.
The dissipated KE of the electrons remains in the crystal lattice in the form of thermal energy.

The bright glow of a light bulb is produced in this way.

The drift velocity that an electron attains in the electric field $E_d = E$.

The electric current is proportional to the average velocity of the electrons and $\Rightarrow$

$\Rightarrow I \propto V_d \propto E$

$I \propto A$ cross-sectional area of the wire

$E = \frac{A V}{e}$

$\Rightarrow I = \frac{A \frac{dU}{e}}{e}$

$p = \frac{I}{e \frac{dU}{e}}$ $p$ - constant of proportionality $p$ depends on the characteristics of the material of the wire.

$p$ - resistivity of the wire

$p = \frac{I}{A}$ resistance of the wire

$I = \frac{AV}{p}$ Ohm's Law
Ohm’s law

The current is proportional to the potential difference between the ends of the conductor.

\[ I = \frac{AV}{R} \]

- for a wire of uniform cross section

\[ R = \rho \frac{L}{A} \]

- for a wire of arbitrary shape Ohm’s law is also valid.

- Ohm’s Law is valid for metallic conductors and for many nonmetallic conductors where the current is carried by a flow of electrons.
- Valid for plasma, electrolytes
- Ohm’s law is not a general law of nature.
- In many materials Ohm’s Law fails when the current is large.
- In semiconductor materials it fails.
- Ohm’s law represents the electrical properties of some conducting materials.
Problem: A conducting wire of length 2.0 m is connected between the terminals of a 12-V battery. The resistance of the wire is 3.0 Ω. What is the electric current in the wire? What is the E-field?

\[ I = \frac{AV}{R} = \frac{12V}{3\Omega} = 4A \]

\[ E = \frac{AV}{L} = \frac{12V}{2m} = 6 \text{ V/m} \quad \text{or} \quad 6 \text{ N/C} \]

Problem: The resistance of a 150-W, 115-V light bulb is 0.732 Ω when the light bulb is at its operating temperature. What current passes through this light bulb when in operation? How many electrons per second does this amount?

\[ I = \frac{AV}{R} = \frac{115V}{0.732\Omega} = 158A \]

158 A = 158 C/s.

One electron is 1.6 \times 10^{-19} C.

Therefore, electrons per second = \[ \frac{158 \text{ C/s}}{1.6 \times 10^{-19} \text{ C}} = 9.88 \times 10^{20} \text{ electrons} \]
The resistivity of materials

\[ R = \sigma \frac{L}{A} \]

- \( R \) - resistance of a wire of unit length
- \( \sigma \) - resistivity

- we can calculate the resistance if \( \sigma \) is known
- we can measure the resistivity if \( R \) has been measured experimentally.

The unit of resistance is 1 volt/ampere:
This unit is called ohm (Ω)

\[ 1 \text{ ohm} = 1 \Omega = 1 \text{V/A} \]

The unit of resistivity is ohm-meters.

\[ \sigma = \frac{R \cdot A}{L} = \frac{\text{ohm} \times \text{m}}{\text{m}} = \text{ohm/m} \]

In ordinary metals, the resistivity increases slightly with temperature \( \Delta \rho = \rho_o \Delta T \)
At high temperatures, the atoms jump violently about their positions in the lattice, and they are more likely to disturb the motion of the electrons.

\[ \Delta R = \Delta R_o \Delta T \]

The increase in resistance is directly proportional to the increment in temperature.
Temperature coefficient of resistance

\[ \alpha_{\text{copper}} = 3.9 \times 10^{-3} \text{ (C}^{-1}) \]

\[ \Delta T \quad \text{in} \ C \]

The change of electrical resistance with temperature is exploited in the operation of the resistance thermometer.

A core of fine platinum wire acts as a sensor.

Calibration
- Ice-water mixture 0°C → \( R_0 \)
- Boiling water 100°C → \( R_{100} \)

Low temperatures
The resistivity of a metal is substantially less than at room temperature.
Superconductivity - the resistance vanishes completely at some critical temperature above absolute zero.

Resistivity of tin as a function of temperature

\[ \rho = 0 \text{ at } T = 3.72 \text{ K} \]
Problem: The electromagnet of a bell is constructed by winding copper wire around a cylindrical core, like thread on a spool. The diameter of the copper wire is 0.45 mm, the number of turns in the winding is 260, and the average radius of a turn is 5.0 mm. What is the resistance of the wire?

\[
\text{Length of wire} = \\
= 260 \text{ turns} \times 2\pi \left(5 \times 10^{-3} \text{ m/turn}\right) = 8.2 \text{ m}
\]

Area of wire = \(\pi R^2 = \pi \left(\frac{0.45 \times 10^{-3}}{2} \text{ m}\right)^2 = 1.6 \times 10^{-7} \text{ m}^2\)

Therefore, \(R = \rho \frac{L}{A} = \left(1.7 \times 10^{-8} \text{ m}\right) \frac{8.2 \text{ m}}{1.6 \times 10^{-7} \text{ m}^2} = 0.875\)
Problem To measure the resistivity of a metal, an experimenter takes a wire of this metal of diameter 0.500mm and length 110m and applies a potential difference of 12.0V to the ends. She finds that the resulting current is 3.75A. What is the resistivity?

\[
\text{Resistance } R = \frac{V}{I} = \frac{12.0V}{3.75A} = 3.2 \Omega
\]

\[
R = \rho \frac{L}{A} \Rightarrow \rho = \frac{RA}{L}
\]

\[
A = \pi r^2 = \pi (0.25 \times 10^{-3})^2 \text{m}^2 = 1.96 \times 10^{-7} \text{m}^2
\]

\[
L = 110 \text{m}
\]

Therefore, \[
\rho = (3.2 \Omega) \left( \frac{1.96 \times 10^{-7} \text{m}^2}{1.10 \text{m}} \right) = 5.71 \times 10^{-7} \Omega \text{m}
\]
Problem. The air conditioner in a home draws a current of 12A.

