PH 202-1E Fall 2006

Electric Forces and Electric Fields

Lectures 1-4

Chapter 18
(Cutnell & Johnson, Physics 6th edition)
Electric Force and Electric Charge

• Qualitatively, a force is any push or pull exerted on a body.
• Force is a vector quantity
• In the metric system of units, the unit of force is the Newton (N)

\[ 1 \text{ Newton} = 1 \text{ N} = 1 \text{ kg} \times \text{m/s}^2 \]

• Newton’s Second Law
  An external force acting on a body gives it an acceleration that is in the direction of the force and has a magnitude directly proportional to the magnitude of the force and inversely proportional to the mass of the body.
  \[ a = \frac{F}{m} \]
  \[ ma = F \]

• Newton’s Third Law
  Any action has an equal in magnitude and opposite reaction.
• Superposition of Forces
  If several forces act simultaneously on a body, then the acceleration is the same as that produced by a single force \( F_{\text{net}} \) given by
  \[ F_{\text{net}} = F_1 + F_2 + F_3 + \ldots + F_n \]
Kinds of Forces

• The gravitational force is a mutual attraction between all masses
  \[ F = \frac{G M m}{r^2} \]
  \[ G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 \]

• The electromagnetic force is an attraction or repulsion between electric charges.
  The electric forces exerted under the condition that the charged particles exerting these forces are at rest or moving very slowly are called **electrostatic forces**.

  When the particles are moving there arises a magnetic force, which depends on the velocities of the particles.

  The combined electrostatic and magnetic forces are called electromagnetic forces.

• Of all the forces, the electric force plays the largest role in our lives.
  • The forces that hold the parts of an atom together, that bind atoms in molecules and hold these building blocks together in large-scale structures such as a rock, a tree, a human body, are electric forces.
The origin of electricity. Structure of an atom.

Structure of an atom of Neon

Ordinary matter consists of atoms, each with a nucleus surrounded by electrons.

Electron

Nucleus

10 electrons are moving around the nucleus and are confined to a roughly spherical region \( \approx 10^{-10} \) m across.

Nucleus of neon is made of 10 protons and 10 neutrons packed very tightly together, confined to a spherical region of \( \approx 6 \times 10^{-15} \) m across.

The atom resembles the solar system

Nucleus – Sun
Electrons – Planets

In the atom, the force that holds an electron near the nucleus is the electric force of attraction between the electron and the protons in the nucleus.

Electric force resembles Gravitation

\[ F_e \sim \frac{1}{r^2} \]

\[ F_{ep}^e/F_{ep}^g = 2 \times 10^{39} \]

\( 10^{-10} \) m
Differences between the gravitational force and the electric force

1) Magnitude
2) Gravitational force between two particles is always attractive

Particles that exert electric forces are said to have an electric charge.

Electric charge is thought of as the source of electric force, just as mass is the source of gravitational force.
For the mathematical formulation of the law of electric force, we assign a positive charge to the proton and a negative charge of equal magnitude to the electron.

<table>
<thead>
<tr>
<th>Particle</th>
<th>Charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proton, p</td>
<td>+ e</td>
</tr>
<tr>
<td>Electron, e</td>
<td>- e</td>
</tr>
<tr>
<td>Neutron, n</td>
<td>0</td>
</tr>
</tbody>
</table>

In the metric system of units the electric charge is measured in **Coulombs (C)**

- $e = 1.6 \times 10^{-19}$ C for proton
- $-e = -1.6 \times 10^{-19}$ C for electron

The net electric charge of a body containing some number of electrons and protons is the algebraic sum of the electron and proton charges.

If $N_e = N_p$ => the atom is electrically neutral

If $N_e \neq N_p$ => atom miss electrons or has extra electrons => **ION**
The electrical forces between two neutral atoms tend to cancel

• Each electron in one atom is attracted by the protons in the nucleus of the other atom and simultaneously it is repelled by the equal number of electrons of that atom.

• Likewise, the electric forces between two neutral macroscopic bodies separated by some distance tend to cancel

• The cancellation of the electric forces between neutral macroscopic bodies explains why we do not see large electric attractions or repulsions between the macroscopic bodies, even though the electric forces between individual e and p are much stronger than the gravitational forces.
Problem.

A coulomb is a fairly large amount of charge. In fact, it is very difficult to assemble that amount of charge in an electrostatic arrangement without a breakdown or discharge.

a) Calculate the number of electrons associated with a charge of \(-1\text{C}\).

b) Determine the mass of this number of electrons.

Solution.

a) \(n = \frac{-1\text{C}}{-1.6 \times 10^{-19} \text{C/electron}} = 6.2 \times 10^{18} \text{ electron}\)

b) \(m = (1.67 \times 10^{-31} \text{ kg/electron}) \times (6.2 \times 10^{18} \text{ electron}) = 1 \times 10^{-12} \text{ kg}\)

Problem.

The electric charge in one mole of protons is called Faraday’s constant. What is its numerical value?

