PH 222-3A Spring 2007

Electric Potential

Lectures 5-6

Chapter 24
(Halliday/Resnick/Walker, Fundamentals of Physics 8th edition)
Chapter 24
Electric Potential

In this chapter we will define the electric potential (symbol $V$) associated with the electric force and accomplish the following tasks:

- Calculate $V$ if we know the corresponding electric field.
- Calculate the electric field if we know the corresponding potential $V$.
- Determine the potential $V$ generated by a point charge.
- Determine the potential $V$ generated by a discrete charge distribution.
- Determine the potential $V$ generated by a continuous charge distribution.
- Determine the electric potential energy $U$ of a system of charges.
- Define the notion of an equipotential surface.
- Explore the geometric relationship between equipotential surfaces and electric field lines.
- Explore the potential of a charged isolated conductor.
Electric Potential Energy

In Chapter 8 we defined the change in potential energy $\Delta U$ associated with a conservative force as the negative value of the work $W$ that the force must do on a particle to take it from an initial position $x_i$ to a final position $x_f$.

$$\Delta U = U_f - U_i = -W = -\int_{x_i}^{x_f} F(x)dx$$

Consider an electric charge $q_0$ moving from an initial position at point $A$ to a final position at point $B$ under the influence of a known electric field $\vec{E}$. The force exerted on the charge is $\vec{F} = q_0\vec{E}$.

$$\Delta U = -\int_{i}^{f} \vec{F} \cdot d\vec{s} = -q_0\int_{i}^{f} \vec{E} \cdot d\vec{s}$$
The Electric Potential $V$

The change in potential energy of a charge $q_0$ moving under the influence of $\vec{E}$ from point A to point B is: $\Delta U = U_f - U_i = -W = -q_0 \int_{i}^{f} \vec{E} \cdot d\vec{s}$.

Please note that $\Delta U$ depends on the value of $q_0$.

We define the electric potential $V$ in such a manner so that it is independent of $q_0$: $\Delta V = \frac{\Delta U}{q_0} = -\frac{W}{q_0}$ Here $\Delta V = V_f - V_i \rightarrow V_f - V_i = -\int_{i}^{f} \vec{E} \cdot d\vec{s}$.

In all physical problems only changes in $V$ are involved. Thus we can define arbitrarily the value of $V$ at a reference point, which we choose to be at infinity: $V_f = V_\infty = 0$. We take the initial position as the generic point $P$ with potential $V_p$: $V_p = -\int_{\infty}^{P} \vec{E} \cdot d\vec{s}$. The potential $V_p$ depends only on the coordinates of $P$ and on $\vec{E}$.
SI Units of $V$:

Definition of voltage: $\Delta V = -\frac{W}{q_0}$

Units of $V$: J/C, known as the volt

**Potential Due to a Point Charge**

Consider a point charge $q$ placed at the origin. We will use the definition given on the previous page to determine the potential $V_p$ at point $P$ a distance $R$ from $O$.

$$V_p = -\int_{\infty}^{R} \vec{E} \cdot d\vec{s} = \int_{R}^{\infty} Edr \cos 0 = \int_{R}^{\infty} Edr$$

The electric field generated by $q$ is:

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

$$V_p = \frac{q}{4\pi\varepsilon_0} \int_{R}^{\infty} \frac{dr}{r^2} = \frac{1}{\int_{\infty}^{\infty} \frac{dr}{x^2}} = -\frac{1}{x}$$

$$\rightarrow V_p = \frac{q}{4\pi\varepsilon_0} \left[ -\frac{1}{r} \right]_{R}^{\infty} = \frac{1}{4\pi\varepsilon_0} \frac{q}{R}$$
Example. The potential of a point charge for a zero reference potential at infinity

A point charge of $q=4.0 \times 10^{-8} \text{C}$ creates a potential at a spot 1.2 m away.

The potential is (a) $V = \frac{kq}{r} = \frac{(8.99 \times 10^9)(4.0 \times 10^{-8})}{1.2} = +300 \text{V}$

when the charge is positive

and (b) $-300 \text{V}$ when the charge is negative
The acceleration of positive and negative charges

A positive charge accelerates from a region of higher electric potential toward a region of lower electric potential.