(a) Suppose that the pair of wires connecting the air conditioner to the fuse box are No. 10 copper wire with a diameter of 0.259 cm and a length of 25 m each. What is the potential drop along each wire? Suppose that the voltage delivered to the house is exactly 110V at the fuse box. What is the voltage delivered to the air conditioner?

(b) Some older homes are wired with No. 12 copper wire with a diameter of 0.205 cm. Repeat the calculation of part (a) for this wire.

\[ R = \pi \left( \frac{d}{2} \right)^2 \times \frac{L}{A} \]

\[ R = \pi \left( \frac{0.259 \times 10^{-2}}{2} \right)^2 \times \frac{25}{5.3 \times 10^{-6}} = 0.08 \Omega \]

\[ \Delta V = IR = 12 \times 0.08 = 0.96V \]

There are 2 segments of wire, one to, and one from the air conditioner. Each is responsible for a drop in voltage of 0.48V. Therefore, potential difference delivered = 110V - 2(0.48V) = 107.04V.
2) Resistance \( R = \frac{L}{A} n = \frac{L}{A^2} \)

where \( d \) is the diameter, and \( A \sim p \).

Therefore \( \Delta V \sim \frac{L}{A^2} \).

Therefore \( \Delta V = 0.97v \left( \frac{0.25g^2}{0.205^2} \right) = 15v \)

\[ \Rightarrow \text{potential difference to air conditioner} = 110 - (2 \times 1.5) v = 107v \]
Problem: What increase in temperature will increase the resistance of a nickel wire from 0.5Ω to 0.6Ω?

\[ \Delta R = 0.1 \Omega \]
\[ \text{Percentage increase } = \frac{0.1}{0.5} \times 100\% = 20\% \]
\[ \Delta R = \Delta R_0 \Delta T \]
\[ \Delta T = \frac{\Delta R / \rho}{\rho} \]
\[ \Delta T = \frac{0.6 - 0.5}{0.5} \times 6 \times 10^{-3} \text{ } ^\circ\text{C}^{-1} = 33^\circ\text{C} \]

Problem: Although aluminium has a somewhat higher resistivity than copper, it has the advantage of having a considerably lower density. Find the weight of a 100 m segment of aluminium cable 3cm in diameter. Compare this weight with that of a copper cable of the same length and the same resistance.

The densities of aluminium and of copper are 2700 kg/m^3 and 8950 kg/m^3, respectively.

Volume of Al = 100 m x \pi (0.015 m)^2 = 0.071 m^3
Mass of Al = Vol. x \rho = 0.071 m^3 x 2700 kg/m^3 = 191 kg.

\[ R_{\text{Al}} = \rho \frac{L}{A} = 2.8 \times 10^{-8} \frac{100}{(0.015 \text{ m})^2} = 4.0 \times 10^{-3} \Omega \]

Area of copper wire at same resistance = \[ A = \frac{\rho L}{\rho} = 4.3 \times 10^{-4} \text{ m}^2 \]

Volume of copper wire = \[ V = 100 \text{ m} \times 4.3 \times 10^{-4} \text{ m}^2 = 0.043 m^3 \]

Mass of Copper = \[ \text{Vol.} \times \rho = 0.043 m^3 \times 8950 \text{ kg/m}^3 \approx 390 \text{ kg} \]
Resistors: Resistors in combination

All conductors have resistance, and any resistive circuit element is called a resistor. This term is usually reserved for an element that is intended to restrict the current.

- the conducting wires in a common circuit have resistance, but it is extremely low.
- other elements in the circuit (toaster element, light bulbs, carbon resistors) are designed to regulate the current, and thus are labeled resistors.

Resistors used in the circuits of electronic devices are often made of a piece of graphite (pure carbon) connected between two terminals.

Carbon has a small resistivity

$$P_c = 3.5 \times 10^{-5} \Omega \cdot m$$ (compare with $$P_{Cu} = 1.7 \times 10^{7}$$).

In circuit diagrams, the symbol for resistor is a zigzag line, reminiscent of the path of an electron inside a conducting material.

Adjustable resistors - rheostats, potentiometers by moving the sliding contact you increase or decrease the length of the wire. $$R = \rho \frac{l}{A}$$
A resistance microphone

If the diaphragm is compressed, the granules come into better contact and $P$ decreases.

- Change of resistance results in a change in the current in the circuit, so the microphone transforms the sound signal into an electric signal.

The two simplest ways of connecting several resistors are in series and in parallel.

\[ I(R) = \Delta V = \Delta V_1 + \Delta V_2 = IR_1 + IR_2 \quad R = R_1 + R_2 \]

- Potential - work per unit charge. The net work done by electric field on a unit charge that moves through the first resistor and then through the second is the sum of the work done in the first and in the second.

- The currents in both resistors are the same.