Solution.

\[Q = N_A \times e = (6.02 \times 10^{23}) \times (1.6 \times 10^{-19}\text{C}) = 9.632 \times 10^4\text{C}\]
Problem.
What is the number of electrons and protons in a human body of mass 73 kg? The chemical composition of the body is roughly 70% oxygen, 20% carbon and 10% hydrogen (by mass).

Solution.
In the human body there are:

(i) 51.1 kg O \quad (73kg \times 0.7 = 51.1kg)
(ii) 14.6 kg C \quad (73kg \times 0.2 = 14.6kg)
(iii) 7.3 kg H \quad (73kg \times 0.1 = 7.3kg)

We divide by atomic mass each element to find how many moles there are.

Number of moles are:

(i) \quad 51.1 / 0.016 = 3194 \text{ mol O}
(ii) \quad 14.6 / 0.012 = 1217 \text{ mol C}
(iii) \quad 7.3 / 0.001 = 7300 \text{ mol H}

Each mol contains \(6.02 \times 10^{23}\) atoms. Each atom has a number of electrons(protons) = atomic number => total amount of electrons (protons) is:

\[(6.02 \times 10^{23}) \times [(3194 \times 8) + (1217 \times 6) + (7300 \times 1)] = 2.4 \times 10^{28} \text{ electrons(protons)}\]
Charge quantization and charge conservation

• All the known particles have charges that are some integer multiple of the fundamental charge. (Charge quantization)

  The charges are always

  \[0 ; \pm e ; \pm 2e ; \pm 3e ; \text{etc.}\]

• Why no other charges exist is a mystery for which classical physics offers no explanation.

• Since charges exist in discrete packets, we may say that charge is quantized and the fundamental charge “e” is called the quantum of charge.

• In a description of the charge distribution on macroscopic bodies, the discrete nature of charge can often be ignored and it is sufficient to treat the charge as a continuous “fluid” with a charge density \((C/m^3)\), that varies more or less smoothly over the volume of the charged body.
Charge Conservation

• The electric charge is a conserved quantity:

In any reaction involving charged particles, the total charges before and after the reaction are always the same. No reaction that creates or destroys net electric charge has ever been discovered.

Conservation of charge in chemical reactions in a lead–acid automobile battery.

The reaction releases electrons at the lead plate, electrons are absorbed at the lead dioxide plate.

Plates of lead and lead dioxide are immersed in an electrolytic solution of sulfuric acid.

Lead Plate: \[ \text{Pb} + \text{SO}_4^{2-} \rightarrow \text{PbSO}_4 + 2[\text{electron}] \]
Charges: \[ 0 + (-2e) \rightarrow 0 + (-2e) \]

Lead-dioxide Plate: \[ \text{PbO}_2 + 4\text{H}^+ + \text{SO}_4^{2-} + 2[\text{el}] \rightarrow \text{PbSO}_4 + 2\text{H}_2\text{O} \]
Charges: \[ 0 + 4e + (-2e) + (-2e) \rightarrow 0 + 0 \]
Problem.

Consider the following hypothetical reactions involving the collision between a high energy proton (from an accelerator) and a stationary proton (in the nucleus of a hydrogen atom serving as a target).

Where

\[ p = \text{proton} \]
\[ n = \text{neutron} \]
\[ \pi^+ = \text{positively charged pion (+e)} \]
\[ \pi^0 = \text{neutral pion (e)} \]
\[ \pi^- = \text{negatively charged pion (-2e)} \]

1) \( p + p \quad n + n + \pi^+ \)  
2) \( p + p \quad n + p + \pi^0 \)  
3) \( p + p \quad n + p + \pi^+ \)  
4) \( p + p \quad p + p + \pi^0 + \pi^0 \)  
5) \( p + p \quad n + p + \pi^0 + \pi^- \)

Which of these reactions are impossible, because they violate the conservation of charge?

1) \( e + e \quad 0 + 0 + e \)  \( \text{Charge is not conserved, reaction is impossible} \)
2) \( e + e \quad 0 + e + 0 \)  \( \text{Charge is not conserved, reaction is impossible} \)
3) \( e + e \quad 0 + e + e \)  \( \text{Charge is conserved} \)
4) \( e + e \quad e + e + 0 + 0 \)  \( \text{Charge is conserved} \)
5) \( e + e \quad 0 + e + 0 + (-e) \)  \( \text{Charge is not conserved, reaction is impossible} \)
Conductors and Insulators. Frictional Electricity

**Conductor.** A conductor – such as copper, aluminum, or iron – is a material that permits the motion of electric charges through its volume.

**Insulator or Dielectric.** Materials that do not permit the motion of charges through them – insulators or dielectrics (glass, inorganic crystals, rubber, nylon, wood,…).

The distinction between conductors and nonconductors arises from the relative mobility of charge within the material.

**Metals.** The atoms of metals hold their own outer electrons weekly and so a bulk sample contains a tremendous number of free electrons, roughly one per atom. The inner electrons are bound to the nucleus. The free electrons (outer) wander through the entire volume of the metal and experience a restraining force only due to the interaction with the surface.

\[
\begin{array}{cccccc}
  e & e & e & e & e \\
  + & + & + & + & + \\
  e & e & e & e & e \\
  + & + & + & + & + \\
  e & e & e & e & e \\
  + & + & + & + & + \\
\end{array}
\]

Al #13 has 13 electrons
The electrons are held inside the metal in much the same way as particles of a gas are held inside a container.