A negative charge accelerates from a region of lower electric potential toward a region of higher electric potential.
Electrostatic Potential of a point charge

The electrostatic potential produced by the point charge is

\[ V = \frac{1}{4\pi \varepsilon_0} \frac{q}{r} \]

\( q' = +1e \)

\( q' = -1e \)

Electrostatic potential of a charge \( q' \) versus distance.
The electric potential energy of a point charge $q$ for non-uniform electric field

$$W = U_1 - U_2$$

Energy $E = K + U = \frac{1}{2} mv^2 + U = \varepsilon \text{const}$

$U = qV$

$E_{\text{energy}} = \frac{1}{2} mv^2 + qV = \varepsilon \text{const}$

$$U = \frac{1}{4\pi \varepsilon_0} \frac{q_1 q_2}{r^2}$$

Potential energy of two interacting point charges

For two charges of equal signs, the electric potential energy is positive and in inverse proportion to the distance.

For two charges of opposite signs, $U < 0$ and this negative potential energy increases with distance from a large negative value to zero.
One electron volt is the magnitude of the amount by which the potential energy of an electron changes when the electron moves through the potential difference of 1 volt.

\[
\text{Electron-volt (eV)}
\]

\[
e_{\text{eV}} = (1.6 \times 10^{-19} \text{C})(1 \text{V}) = 1.6 \times 10^{-19}
\]

In chemical reactions among atoms or molecules, the energy released or absorbed by each atom or molecule is typically 1 to 2 eV.

The mechanical energy of a point charge moving in an electric field

\[
\text{Energy} = K + U = \frac{1}{2} m v^2 + \frac{1}{4 \pi \varepsilon_0} \frac{q^2}{2} = E_{\text{mech}}
\]

The law of conservation of energy for the motion of a point charge \( q \) in an electric field \( E' \): The total energy remains constant during the motion.
Problem: The electric potential difference between the positive and negative poles of an automatic battery is 12 volts. In order to charge the battery fully, the charging device must force $2.0 \times 10^5$ coulombs from the negative terminal to the positive terminal. How much work must the charging device do during this process?

\[ W = U_1 - U_2 \]
\[ U_2 = U_1 + W \]
\[ U_1 = 0 \Rightarrow W = U_2 \]
\[ U_2 = QV \]

\[ \Rightarrow W = QV = 2 \times 10^5 \text{C} \times 12 \text{volts} = \]
\[ = 2.4 \times 10^6 \text{J} \]
A headlight connected to a 12-V battery
Problem. A proton sits at the origin of coordinates. How much work must you do against the electric force of the proton to push an electron from the point $x = 10 \times 10^{-10} \text{ m}, y = 0$ in the $x$-$y$ plane to the point $x = 2.5 \times 10^{-10} \text{ m}, y = 2.5 \times 10^{-10} \text{ m}$?

\[ U_1 = -\frac{1}{4\pi \varepsilon_0} \frac{e^2}{x_1} = -\frac{(9 \times 10^9)(1.6 \times 10^{-19})^2}{1 \times 10^{-10}} = -2.3 \times 10^{-18} \text{ J} \]

\[ U_2 = -\frac{1}{4\pi \varepsilon_0} \frac{e^2}{x_2} = -\frac{(9 \times 10^9)(1.6 \times 10^{-19})^2}{\sqrt{(2.5 \times 10^{-10})^2 + (2.5 \times 10^{-10})^2}} = -6.5 \times 10^{-19} \text{ J} \]

Work done by the electric force

\[ W = U_1 - U_2 = -2.3 \times 10^{-18} + 6.5 \times 10^{-19} = -1.6 \times 10^{-18} \text{ J} = -10 \text{ eV} \]

Work to be done against the electric force = +10 eV
Problem

A proton is accelerated from rest through a potential of $2.5 \times 10^5 \text{V}$. What is its final speed?