- Any change that flows through the first resistor also flows through the second.
Two resistors connected in parallel:

The potential difference across each resistor is the same as the potential difference across the combined resistance.

\[ \frac{\Delta V}{R} = \Delta V \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \]

\[ \frac{\Delta V}{R} = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} \]

\[ R = R_1 + R_2 \]

\[ T = \frac{\Delta V}{R} \]

\[ T_1 = \frac{\Delta V}{R_1} \]

\[ T_2 = \frac{\Delta V}{R_2} \]

\[ \sum \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots \]

Series combination of resistances:

\[ P = \sum \left( \frac{1}{R_1} + \frac{1}{R_2} + \cdots \right) \]

Parallel combination of resistances:

\[ \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots \]
Problem: Three resistors 4 Ω, 6 Ω, and 8 Ω respectively, are connected in series. What is the resistance of this combination? If this combination is connected to a 12-V battery, what is the current?

\[ R = R_1 + R_2 + R_3 = (4 + 6 + 8) \Omega = 18 \Omega \]
\[ I = \frac{V}{R} = \frac{12 \text{ V}}{18 \Omega} = 0.67 \text{ A} \]

Problem: A brass wire and iron wire of equal diameters and of equal lengths are connected in parallel. Together they carry a current of 6.0 A. What is the current in each resistor?

\[ I_1 + I_2 = I_0 = 6.0 \text{ A} \]
\[ I_1 = \frac{I_0}{(1 + \frac{R_1}{R_2})} \]
\[ I_2 = \frac{I_0}{R_2} \]

where \( I_1, I_2 \) are currents through \( R_1, R_2 \) respectively.

\[ R_1, \text{ resistance of brass} \]
\[ R_2 = \text{resistance of iron} \]

\[ I_{\text{Iron}} = \frac{I_0}{1 + \frac{0.010}{10^{-5}}} = 6.0 \text{ A} / (1 + \frac{10^{-2}}{10^{-5}}) = 2.5 \text{ A} \]

\[ I_2 \text{ Brass} = I_0 - I_1 = 3.5 \text{ A} \]
Problem. Three resistors with resistances of 3.0Ω, 5.0Ω, and 8.0Ω, are connected in parallel. If this combination is connected to a 12.0V battery, what is the current through each resistor? What is the current through the combination?

Let $I_1$, $I_2$, $I_3$ be currents through $R_1$, $R_2$, $R_3$. Let $\Delta V$ be potential difference through all three.

Then $\Delta V = I_1 R_1 = I_2 R_2 = I_3 R_3$

$I_1 = \frac{\Delta V}{R_1} = \frac{12}{3} = 4.0A$

$I_2 = \frac{12}{5} = 2.4A$; $I_3 = \frac{12}{8} = 1.5A$

Total current

$I = I_1 + I_2 + I_3 = 7.9A$. 
Problem: Three resistors with $R_1 = 2.0 \Omega$, $R_2 = 4.0 \Omega$, and $R_3 = 6.0 \Omega$ are connected as shown in Fig.

(a) Find the net resistance of the combination.

(b) Find the current that passes through the combination if a potential difference of 8.0 V is applied to the terminals.

(c) Find the potential difference and the current for each individual resistor.

$$R_{eq} = R_1 + \frac{R_2 R_3}{R_2 + R_3} = 2 + \frac{4 \times 6}{10} = 4.0 \Omega$$

$$I = \frac{V}{R_{eq}} = \frac{8}{4.0} = 1.82 A$$

$$V_{R_1} = 3.64 V = IR_1 = 1.82 \times 2 = 3.64 V$$

$$V_{R_2} = V_{R_3} = 4.36 V; \quad (P=3.64)$$

$$I = \frac{V}{R_2} \Rightarrow I_{R_2} = 1.82 A; \quad I_{R_3} = 1.1 A; \quad I_{R_1} = 0.72 A$$

$$V_{AB} \quad V_{BC} = V_{AB} - V_{AC}$$
Internal resistance

• Generator – the internal resistance is the resistance of wires in generator
• In a battery – the internal resistance is due to the chemicals within the battery
• When external resistance is connected to the battery it is connected in series with the internal resistance

Problem. Given: \( r = 0.010 \, \Omega \); \( E = 12 \, V \); \( I = 100 \, A \). Find: Terminal voltage (TV) – actual voltage between the terminals of a battery

\[ Ir = 100 \times 0.010 = 1.0 \, V \]
\[ TV = 12 \, V - 1 \, V = 11 \, V \]

In order to keep a current flowing in a circuit, the force of EMF must do work

\[ \Delta W = \varepsilon \Delta Q \]

\[ \Rightarrow \frac{\Delta W}{\Delta t} = \varepsilon \frac{\Delta Q}{\Delta t} \]

The rate of work is the power; \( \frac{\Delta Q}{\Delta t} \) - current

Power delivered by source of EMF

\[ P = IE \]

P is large if it pumps a large current through a large potential difference.

P is positive if the current passes through the source in the forward direction.

The electric potential energy is carried to the resistors and is dissipated there.

If within a resistor the charge \( \Delta Q \) supplies potential change \( \Delta V \)

\[ \frac{\Delta U}{\Delta t} = \Delta V \frac{\Delta Q}{\Delta t} \]

Power dissipated in the resistor is

\[ P = (\Delta V I) \]

\[ P = I^2 R \] or \[ P = \frac{(\Delta V)^2}{R} \]
The conversion of electric energy into thermal energy in a resistor is called Joule heating.

Simple electric appliances merely convert electric energy into thermal energy by means of a heating element, consisting of a coiled wire having resistance greater than that of the wires connecting the device to the outlet.