=>

Electrons in metals form free electron gas

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**Conductors**

- Metals
- Electrolytes or liquid conductors containing ions of impurity.
  - Solution of salt in water
    - Na\(^+\), Cl\(^-\)
- Ionized gases
  - Plasma
  - Gases containing a mixture of ions and free electrons

**Ordinary gases are insulators.**

Ionization of a gas occurs whenever the gas molecules are subjected to large electric forces, that produce a sudden catastrophic ionization of the gas.
The ions of dielectric crystals hold their electrons strongly and so a sample doesn’t contain free electrons. Free electrons can appear only when we ionize our ions and provide electrons with KE > Potential energy of attraction to the ions.

This process can be described with the help of an energy level diagram.

Conduction band

Forbidden band

3 ÷ 15 eV

Valence band

Insulator

Metal (conductor)

1 eV = 1.6 x 10^{-19} J

Insulator or Dielectric. Energy level diagram.

\[
\begin{array}{c|c|c|c|c|c}
\text{Na}^+ & \text{Cl}^- & + & - & + & - \\
- & + & - & + & - & + \\
+ & - & + & - & + & - \\
- & + & - & + & - & + \\
\end{array}
\]
Direct transfer of electrons

Charging a sphere of pith positively by direct transfer of electrons from ball to rod. This leaves the ball positively charged, and it is immediately repelled from the rod.
Frictional electricity. Charging by rubbing.

Electrons are transferred in contact from the asbestos to the glass, and from the glass to the silk.
Charging by induction

(a) Negative rod far away from metallic sphere

(b) Negative rod brought close to, but not touching, metal sphere

(c) Grounding the metal sphere

(d) The sphere is charged positively
No matter what the shape of the conductor, excess charge always resides on its outer surface.

The distribution of charge placed on the surfaces of a conductor and a nonconductor.

Charge tends to bunch up on the pointed regions of a conductor.

The distribution of charge on the surfaces of identical conductors. The charge divides evenly on the two spheres.

Electroscopes homemade and otherwise. No longer used in the laboratory, the electroscope is now primarily a teaching device. To make one, remove the thin aluminum foil from the wrapper on a stick of gum. Hang it on a thick wire, lower it into a bottle, and seal the bottle with wax or clay.
Coulomb’s Law

Coulomb investigated the repulsion between small balls charged by rubbing.

1. The magnitude of the electric force that a particle exerts on another particle is directly proportional to the product of their charges and inversely proportional to the square of the distance between them.
2. The direction of the force is along the line from one particle to the other.
3. Like charges repel, and unlike charges attract.

\[ F = F' = \text{[const]} \times \frac{(q' \times q)}{r^2} \]

Positive value \( F \) – repulsive
Negative value \( F \) – attractive

In SI units, the constant of proportionality:

\[ [\text{const}] = 8.99 \times 10^9 \text{ (N \cdot m^2)/c^2} \]

\[ [\text{const}] = \frac{1}{4\pi \varepsilon_0} \]

\[ \varepsilon_0 = 8.85 \times 10^{-12} \text{ c^2/(N \cdot m^2)} \]

Permittivity constant
F = \frac{1}{(4\pi \varepsilon_0)} \left( q' \cdot q \right) / r^2

Assumption: Coulomb’s Law applies to particles – electrons and protons – and also to any small charged bodies, provided that the sizes of those bodies are much smaller than the distance between them – point charges.

F = \frac{1}{(4\pi \varepsilon_0)} \left( q' \cdot q \right) / r^2 \quad F = G \frac{(Mm)}{r^2}

→ \frac{1}{(4\pi \varepsilon_0)} \text{ is analogous to } G

→ q \quad \text{is analogous to } m

→ r \quad \text{is analogous to } r

**Definition of Coulomb**

The coulomb is defined in terms of a standard electric current:

One coulomb is the amount of electric charge that a current of one ampere delivers in one second.
**Problem.** The electric force of attraction between an electron and a proton separated by a distance of 0.53x10^{-10} m is 8.2x10^{-8} N. What is the force if the separation is twice as large? Three times as large. Four times as large?

\[
F = \frac{1}{(4\pi \varepsilon_0)} \frac{(q_1 \cdot q_2)}{r^2} = 8.2 \times 10^{-8} \text{ N}
\]

\[
F' = \frac{1}{(4\pi \varepsilon_0)} \frac{(q_1 \cdot q_2)}{(r')^2} = ?
\]

\[
r' = 2r: \quad F' = \frac{1}{(4\pi \varepsilon_0)} \frac{(q_1 \cdot q_2)}{(r')^2} = 1/(4\pi \varepsilon_0) \frac{(q_1 \cdot q_2)}{4r^2} = \frac{1}{4} F =
\]

\[
= 2.05 \times 10^{-8} \text{ N}
\]

\[
r' = 3r: \quad F' = \frac{1}{(4\pi \varepsilon_0)} \frac{(q_1 \cdot q_2)}{(r')^2} = 1/(4\pi \varepsilon_0) \frac{(q_1 \cdot q_2)}{9r^2} = \frac{1}{9} F =
\]

\[
= 0.91 \times 10^{-8} \text{ N}
\]

\[
r' = 4r: \quad F' = 1/16 F = 0.51 \times 10^{-8} \text{ N}
\]
Problem.