$E_0 = \frac{1}{2} m v_0^2 + e V_0$

$E_1 = \frac{1}{2} m v^2 + e V_1$

$E_0 = E_1$

$e V_0 - e V_1 = \frac{m v^2}{2}$

$\frac{m v^2}{2} + e V_0 = \frac{m v^2}{2} + e V_1$

$\frac{m v^2}{2} = e (V_0 - V_1)$

$v_f = \sqrt{\frac{2 e (V_0 - V_1)}{m}} = \sqrt{\frac{2(1.6 \times 10^{-19} \text{C})(2.5 \times 10^5 \text{V})}{1.67 \times 10^{-27} \text{kg}}}$

$= 6.9 \times 10^6 \text{ m/s}$
Problem: A positive point charge \( q \) with mass \( m \) is released at a distance \( d \) from a fixed positive point charge \( Q \). How fast is the charge \( q \) moving when the distance has grown to three times the initial value?

\[ \begin{align*}
1) & \quad \text{Diagram} \\
2) & \quad \text{Equation} \\
C_1 &= \frac{m v_1^2}{2} + \frac{1}{4\pi\varepsilon_0} \frac{qQ}{d} \\
C_2 &= \frac{m v_2^2}{2} + \frac{1}{4\pi\varepsilon_0} \frac{qQ}{3d} \\
C_1 &= C_2 = \text{const} \\
\frac{1}{4\pi\varepsilon_0} \frac{qQ}{d} &= \frac{m v_2^2}{2} + \frac{1}{4\pi\varepsilon_0} \frac{qQ}{3d} \\
\frac{m v_2^2}{2} &= \frac{2 q Q}{4\pi\varepsilon_0 3d} \\
U &= \sqrt{\frac{qQ}{3\pi\varepsilon_0 m d}}
\end{align*} \]
Potential Due to a Group of Point Charges

Consider the group of three point charges shown in the figure. The potential $V$ generated by this group at any point $P$ is calculated using the principle of superposition.

1. We determine the potentials $V_1$, $V_2$, and $V_3$ generated by each charge at point $P$:

$$V_1 = \frac{1}{4\pi \varepsilon_0} \frac{q_1}{r_1}, \quad V_2 = \frac{1}{4\pi \varepsilon_0} \frac{q_2}{r_2}, \quad V_3 = \frac{1}{4\pi \varepsilon_0} \frac{q_3}{r_3}$$

2. We add the three terms:

$$V = V_1 + V_2 + V_3$$

$$V = \frac{1}{4\pi \varepsilon_0} \frac{q_1}{r_1} + \frac{1}{4\pi \varepsilon_0} \frac{q_2}{r_2} + \frac{1}{4\pi \varepsilon_0} \frac{q_3}{r_3}$$

The previous equation can be generalized for $n$ charges as follows:

$$V = \frac{1}{4\pi \varepsilon_0} \frac{q_1}{r_1} + \frac{1}{4\pi \varepsilon_0} \frac{q_2}{r_2} + \ldots + \frac{1}{4\pi \varepsilon_0} \frac{q_n}{r_n} = \frac{1}{4\pi \varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i}$$
**Example: Potential Due to an Electric Dipole**

Consider the electric dipole shown in the figure. We will determine the electric potential \( V \) created at point \( P \) by the two charges of the dipole using superposition. Point \( P \) is at a distance \( r \) from the center \( O \) of the dipole. Line \( OP \) makes an angle \( \theta \) with the dipole axis:

\[
V = V_+ + V_- = \frac{1}{4\pi\varepsilon_0} \left( \frac{q}{r_+} - \frac{q}{r_-} \right) = \frac{q}{4\pi\varepsilon_0} \frac{r_- - r_+}{r_- r_+}.
\]

We assume that \( r \gg d \), where \( d \) is the charge separation. From triangle \( ABC \) we have: \( r_- - r_+ \approx d \cos \theta \).