- Incandescent light bulbs rely on Joule heating.
- Joule heating is used for a crude control of currents by means of fuses and circuit breakers.

Ordinary fuses. If the current is excessive, the metal ribbon melts and cuts off the current.

A circuit breaker. When heated, the bimetallic strip bends up and pushes the lever, which releases the spring loaded contact arm and breaks the circuit.
Problem. An electric water heater of resistance 8Ω draws 15A of current when connected to the voltage supply. What is the cost of operating the water heater for 4 hours if the electric company charges 8¢ per kilowatt-hour?

\[ P = I^2R = (15^2) \times 8 = 1800 \text{ W} \]

Energy = \( Pt = (1800 \text{ W})(4 \text{ hr}) = 7200 \text{ kW} \cdot \text{hr} \)

Cost = \( (7200 \text{ kW} \cdot \text{hr})(8\text{¢}/\text{kW} \cdot \text{hr}) = 58 \text{¢} \)

Problem

A small electric motor operating on 115V delivers 0.75 hp of mechanical power. Ignoring friction losses within the motor, what current does this motor require?

\[ 0.75 \text{ hp} = 0.75 \times 745.7 \text{ W} = 560 \text{ W} \]

\[ I = \frac{P}{\Delta V} = \frac{560 \text{ W}}{115 \text{ V}} = 4.9 \text{ A} \]
Problem. To heat 1 kg of water \((p=1000\text{ kg/m}^3)\) \(^{1}\)\(^{2}\) takes 4.18 J/°C. How long does it take to heat one liter of water by 50°C with an electric heating element of resistance 2.05Ω carrying a current of 32A? 

\[
P = I^2R = (32^2) \times 2 = 2 \times 10^3 W = 2 \times 10^3 \frac{W}{V}
\]

\[
Q = mcAT = (10^3 g) \times (4.18 \frac{J}{g} \cdot ^\circ C) \times 50^\circ C = 2 \times 10^5
\]

\[
t = \frac{Q}{P} = \frac{2 \times 10^5}{2 \times 10^3} = 100s
\]

Problem. The aluminum cable of a high-voltage transmission line carries a current of 600 A. The cable is 60 km long and it has a diameter of 2.5 cm. What is the power lost to Joule's heat in this cable?

\[
R = \frac{\rho L}{A} = \frac{(2.8 \times 10^{-5} \Omega \cdot \text{m})(6 \times 10^4 \text{ m})}{\pi (0.0125 \text{ m})^2} = 3.45 \Omega
\]

\[
P_{\text{loss}} = I^2R = (600 A)^2 \times 3.45 \Omega = 1.2 \times 10^6 W
\]

\[
\text{Power transmitted} = I\Delta V = 600 \times 240 \times 10^{-3} = 144 \times 10^3 W
\]

\[
\frac{P_{\text{loss}}}{P_{\text{trans}} = \frac{1.2 \times 10^6}{1.44 \times 10^6} \times 100\% \approx 1%}
\]
An electric clothes dryer operates on a voltage of 220V and draws a current of 20A. How long does the dryer operate take to dry a full load of clothes? The clothes weigh 6.0 kg when wet and 3.7 kg when dry. Assume that all the electric energy going into the dryer is used to evaporate water (the heat of evaporation is 539 kcal/kg).

Mass of water evaporated
\[ = 6.0 - 3.7 = 2.3 \text{ kg.} \]

Energy needed
\[ = 2.3 \times (539 \times 10^3) \times 4.185 \approx 5187165 \text{ erg} \]

Power
\[ = VI = 220V \times 20A = 4400 \text{ W} \]

Time taken
\[ = \frac{\text{Energy}}{\text{Power}} = \frac{5.18 \times 10^6}{4400} \approx 1180 \text{ s} = 19.6 \text{ min} \]
Alternating current (AC)

- The direction of the charge flow reverses periodically.
- In an ac circuit, ac generators serve the same purpose as a battery serves in a dc circuit. Give energy to the moving charge.
- Current also oscillates:
  \[ I = \left( \frac{V_o}{R} \right) \sin 2\pi ft = I_o \sin 2\pi ft \]
- Power:
  \[ P = IV = I_o V_o \sin^2 2\pi ft \]
- Average power:
  \[ \bar{P} = \frac{1}{2} I_o V_o \]
  - Average current & average voltage:
    \[ \bar{P} = \left( \frac{I_o}{\sqrt{2}} \right) \left( \frac{V_o}{\sqrt{2}} \right) = I_{rms} \cdot V_{rms} \]
  - Peak square current & voltage:
    \[ V_{rms} = \frac{V_o}{\sqrt{2}} \]
    \[ I_{rms} = \frac{I_o}{\sqrt{2}} \]

The voltage produced between the terminals of the generator fluctuates sinusoidally in time.

Heating element (thin wire of resistance \( R \))

\[ P_{rms} = \frac{1}{2} I_{rms} V_{rms} = \frac{1}{2} \cdot I_{rms} \cdot V_{rms} = \frac{V_{rms}^2}{2R} \]

\[ V = V_o \sin 2\pi ft \]

\[ f = \frac{2\pi}{2\pi ft} \]

\[ V_o = 170 \text{ volts} \]

\[ f = 60 \text{ Hz} \]
Single-loop circuits

The electric circuits in automobiles and in battery operated appliances or tools consist of one or several sources of EMF connected to one or several resistors.