Suppose that two grains of dust of equal masses each have a single electron charge. What must be the masses of the grains if their gravitational attraction is to balance their electric repulsion?

\[ F_{\text{grav}} = F_{\text{elec}} ; \quad G \frac{m^2}{r^2} = \frac{1}{4\pi \varepsilon_0} \frac{e^2}{r^2} \]

\[
\begin{align*}
m &= e \sqrt{\frac{1}{G 4\pi \varepsilon_0}} \quad = (1.6 \times 10^{-19} \text{ C}) \sqrt{\frac{9 \times 10^9 \text{ Nm}^2/\text{c}^2}{6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2}} \\
&= (1.6 \times 10^{-19} \text{ C}) \times (1.16 \times 10^{10} \text{ kg/c}) = 2 \times 10^{-9} \text{ kg}
\end{align*}
\]

Problem.

Suppose that the two protons in the nucleus of a helium atom are at a distance of 2 x 10^{-15} m from each other. What is the magnitude of the electric force of repulsion that they exert on each other? What would be the acceleration of each if this were the only force acting on them? Treat the protons as point particles.

\[
F = \frac{1}{4\pi \varepsilon_0} \frac{q_1 \cdot q_2}{r^2} = \frac{(9 \times 10^9 \text{ Nm}^2/\text{c}^2)}{(2 \times 10^{-15})^2 \text{ m}^2} = 58 \text{ N}
\]

\[
a = \frac{F}{m} = \frac{58 \text{ N}}{1.67 \times 10^{-27} \text{ kg}} = 3.4 \times 10^{28} \text{ m/s}^2
\]
Problem. The electric charge flowing through an ordinary 115-Volt, 150 Watt light bulb is 1.3 C/s. How many electrons/second does this amount to?

1 electron has $1.6 \times 10^{-19}$ C. Therefore

$$1.3 \, \text{C/s} = \left(\frac{1.3 \, \text{C}}{1.6 \times 10^{-19} \, \text{C/electron}}\right) / \text{s} =$$

$$= 8.125 \times 10^{18} \, \text{electrons/s}$$

Problem: A moveable charge of $2 \times 10^{-6}$ C is at the point $x = 2$ m, $y = 0$ on the x-axis. A second moveable charge of $-3 \times 10^{-6}$ C is at the point $x = 0$, $y = -3$ m on the y-axis.

1) What is the electric force that the first charge exerts on the second $F_{21}$? 2) $F_{21} = ?$ Express the answer as a vector with x and y components.

$$F_{21} = -F_{21}$$

$$F_{21} = \frac{1}{4 \pi \varepsilon_0} \frac{q_1 q_2}{r^2} = \frac{9 \times 10^9 \, \text{Nm}^2/\text{C}^2}{x (2 \times 10^{-6} \, \text{C})(3 \times 10^{-6} \, \text{C})} = 4.2 \times 10^{-5} \, \text{N}$$

$$\tan \theta = \frac{3}{2} ; \quad \theta = \arctan \left( \frac{3}{2} \right) = 53.7^\circ$$

$$(F_{21})_x = F \sin \theta = 2.3 \times 10^{-3} \, \text{N}; \quad (F_{21})_y = -F \cos \theta = -3.5 \times 10^{-3} \, \text{N}$$

$$(F_{21})_x = -(F_{21})_x = -2.3 \times 10^{-3} \, \text{N}; \quad (F_{21})_y = -(F_{21})_y = 3.5 \times 10^{-3} \, \text{N}$$
Problem. Under the influence of the electric force of attraction, the electron in a hydrogen atom orbits around the proton on a circle of radius $0.53 \times 10^{-10}$ m. What is the orbital speed? What is the orbital period?

Let "v" be the velocity of electron.
To remain in circular orbit, we need $m \frac{v^2}{r} = \frac{F}{c}$; $\frac{v^2}{r} = \frac{F}{m}$.

$\Rightarrow \frac{-mv^2}{r} = -F$;

$\Rightarrow v = \sqrt{\frac{Fr}{m}} = \sqrt{(8.2 \times 10^{-22} \text{ N})(0.53 \times 10^{-10} \text{ m})}$

$= 2.2 \times 10^6 \text{ m/s}$

Orbital period $T = \frac{2\pi r}{v} = \frac{2\pi (0.53 \times 10^{-10} \text{ m})}{2.2 \times 10^{-6} \text{ m/s}} = 6.5 \times 10^{-5}$
Problem. Two Styrofoam balls each of mass 4 g, are hung from a common point in the ceiling by silk threads 1 m long. After being given identical charges, the balls repel each other and hang so that each thread makes an angle of 15° with the vertical. Find the charge given to each Styrofoam ball. The acceleration of gravity g equals 9.8 m/s².

\[ r = 0.52 \text{ m} \]

1) In a problem like this, carefully draw diagrams especially "free-body" diagrams are most important.