Also: \( r_- r_+ \approx r^2 \rightarrow V \approx \frac{q}{4\pi\varepsilon_0} \frac{d \cos \theta}{r^2} = \frac{1}{4\pi\varepsilon_0} \frac{p \cos \theta}{r^2} \),

where \( p = qd = \) the electric dipole moment.

\[
V = \frac{1}{4\pi\varepsilon_0} \frac{p \cos \theta}{r^2}
\]
Potential Due to a Continuous Charge Distribution
Consider the charge distribution shown in the figure. In order to determine the electric potential $V$ created by the distribution at point $P$ we use the principle of superposition as follows:

1. We divide the distribution into elements of charge $dq$.
   For a volume charge distribution, $dq = \rho dV$.
   For a surface charge distribution, $dq = \sigma dA$.
   For a linear charge distribution, $dq = \lambda d\ell$.

2. We determine the potential $dV$ created by $dq$ at $P$: $dV = \frac{1}{4\pi\varepsilon_0} \frac{dq}{r}$.

3. We sum all the contributions in the form of the integral: $V = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{r}$.

Note 1: The integral is taken over the whole charge distribution.
Note 2: The integral involves only scalar quantities.
Example: Potential created by a line of charge of length $L$ and uniform linear charge density $\lambda$ at point $P$. Consider the charge element $dq = \lambda dx$ at point $A$, a distance $x$ from $O$. From triangle $OAP$ we have:

$$r = \sqrt{d^2 + x^2}.$$ Here $d$ is the distance $OP$.

The potential $dV$ created by $dq$ at $P$ is:

$$dV = \frac{1}{4\pi\varepsilon_0} \frac{dq}{r} = \frac{1}{4\pi\varepsilon_0} \frac{\lambda dx}{\sqrt{d^2 + x^2}}$$

$$V = \frac{\lambda}{4\pi\varepsilon_0} \int_0^L \frac{dx}{\sqrt{d^2 + x^2}}$$

$$\int \frac{dx}{\sqrt{d^2 + x^2}} = \ln \left( x + \sqrt{d^2 + x^2} \right)$$

$$V = \frac{\lambda}{4\pi\varepsilon_0} \left[ \ln \left( x + \sqrt{d^2 + x^2} \right) \right]_0^L$$

$$V = \frac{\lambda}{4\pi\varepsilon_0} \left[ \ln \left( L + \sqrt{L^2 + d^2} \right) - \ln d \right]$$
Induced Dipole Moment

Many molecules such as H$_2$O have a permanent electric dipole moment. These are known as "polar" molecules. For others, such as O$_2$, N$_2$, etc. the electric dipole moment is zero. These are known as "nonpolar" molecules. One such molecule is shown in fig. a. The electric dipole moment $\bar{p}$ is zero because the center of the positive charge coincides with the center of the negative charge.

In fig. b we show what happens when an electric field $\vec{E}$ is applied to a nonpolar molecule. The electric forces on the positive and negative charges are equal in magnitude but opposite in direction.

As a result the centers of the positive and negative charges move in opposite directions and do not coincide. Thus a nonzero electric dipole moment $\bar{p}$ appears. This is known as "induced" electric dipole moment, and the molecule is said to be "polarized." When the electric field is removed $\bar{p}$ disappears.
A collection of points that have the same potential is known as an equipotential surface. Four such surfaces are shown in the figure. The work done by $\mathbf{E}$ as it moves a charge $q$ between two points that have a potential difference $\Delta V$ is given by $W = -q\Delta V$.

For path I: $W_I = 0$ because $\Delta V = 0$.
For path II: $W_{II} = 0$ because $\Delta V = 0$.
For path III: $W_{III} = -q\Delta V = q(V_1 - V_2)$.
For path IV: $W_{IV} = -q\Delta V = q(V_1 - V_2)$.

**Note**: When a charge is moved on an equipotential surface ($\Delta V = 0$) the work done by the electric field is zero: $W = 0$. 

$W = -q\Delta V$
The Electric Field $\vec{E}$ is Perpendicular to the Equipotential Surfaces

Consider the equipotential surface at potential $V$. A charge $q$ is moved by an electric field $\vec{E}$ from point $A$ to point $B$ along a path $\Delta \vec{r}$.

Points $A$ and $B$ and the path lie on $S$.

Let's assume that the electric field $\vec{E}$ forms an angle $\theta$ with the path $\Delta \vec{r}$.