EMF source is represented by a stack of parallel short and long lines (similar to the plates of a lead-acid battery)

1) Mark the long line as +
2) Mark the short line as -

Find $I = ?$

1) Apply Ohm's Law

$\Delta V$ across the resistor

$\Delta V = -IR$ (potential decreases across the resistor)

2) Apply Kirchhoff Rule

Around any closed loop in a circuit the sum of all the EMFs and all the potential changes across the resistors and other circuit elements must equal zero.

- EMF - positive if the current flows in the forward direction (from + to -)
- EMF - negative if the current flows in the backward direction (from - to +)

$E = -IR = 0$ => $E = IR$ => $I = \frac{E}{R}$
Problem. Four resistors, with $R_1 = 25 \Omega$, $R_2 = 15 \Omega$, $R_3 = 40 \Omega$, and $R_4 = 20 \Omega$, are connected to a 12 V battery as shown. (a) Find the combined resistance. (b) The current in each resistor.

4. Kirchhoff's rule for this single-loop circuit with resistances $R_1$, $R_2$, and $R_3$ in series with an emf of 12 V tells us:

$$E = IR_2 + IR - IP_1 = 0$$

$$I = \frac{E}{R_2 + R + P_1} = \frac{12}{15 \Omega + 13.3 \Omega + 25 \Omega} = \frac{12}{53.3} = 0.23 A$$

Current through resistors $R_1$, $R_2$, and $R_3$.

$$I_{R_3} = \frac{V_A - V_0}{R_3} = \frac{IR}{R_3} = \frac{0.23 \times 13.3}{40} = 0.08 A$$

$$I_{R_4} = \frac{V_A - V_0}{R_4} = \frac{IR}{R_4} = \frac{0.23 \times 13.3}{20} = 0.15 A$$
Single loop circuits and Kirchhoff’s rules. The statement just made, that around any closed loop the sum of all the emf’s and all the potential drops across resistors and other circuit elements must equal zero, is often referred to as Kirchhoff’s second rule of circuit analysis. The emf is taken to be positive if the current flows through the source in the forward direction. Fig. shows a possible closed-loop circuit.

Let us assume that the current flow in the circuit is clockwise. Then from Kirchhoff’s second rule, since the current I is the same everywhere in the circuit,

\[ e_1 + e_2 - e_3 - IR_1 - IR_2 - IR_3 = 0 \]
\[ e_1 + e_2 - e_3 = I(R_1 + R_2 + R_3) \]
\[ I = \frac{e_1 + e_2 - e_3}{R_1 + R_2 + R_3} \]

If our result for the current I is positive, our choice that the current flow is clockwise was correct. A negative value for I means that our choice was wrong; the direction of I is counterclockwise.
It is also possible to have multiloop circuits, in which currents can flow along several alternate paths. Such a circuit is shown in Fig. Unless we allow charge to build up in a circuit, the sum of currents entering and leaving any branch point in the circuit must equal zero. This is really a statement of charge conservation, equivalent to stating that the current is the same everywhere in a single-loop circuit, but it is usually referred to as Kirchhoff’s first rule of circuit analysis.

Kirchhoff’s rules allow us to solve for the currents flowing in the various loops of multiloop circuits. For example, we can solve for the currents in the circuit of Fig. as follows: first, label the three possible values of current $I_1$, $I_2$, and $I_3$, as shown in Fig., guessing at the most likely direction of positive current flow. If we are wrong, the sign of the current will turn out negative but the magnitude will be correct. Next apply Kirchhoff's first rule to one of the branch points, giving

$$I_1 + I_2 = I_3$$

Now apply Kirchhoff's second rule to any two closed loops:

$$\frac{\varepsilon_1 - I_1 R_1 - I_2 R_3 - I_1 R_4}{R_1} = 0$$
$$\frac{\varepsilon_2 - I_2 R_2 - I_3 R_3}{R_2} = 0$$

We have three equations and three unknowns, $I_1$, $I_2$, and $I_3$, so that we can determine the values of current, provided that we know the resistances $R_1$, $R_2$, $R_3$, and $R_4$ and the values of emf. No additional information would be obtained by applying the first rule to branch point $B_2$ in addition to $B_1$, or by applying the second rule to a different closed loop—for example, to the loop containing both sources of emf. The results would be the same. Kirchhoff’s rules provide a
When calculating the potential change across a resistor we must take the product of the resistance and the net current through the resistor. The net current is the algebraic sum of all the currents flowing through the resistor.
Problem. Consider the circuit shown in Fig.

Given that \( E_1 = 6.0 \text{V} \), \( E_2 = 10 \text{V} \), and \( R_1 = 2.0 \Omega \), what must be the value of the resistance \( R_2 \) if the current through this resistance is to be 2.0 A?

1) Variant

2) 2 loops

3) Apply Kirchhoff's Rule to the first loop

\[ E_1 - (I_1 - I_2)R_1 = 0 \quad (0) \]

for the second

\[ E_2 - I_2 R_2 - (I_2 - I_1)R_1 = 0 \quad (0) \]

Substitute the numeric values in (0) and (0)

\[ 6.0 - (I_1 - I_2)2.0 = 0 \]

\[ 10.0 - 2.0R_2 + (I_2 - I_1)2.0 = 0 \]

\[ 16 - 2.0R_2 = 0, \]

\[ R_2 = \frac{16}{2.0} = 8.0 \Omega \]
2. variant

\[ E_1 \quad R_1 \quad R_2 \quad E_2 \]

1) Label three possible values of current \( I_1, I_2, I_3 \) guessing at the most likely direction of positive current flow.