2) The electric force of repulsion
\[ F_e = \left( \frac{1}{4\pi\varepsilon_0} \frac{q^2}{r^2} \right)N, \quad \text{where} \quad r = 2L \times \sin 15° = 0.52 \text{ m} \]

3) Equilibrium condition
\[ F_{ex} = 0; \quad F_{ey} = 0 \]
\[ F_x = T \sin 15° = 0 \quad (i) \]
\[ T \cos 15° - mg = 0 \quad (ii) \]
\[ \frac{(i)}{(ii)} \Rightarrow \frac{F_e}{mg} = \frac{T \sin 15°}{T \cos 15°} = \tan 15° \]
\[ F_e = mg \tan 15° = (4 \times 10^{-3})(9.8) \text{ N} \]
\[ F_e = 1.05 \times 10^{-2} \text{ N} \]

4) We already know that \[ F_e = \frac{1}{4\pi\varepsilon_0} \left( \frac{q^2}{r^2} \right) \] where solving for \( q \), \[ q = \sqrt{\frac{F_e \cdot 4\pi\varepsilon_0 r^2}{2}} = 5.5 \times 10^{-7} \text{ C} \]
The Superposition of Electric Forces

If several point charges $q_1, q_2, q_3, ...$ simultaneously exert electric forces on the charge $q$, then the net force on $q$ is obtained by taking the vector sum of the individual forces.

$$ F = F_1 + F_2 + F_3 + ... $$

Principle of Superposition of Electric Forces.

The force contributed by each charge is independent of the presence of the other charges. Charge $q_2$ doesn't affect the interaction of $q_1$ with $q$. 
Problem (superposition principle). Figure shows the arrangement of nuclear charges (positive) in HCl molecule. The magnitude of these charges are e and 17e, respectively, and the distance between them is $1.28 \times 10^{-10}$ m. What is the net electric force that these charges exert on an electron placed $0.5 \times 10^{-10}$ m above the H nucleus?

\[ F_x = F_{1x} + F_{2x} = -2.0 \times 10^{-7} \text{ N}; \quad F_y = F_{1y} + F_{2y} = -9.2 \times 10^{-8} \text{ N} - 7.6 \times 10^{-8} \text{ N} = -1.7 \times 10^{-7} \text{ N} \]
The Electric Field

Action at a distance - particle exerts a direct gravitational or electric force on another particle even though these particles are not touching.

Field acts as a mediator of the force. A gravitating or electrically charged body generates a gravitational or electric field which is distributed over the empty space around the body.

In contact with another body it pushes or pulls conveying forces from one body to another through action by contact.

Charges exert forces on one another by means of disturbances that they generate in the space surrounding them. These disturbances are called Electric fields.
Electric interaction between charges is action at a distance: a charge $q'$ generates an electric field which permeates the surrounding space and exerts a force on any other charge that it touches.

Field is the fifth state of matter. Solids, liquids, gases, plasma, fields. Fields possess energy and momentum. The conceptual scheme of action by contact provided by electric field.

Charge $q'$ - electric field of charge $q' ightarrow$ force on charge $q$.

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q'}{r^2}$$

$$F = \frac{1}{4\pi\varepsilon_0} \frac{q \cdot q'}{r^2} = q' E$$

$E$ is a vector. Direction: $E$ is directed radially outward if $q'$ is positive, and radially inward if $q'$ is negative.

$$F = q' E$$
General definition of electric field

The electric field \( \mathbf{E} \) is equal to the electric force \( \mathbf{F} \) divided by the magnitude \( q \) of the charge.

\[
\mathbf{E} = \frac{\mathbf{F}}{q}
\]

The electric field is the force per unit charge.

The unit of Electric field

newton/coulomb (N/C)

Superposition Principle for Electric Field

The net electric field generated by any distribution of point charges with specified positions can be calculated by forming the vector sum of the individual electric fields of the point charges.
Problem (Electric field). To measure the magnitude of the horizontal electric field, an experimenter attaches a small charged cork ball to a string and suspends this device in the electric field. The electric force pushes the cork ball to one side, and the ball attains equilibrium when the string makes an angle of 35° with the vertical. The mass of the ball is $3 \times 10^{-5}$ kg, and the charge on the ball is $4 \times 10^{-7}$ C. What is the magnitude of the electric field?

\[
T \cos \theta - mg = 0 \quad (\text{1)} \\
T \sin \theta - qE = 0 \quad (\text{2)}
\]

\[
tan \theta = \frac{qE}{mg} = \frac{qE}{(3 \times 10^{-5} \text{ kg})(9.8 \text{ m/s}^2)} = \frac{4 \times 10^{-7} \text{ C}}{mg}
\]

\[
E = \frac{mg \tan \theta}{q} = \frac{(3 \times 10^{-5} \text{ kg})(9.8 \text{ m/s}^2) \cdot \tan 35°}{(4 \times 10^{-7} \text{ C})} = 5.1 \times 10^2 \text{ N/C}
\]
Problem: An electron moving through an electric field is observed to have an acceleration of \(10^{-16}\) m/s\(^2\) in the \(x\) direction. What must be the magnitude and the direction of the electric field that produces this acceleration?