2) Apply Kirchhoff's first rule

\[ I_1 = I_3 + I_2 \]

3) Apply Kirchhoff's second rule to any two closed loops

\[ E_1 - I_3 R_1 = 0 \]

\[ E_2 - I_2 R_2 + I_3 R_1 = 0 \]

\[ E_1 + E_2 - I_2 R_2 = 0 \]

\[ I_2 = \frac{E_1 + E_2}{R_2} = \frac{16}{8} = 2 \, \text{amps} \]
Label 3 possible directions of current

2) Apply Kirchhoff's first rule

\[ I_2 = I_1 + I_3 \]

3) Apply Kirchhoff's second rule

\[ E_2 - I_2 R_2 - I_1 R_1 - E_1 = 0 \]
\[ E_1 + I_1 R_1 - I_3 R_3 = 0 \]
\[ E_2 - I_2 R_2 - \left(\frac{E_1 + I_1 R_1}{R_3}\right) R_3 - I_3 R_3 = 0 \]
\[ I_1 \left( R_2 R_3 + R_1 R_2 + R_2 R_3 \right) = \frac{E_1 R_1 - E_2 R_2}{R_3} \]

\[ I_1 = \frac{E_1 R_1 - E_2 R_2}{R_3} \]

Find \( I_1 \): \( I_{R_3} = ? \)

\( I_1 = \frac{12 \times 0.5 - 6.0 \times 0.2 - 6.0 \times 0.5}{0.5 \times 0.5 + 0.7 \times 0.2 + 0.5 \times 0.5} \approx 16.58 \text{A} \)

\( I_2 \approx 6.6 \text{A} \)
Problem

2 variants

\[ E_2 - I_1 R_2 - (I_1 - I_2) R_1 - E_1 = 0 \] (1)

\[ E_1 - (I_2 - I_1) R_1 - I_2 R_3 = 0 \] (2)

Flow (2) \[ E_1 - I_2 R_1 + I R_1 - I_2 P_3 = 0 \rightarrow I = \frac{I_2 (R_1 + R_3 - R_2)}{R_1} \]

Substituting (1) \[ E_2 - \left[ \frac{\frac{I_2 (R_1 + R_3 - R_2)}{R_1} - E_1 - I R_1 - I_2 R_3 + E_{2 R_2}}{R_2} \right] R_2 = 0 \]

\[ E_2 R_1 - I_2 R_2 R_3 - I_2 R_2 R_3 + E_{2 R_2} + E_{2 R_2} = 0 \]

\[ I_2 (R_1 R_2 + R_2 R_3 + R_1 R_3) = \frac{E_2 R_2}{R_1 R_2 + R_2 R_3 + R_1 R_3} \]

\[ I_2 = \frac{E_2 R_2}{R_1 R_2 + R_2 R_3 + R_1 R_3} = \frac{12 \times 0.25 + 6 \times 0.02}{0.25 \times 0.21 + 0.02 \times 0.05 + 0.25 \times 0.05} = 15.3 \text{A} \]

\[ I_1 = \frac{E_2 (R_1 + R_3) - E_1}{R_1} = \frac{15.3 (0.25 + 0.05) - 6}{0.25} = 21.9 \text{A} \]

\[ I_{I_1} = I_1 - I_2 = 21.9 - 15.3 = 6.6 \text{A} \]
Problem

\[ E_1 = 12.0 \text{V}, \quad E_2 = 8.0 \text{V} \]

\[ R_1 = 4.0 \Omega; \quad R_2 = 4.0 \Omega \]

\[ R_3 = 2.0 \Omega \]

Find \( I_1 \) and \( I_2 \) and \( I_3 \).

Currents in \( R_1, R_2, R_3 \):

\[ E_1 - (I_1 + I_2)R_1 - I_1R_2 = 0 \]
\[ E_1 - (I_1 + I_2)R_1 - E_2 - I_2R_3 = 0 \]

\[ 12 - (I_1 + I_2)4 - 4I_1 = 0 \]
\[ 12 - (I_1 + I_2)4 - 8 - 2I_2 = 0 \]

\[ 12 - 4I_1 - 4I_2 - 4I_1 = 0 \]
\[ 12 - 8I_1 - 4I_2 = 0 \]
\[ 4 - 4I_1 - 6I_2 = 0 \]
\[ 4 - 4I_1 - 8I_2 = 0 \]

\[ 2 - 2I_1 - 3I_2 = 0 \]

\[ 1 + 4I_2 = 0 \]

\[ I_3 = -0.5 \text{A} \]

\[ 3 - 2I_1 + 0.5 = 0 \]

\[ I_1 = \frac{3 + 0.5}{2} = 1.75 \text{A} \]

Current in \( R_1 \):

\[ I_1 + I_2 = 1.75 - 0.5 = 1.25 \text{A} \]

Current in \( R_2 \):

\[ I_2 = 1.75 \text{A} \]

Current in \( R_3 \):

\[ I_3 = -0.5 \text{A} \]
Electrical measurements. Ammeter and voltmeter.

The ammeter measures the electric current flowing into its terminals.

The voltmeter measures the potential difference applied to its terminals.

The ammeter has a low internal resistance and permits the passage of whatever current enters its terminals with little hindrance, whereas the voltmeter has a very large internal resistance and draws only an extremely small current.

An electric circuit

Correct connection of the voltmeter

Correct connection of the ammeter
Problem

A voltmeter of internal resistance 5000 ohms is connected across the poles of a battery of internal resistance 0.2 ohm. The voltmeter reads 1.4993 volts. What is the actual zero-current emf of the battery?

\[
\begin{align*}
\epsilon - 1.4993 - I \cdot 0.2 &= 0 \\
\epsilon &= 1.4993 + \frac{1.4993}{5000} = 1.49936\ \text{V}.
\end{align*}
\]
Capacitors in combination

Schematically, commonly used two conductors for capacitors are represented in a circuit diagram as two parallel lines with terminals attached to their middles.

\[
\text{Symbol for a capacitor}
\]

There are two simplest ways of wiring capacitors together: parallel in series.

\[
C_1 = C_2 = C_3 = \ldots
\]

Several capacitors connected in parallel

- Let us feed charge \( Q \) into this combination
  \( Q \) will be stored on \( C_1 \); \( Q_2 \) on \( C_2 \); \( Q_3 \) on \( C_3 \); \( Q_4 \) on \( C_4 \); \( Q_5 \) on \( C_5 \); \ldots
  \[
  Q = Q_1 + Q_2 + Q_3 + \ldots
  \]
- The upper plates of each capacitor are connected by conductor: upper plates have same potential \( V_u \)
- The lower plates are connected by conductor: a the lower plates have same potential \( V_l \)
- The same potential difference \( V_u - V_l = \Delta V \) exists for all the capacitors
- \( Q = Q_1 + Q_2 + \ldots = C_1 \Delta V + C_2 \Delta V + \ldots = (C_1 + C_2 + \ldots) \Delta V \)

\[
C = \frac{Q}{\Delta V} = C_1 + C_2 + C_3 + \ldots + C_n
\]
Capacitors connected in series

- fed charge into combination via the two terminals A, B
- the plates are forced to have the same charge
- the total potential difference $\Delta V = V_A - V_B = \frac{Q}{C}$ must be the sum of the individual potential differences

$$\frac{Q}{C} = \Delta V = \Delta V_1 + \Delta V_2 = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C = \frac{C_1 C_2}{C_1 + C_2}$$

For series combination of capacitors

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$

The net capacitance is always less than the individual capacitance connected in series. Ex. $C_1 = C_2 = C$

$$C_{net} = \frac{C_2}{2C} = \frac{C}{2}$$
Problem: Three capacitors with capacitances $C_1 = 5.0 \mu F$, $C_2 = 30 \mu F$, and $C_3 = 80 \mu F$ are connected as shown in Fig. Find the combined capacitance.

1) The combined branch of $C_1$ and $C_2$ has capacitance given by

$$C' = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1 C_2}{C_1 + C_2} \Rightarrow C' = \frac{5.0 \mu F \cdot 30 \mu F}{5.0 \mu F + 30 \mu F} = 18.75 \mu F$$

2) $C'$ is parallel with the other capacitor $C_3$

$$C = C' + C_3 = 18.75 \mu F + 80 \mu F = 99 \mu F$$
Problem: What is the total charge stored on the three capacitors connected to a 30-V battery as shown in Fig.

\[ C = C_1 + C_2 + C_3 = (30 + 15 + 10) \times 10^{-6} \text{ F} = 55 \times 10^{-6} \text{ F} \]

\[ Q = CV = \left(55 \times 10^{-6} \text{ F}\right)(30 \text{ V}) = 1.7 \times 10^{-3} \text{ C} \]
Problem. Six identical capacitors of capacitance $C$ are connected as shown in Fig. What is the total capacitance of the combination?

1) $C_{1} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C}$, $\Rightarrow C' = \frac{C}{2}$

for each pair in series

2) $C'' = C' + C' + C' = 3C' = \frac{3C}{2}$

for the 3 pairs in parallel
Problem: Six identical capacitors of capacitance $C$ are connected as shown in Fig. What is the total capacitance of the combination?

1) $C' = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} = \frac{3}{C}$ and $C' = \frac{C}{3}$ for each triplet in series.

2) $C'' = C' + C' = 2C' = \frac{2C}{3}$ for the pair of triplets in parallel.
The RC circuit

Kirchhoff’s Rules and the methods for solving circuits apply also to time-dependent currents. (Currents and cuts must not vary too quickly.)

The simple case of the time-dependent current in a circuit — resistor and capacitor connected in series and charged by a battery.

\[
\begin{align*}
E - IR - \frac{Q}{C} &= 0 \\
E - IR &= \frac{Q}{C}
\end{align*}
\]

1) \( t=0 \quad Q=0 \)
   \( \Rightarrow I = \frac{E}{R} \)

2) \( t = \text{time of charging} \quad I = 0 \)
   \( Q = CE \)

Characteristic time for RC circuit (time for the charging process):

\[
\tau = \frac{Q}{I} = \frac{CE}{E/R} = \frac{RC}{E}
\]
(a) Current in the RC circuit as a function of time.
\[ I_0 = \frac{E}{R}; \quad I_{\text{final}} (t=\infty) = 0 \]

(b) Charge on the capacitor as a function of time.
\[ Q = 0; \quad Q(t=\infty) = CE \]
\[ Q = CV (1 - \exp\left(-\frac{t}{RC}\right)) \]
At the characteristic time \( t = RC \), the charge on the capacitor reaches 63% of its final value.
Problem. A capacitor with \( C = 20 \mu F \) and a resistor with \( R = 100 \Omega \) are suddenly connected in series to a battery with \( \varepsilon = 6.0 \text{ V} \).