\[
\begin{align*}
F &= qE \\
\frac{F}{m} &= \vec{a} = (a, 0) \\
\vec{a} &= \frac{E}{m} = \frac{qE}{m} \\
E &= \frac{ma}{q} = \left(\frac{mg}{q}, 0\right) \\
\vec{a} &= 1 \times 10^{-16} \text{ m/s}^2 \text{ (positive)} \\
\frac{ma}{q} &= \left(9.1 \times 10^{-31} \text{ kg}\right) \left(1 \times 10^{16} \text{ m/s}^2\right) \\
&= -9.1 \times 10^{-6} \text{ C} \\
\vec{E} &= (-5.7 \times 10^4 \text{ N/C}, 0) \\
\text{negative sign means \(E\) points in the direction opposite to \(a\).}
\end{align*}
\]
Problem: The electric field in the electron gun of a TV tube is supposed to accelerate electrons uniformly from 0 to \(3.3 \times 10^7\) m/s within a distance of 1.0 cm. What electric field is required?

\[ v^2 = v_o^2 + 2ax \]  (if \(x_o=0\) at \(t_o=0\))

\[ a = \frac{1}{2} \left( \frac{v^2 - v_o^2}{x-x_o} \right) = \frac{1}{2} \left( \frac{(3.3 \times 10^7 \text{m/s})^2}{0.01 \text{m}} \right) = 5.45 \times 10^6 \text{ m/s}^2 \]

\[ E = \frac{F}{q} = \frac{m \alpha}{q} \]

\[ E = \left(9.1 \times 10^{-31} \text{kg}\right)(5.45 \times 10^6 \text{m/s}) / \left(1.6 \times 10^{-19} \text{C} \right) = -3.1 \times 10^5 \text{N/C} \]

Direction opposite to \(F\) and \(a\)
Problem: Three point charges \(-Q, 2Q,\) and \(-Q\) are arranged on a straight line as illustrated. What is the electric field that these charges produce at a distance \(x\) to the right of the central charge?

\[
\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = (E_x, E_y)
\]

\[
E_{1x} = -\frac{Q}{4\pi\varepsilon_0 (d+x)^2}
\]

\[
E_{2x} = \frac{2Q}{4\pi\varepsilon_0 x^2}
\]

\[
E_{3x} = -\frac{Q}{4\pi\varepsilon_0 (x-d)^2}
\]

\[
E_x = E_{1x} + E_{2x} + E_{3x} = \frac{Q}{4\pi\varepsilon_0} \left[ \frac{-1}{(d+x)^2} + \frac{2}{x^2} - \frac{1}{(x-d)^2} \right]
\]
Lines of Electric Field

The electric field can be represented graphically by drawing at any given point of space, a vector whose magnitude and directions are those of the electric field at that point.

\[ \mathbf{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \]

Electric field vectors surrounding a positive point charge. The field vectors are directed radially outward.
The electric field can be represented graphically by field lines.

- **Direction**: the tangent to the field lines has the direction of the electric field.
- **Density**: the density of lines is directly proportional to the magnitude of the electric field.

To draw a pattern of field lines we have to continue each line indefinitely except where it begins on a positive charge or where it ends on a negative charge.

\[ E = k \frac{q}{r^2} \implies \text{the number (density) of field lines} \sim \frac{1}{q}. \]

The convention or “normalization”:

- the number of electric field lines emerging from a charge \( q \) is \( \frac{q}{E} \).

\[ \text{if } q = 1 \text{C, the number of field lines is } \frac{1}{E} = \left(8.90 \times 10^{-12}\right) = 1.12 \times 10^{10} \text{ lines.} \]

The density of field lines is \( E \).

Electric field lines of a positive point charge. The arrows on these lines indicate the direction of the electric field along each line.
Major properties of the field lines

1. The field lines start on positive charges and end (if they end) on negative charges - the positive charges are sources of field lines, the negative - are sinks.

2. Field lines never intersect (except where they start or end on point charges).

3. The field lines are not physical objects, are not a form of matter. They represent merely mathematical approach helping us in understanding of the spatial dependence of the electric fields surrounding electric charges.

4. Tangent to the field line has the direction of the electric field.

5. The density of the field lines equal to the magnitude of the electric field.
Two graphical methods of graphical representation of electric field: a) electric field vectors; b) electric field lines (directed radially outward from the positive charge $+q$)
The electric field lines are directed radially inward toward a negative point charge
The electric field lines of an electric dipole are curved and extend from the positive to the negative charge. At any point, such as 1, 2, or 3, the field created by the dipole is tangent to the line through the point.
The electric field lines for two identical positive point charges. If the charges were both negative, the directions of the lines would be reversed.
The concept of the field lines for the computation of the electric field of a large, flat, charged sheet.