(a) What is the charge on the capacitor at \( t = 0 \)? At \( t = 0.002 \text{ s} \)?

(b) What is the final value of the charge?

(c) What is the rate of increase of the charge at \( t = 0 \)?

\[
Q = CV \left( 1 - e^{-\frac{t}{RC}} \right)
\]

\[
RC = 100 \times 20 \times 10^{-6} = 2 \times 10^{-3} \text{ s}
\]

(a) At \( t = 0 \), \( Q = 0 \)

At \( t = 0.002 \text{ s} \)

\[
Q = 20 \times 10^{-6} \times 6 \left( 1 - e^{-1} \right) = 764 \mu \text{C}
\]

(b) At \( t \to \infty \)

\[
Q = CV = 120 \mu \text{C}
\]

(c) At \( t = 0 \), the voltage drop is entirely across the resistor:

\[
\frac{\Delta Q}{\Delta t_{t=0}} = I = \frac{\varepsilon}{R} = \frac{6.0}{100} = 60 \times 10^{-3} \text{ A}
\]

\[
\varepsilon - \mathcal{E} - \frac{Q}{C} = 0
\]
3. A current of 3A flows through the wire shown in Fig. What will a voltmeter read when connected from:
   (a) A to B
   (b) A to C
   (c) A to D

(a) Point A is at the higher potential because current always flows "downhill" through a resistor. There is a potential drop of
   \[ IR = I(A)(6 \Omega) = 18V \] from A to B.
   Voltmeter will read \( V_B - V_A = 18V \) \( V_B - V_A < 0 \)

(b) In going from B to C one goes from the positive to the negative side of the battery; hence there is a potential drop of 8V from B to C. The drop adds to the drop of 18V from A to B found in (a).
   Voltmeter will read \( 26V \) from A to C.

(c) From C to D there is first a drop of
   \[ IR = I(A)(3 \Omega) = 9V \] through the resistor. Then because one goes from negative to the positive terminal of the 7V battery there is a 7V rise through the battery. The voltmeter will read
   \[ -18V - 8V - 9V + 7V = -28V \]
4. For the circuit shown in Fig. determine:
(a) the net resistance of the combination
(b) the current that passes through the combination
(c) the potential difference and the current for each individual resistor

\[ \begin{align*}
\text{A} & \quad 5\Omega \quad \text{B} \\
1 & \quad 30\text{V} \quad \text{1} \quad 3\Omega
\end{align*} \]

(a) The 3Ω and 7Ω resistances are in parallel.
Their joint resistance \( R_j \) is found from
\[ R_j = \frac{1}{\frac{1}{3} + \frac{1}{7}} = \frac{3 \times 7}{3 + 7} = \frac{21}{10} \Omega = 2.1\Omega \]
Thus the equivalent resistance of the entire circuit is
\[ R_{eq} = 2.1 + 5 = 7.1\Omega \]

(b) The current that passes through the combination is
\[ I = \frac{30}{7.1} = 4.2\text{A} \]

(c) \[ I_3 = 4.2\text{A} \]
\[ V_{AB} = I_3 R_3 = 4.2 \times 5 = 21\text{V} \]

\[ V_a - V_b = I_3 R_3 = I_2 R_1 = 4.2 \times 2.1 = 8.8\text{V} \]

\[ I_7 = \frac{8.8}{7} = 1.3\text{A} \]

\[ I_3 = \frac{8.8}{3} = 2.9\text{A} \]
5. (a) Find the current in the three resistors shown in Fig.
(b) Find the power delivered by the battery $E_i$.

\[ 0 \quad I_1 = I_2 + I_3 \]

\[ 0 \quad E - I_1 R_1 - E_2 - I_2 R_2 = 0 \]

\[ 0 \quad E - I_1 R_1 - E_2 - I_2 R_2 = 0 \]

\[ 0 \quad E - I_1 R_1 - E_2 - I_2 R_2 = 0 \]

\[ E_1 - I_1 R_1 - (I_1 + I_2) R_2 = 0 \]

\[ E_2 - (I_1 + I_2) R_2 - I_2 R_3 = 0 \]

\[ 12 - 9(I_1 + I_2) - 2 I_2 = 0 \]

\[ 2 - 2 I_1 - 3 (I_1 + I_2) = 0 \]

\[ 2 - 2 I_1 - 3 (I_1 + I_2) = 0 \]

\[ 12 - 2 I_2 - 9 (I_1 + I_2) = 0 \]

\[ 12 - 2 I_2 - 9 (I_1 + I_2) = 0 \]

\[ 2 - 5 I_1 - 3 I_2 = 0 \]

\[ I_1 = \frac{2 - \sqrt{52}}{3} \]

\[ I_2 = \frac{2 + \sqrt{52}}{3} \]

\[ 14 + 28 I_1 = 0 \]

\[ I_1 = -0.5 A \]

\[ I_2 = \frac{2 - \sqrt{52}}{3} = \frac{2 + 2 \sqrt{5}}{3} \]

\[ I_{R1} = I_1 = -0.5 A \]

\[ I_{R2} = I_1 + I_2 = -0.5 + 1.5 = 1.0 A \]

\[ I_{R3} = I_2 = 1.5 A \]

\[ P = E_1 I_1 = 6 \times 0.5 = 3 \text{ W} \]