- **Assumption**: Positive charge is uniformly distributed on a very large horizontal sheet. Density of charge \( \sigma \) (C/m²).
- **Symmetry arguments**:
  - The pattern of field lines must respect the symmetry of the charge distribution. The pattern of field lines in the space above the sheet is the mirror image of the pattern below it.
  - One half of the upper-ward direction, one half-downward.
- **Consider one of the field lines starting at the top and directed to the right of the line.**
- **The field lines are uniformly distributed since the charge is uniformly distributed.**

**Electric field of flat sheet**:
- Consider an area \( A \) of the sheet.
- Number of field lines intersecting is \( \sigma A \). The upward direction \( \frac{\sigma A}{2\varepsilon} \).
- Density of lines \( \frac{\sigma A}{2\varepsilon} \).
- Electric field at flat sheet: \( E = \frac{\sigma}{2\varepsilon} \).
The Electric Field of the Two Charged Sheets

Individual electric fields of two sheets of charge of opposite signs.

The net electric field of the two sheets.

\[ E = \frac{\sigma}{\varepsilon_0} + \frac{\sigma}{\varepsilon_0} = \frac{2\sigma}{\varepsilon_0} \]
In the central region of a parallel plate capacitor, the electric field lines are parallel and evenly spaced, indicating that the electric field there has the same magnitude and direction at all points.
Problem: Electric charge is uniformly distributed over each of three large, parallel sheets of paper. The charges per unit area of the sheets $\sigma = 2 \times 10^{-6} \text{C/m}^2$, $2 \times 10^{-6} \text{C/m}^2$ and $-2 \times 10^{-6} \text{C/m}^2$, respectively. The distance between one sheet and the next is 10 cm. Find the strength of the electric field $E$ above the sheets, below the sheets, and in space between the sheets. Find the direction of $E$ at each place.

\[E = \frac{\sigma}{2\varepsilon_0} = \frac{2 \times 10^{-6}}{2 \times 8.85 \times 10^{-12} \text{C}^2/\text{N} \cdot \text{m}^2} = 1.1 \times 10^5 \text{ N/C}\]

\[E_A = (1.1 \times 10^5 \text{ N/C}) + (1 \times 10^5 \text{ N/C}) = 2.1 \times 10^5 \text{ N/C}\]

\[E_B = (1.1 \times 10^5 \text{ N/C}) - (1 \times 10^5 \text{ N/C}) = 0.1 \times 10^5 \text{ N/C}\]

\[E_C = (1 \times 10^5 \text{ N/C}) + (1 \times 10^5 \text{ N/C}) = 2 \times 10^5 \text{ N/C}\]

\[E_D = (1.1 \times 10^5 \text{ N/C})[-1-1+1] = -1.1 \times 10^5 \text{ N/C}\]
Electric Flux.

The electric field and the surface through which it passes are brought together to introduce the concept of electric flux.

Suppose that a surface in the shape of a rectangle or triangle is immersed in a constant electric field \( E \).

The electric flux \( \Phi \) through the surface is defined as the product of the area \( A \) of the surface, the magnitude of the normal component of the electric field, and the angle \( \theta \) between the field and normal.

\[
\Phi = E \cdot A \cdot \cos \theta
\]
(a) Area $A$ is L field lines. $\Phi = EA \cos \theta$.
$\theta = 0^\circ$; $\cos 0^\circ = 1$
$\Rightarrow \Phi = EA$
The area $A$ intercepts the maximum possible field lines.

The electric flux through the area $A$ is numerically equal to the number of field lines intercepted by the area.

(b) $\theta = 90^\circ$; $\Phi = EA \cos 90^\circ = 0$
All the field lines pass the area without interception.
We define the flux of $E$, which we call $\Phi$, through $A$ as $\Phi = E_n A = EA \cos \theta$, in which $E_n$ is the component of the electric field vector perpendicular to the surface. From this definition we see that $\Phi$ is positive if $0^\circ < \theta < 90^\circ$, and negative if $90^\circ < \theta < 180^\circ$. We have shown that the electric field line density is equal in magnitude to the electric field, $E$. The flux through the element $A$ is therefore exactly equal to the number of electric field lines through $A$. What does it mean for $\Phi$ to be negative? Consider an arbitrary closed surface that encloses a well-defined volume, as shown in Fig. A negative value of $\Phi$ implies that flux lines are entering the volume enclosed by the surface, whereas a positive value of $\Phi$ means that the flux lines are leaving the volume. If we evaluate the electric flux $\Phi$ for a closed surface and obtain a positive result, the answer tells us that more field lines are leaving the surface than entering. Because field lines originate on positive charge, we can infer that the enclosed volume must contain some positive charge. Similarly, a negative answer implies that the volume encloses some negative charge. If $\Phi$ is zero, then this volume must enclose a net charge of zero. Further, because field lines originate on charges and are directed radially away from or toward a point charge, field lines from charges outside the enclosed volume enter the surface at some point and pass right through it, making no net contribution to the electric flux enclosed by the surface.
Gauss' Law. The preceding statements are quantified in Gauss' Law, which states that the net electric flux, \( \Phi \) through a closed surface, defined by \( \Phi = E \cdot A \), is numerically equal to \( Q/\varepsilon_0 \), where \( Q \) is the total charge enclosed by the surface. This follows directly from the definition that the number of field lines emerging from a charge \( q' \) is \( q'/\varepsilon_0 \). It does not matter how many small charges \( q' \) are contained within the volume; each contributes \( q'/\varepsilon_0 \) field lines that pass out through the closed surface. Gauss' Law is customarily written in the form

\[
\Phi = E \cdot A = Q/\varepsilon_0
\]

When Gauss' Law is applied to such a surface, the surface is usually referred to as a Gaussian surface. The most common use of Gauss' Law is in the calculation of electric field strength and is the most useful when there is a high degree of symmetry in the charge distribution. For example, a point charge has spherical symmetry, so we expect the electric field of this charge to have spherical symmetry also. Lines and sheets of charge are, respectively, characterized by single directions and planes in space, so that electric fields resulting from these configurations should have symmetries with respect to these geometries. In the case of the plane of charge, this means that the field is oriented perpendicular to the plane. Gauss' Law allows rapid and easy calculations of electric field strength for any of these charge distributions.
Consider a point charge $q$.
- The amount of lines emerging $\frac{q}{\varepsilon_0}$.
- The arrangement of field lines must be spherically symmetric with a uniform distribution over all radial directions.
- At a distance $r$ from the point charge, the lines are uniformly distributed over the area $4\pi r^2$ of a concentric sphere.

$\Rightarrow$ there are $\left(\frac{q}{\varepsilon_0}\right) / 4\pi r^2$ lines per unit area.

$\Rightarrow E = \frac{1}{4\pi \varepsilon_0} \frac{q}{r^2}$
Problem 54  The magnitude of the electric field produced by a long thin straight line of charge is

\[ E = \frac{1}{2\pi \varepsilon_0} \frac{\lambda}{r} \]

To obtain this result, consider a segment of the straight charged line of length \( h \).

(a) What is the amount of \( Q \) on this segment?

(b) How many field lines start on this segment?

(c) Consider a cylindrical surface of radius \( r \) concentric with the straight charged line. What is the area of this surface? What is the density of field lines intercepted by this surface?

(d) What is the magnitude of \( E \) at this surface?

\[ (a) \ Q = \lambda h \]
\[ (b) \ Number \ of \ field \ lines = \frac{Q}{E_0} = \frac{1}{\varepsilon_0} \]
\[ (c) \ \text{Density of lines} = \frac{\text{Number of lines}}{A} = \frac{Q}{E_0 A} = \frac{\lambda h}{2\pi \varepsilon_0} \]

(d) \[ E = \text{[density of lines]} = \lambda/(2\pi r \varepsilon_0) = (1/2\pi \varepsilon_0) \times (\lambda/r) \]
Conductors and electric fields An electrical conductor has rather special electrostatic properties. At all points inside an electrical conductor, the electric field must be zero, because if it were not, the conduction electrons would move. When we first place a conductor in an electric field, the conduction electrons do move and redistribute themselves on the surface of the conductor such that the field set up by the surface charges on the conductor exactly cancels the external electric field inside the conductor. When this state occurs, the charge distribution on the conductor is said to be in static equilibrium. We can use Gauss' Law to prove that for a conductor in static equilibrium, the following are true: (a) any net electric charge resides on the surface of the conductor, and (b) the electric field at the surface of the conductor is normal to the surface.

\[ \Phi = \frac{Q}{\varepsilon_0} \]

Since \( E = 0 \); \( \Phi = 0 \) and \( \rightarrow Q \) in the volume \( = 0 \)
An excess charge within a conductor (copper) moves quickly to the surface. At equilibrium under static conditions, any excess charge resides on the surface of the conductor.
The electric field inside a conductor: Shielding

A) A cylindrical conductor (shown as an end view) is placed between the oppositely charged plates of a capacitor. The electric field lines do not penetrate the conductor. The blow-up shows that, just outside the conductor, the electric field lines are perpendicular to its surface. B) The electric field is zero in a cavity within the conductor.
Problem 51 A point charge of $2 \times 10^{-12} \text{C}$ is located at the center of a cubic Gaussian surface. What is the electric flux through each of the faces of the cube?

[Diagram of a cube]

Apply Gauss' Law

- If the volume within an arbitrary closed surface holds a net electric charge $Q$, then the electric flux through the surface is $\Phi = \frac{Q}{\varepsilon_0}$.

- $\Phi = 6 \Phi_{\text{face}}$

- $\Phi_{\text{face}} = \frac{Q}{6\varepsilon_0} = \frac{4 \pi}{6} \left( \frac{9.0 \times 10^9}{2 \times 10^{-12} \text{C}} \right) \left( \frac{2 \mu \text{m}}{2 \mu \text{m}} \right) = 3.8 \times 10^{-2} \text{N m}^2\text{C}^{-1}$