The work done by the electric field is: $W = \vec{F} \cdot \Delta \vec{r} = F \Delta r \cos \theta = qE \Delta r \cos \theta$.

We also know that $W = 0$. Thus: $qE \Delta r \cos \theta = 0$, where $q \neq 0$, $E \neq 0$, $\Delta r \neq 0$. Thus: $\cos \theta = 0 \rightarrow \theta = 90^\circ$.

The correct picture is shown in the figure below.
Examples of Equipotential Surfaces and the Corresponding Electric Field Lines

Uniform electric field

Isolated point charge

Electric dipole

Equipotential surfaces for a point charge $q$:

$$V = \frac{q}{4\pi\varepsilon_0 r}.$$  Assume that $V$ is constant $\rightarrow r = \frac{q}{4\pi\varepsilon_0 V}$ = constant.

Thus the equipotential surfaces are spheres with their center at the point charge and radius $r = \frac{q}{4\pi\varepsilon_0 V}$. 
Calculating the Electric Field $\vec{E}$ from the Potential $V$

Now we will tackle the reverse problem, i.e., determine $\vec{E}$ if we know $V$.
Consider two equipotential surfaces that correspond to the values $V$ and $V + dV$ separated by a distance $ds$ as shown in the figure. Consider an arbitrary direction represented by the vector $d\vec{s}$. We will allow the electric field to move a charge $q_0$ from the equipotential surface $V$ to the surface $V + dV$.

The work done by the electric field is given by:

$$W = -q_0 dV \quad (\text{eq. 1}).$$

Also

$$W = F ds \cos \theta = E q_0 ds \cos \theta \quad (\text{eq. 2})$$

If we compare these two equations we have:

$$Eq_0 ds \cos \theta = -q_0 dV \rightarrow E \cos \theta = -\frac{dV}{ds}.$$

From triangle $PAB$ we see that $E \cos \theta$ is the component $E_s$ of $\vec{E}$ along the direction $s$. Thus:

$$E_s = -\frac{\partial V}{\partial s}.$$ 

We have switched to the partial derivative symbols to emphasize that $E_s = -\frac{\partial V}{\partial s}$ involves only the variation of $V$ along a specific $s$ axis.
We have proved that \( E_s = -\frac{\partial V}{\partial s} \).

The component of \( \vec{E} \) in any direction is the negative of the rate at which the electric potential changes with distance in this direction.

If we take \( s \) to be the \( x-, y-, \) and \( z- \)axes we get:

\[
E_x = -\frac{\partial V}{\partial x}; \quad E_y = -\frac{\partial V}{\partial y}; \quad E_z = -\frac{\partial V}{\partial z}
\]

If we know the function \( V(x, y, z) \) we can determine the components of \( \vec{E} \) and thus the vector \( \vec{E} \) itself:

\[
\vec{E} = -\frac{\partial V}{\partial x} \hat{i} - \frac{\partial V}{\partial y} \hat{j} - \frac{\partial V}{\partial z} \hat{k}
\]

For the simple situation in which the electric field \( \vec{E} \) is uniform \( E = -\frac{\Delta V}{\Delta s} \), where \( s \) is perpendicular to the equipotential surface.
Potential Energy $U$ of a System of Point Charges

We define $U$ as the work required to assemble the system of charges one by one, bringing each charge from infinity to its final position.

Using the above definition we will prove that for a system of three point charges $U$ is given by:

$$U = \frac{q_1 q_2}{4\pi \varepsilon_0 r_{12}} + \frac{q_2 q_3}{4\pi \varepsilon_0 r_{23}} + \frac{q_1 q_3}{4\pi \varepsilon_0 r_{13}}$$

**Note:** Each pair of charges is counted only once.

For a system of $n$ point charges $\{q_i\}$ the potential energy $U$ is given by:

$$U = \frac{1}{4\pi \varepsilon_0} \sum_{i<j}^{n} \frac{q_i q_j}{r_{ij}}.$$  

Here $r_{ij}$ is the separation between $q_i$ and $q_j$.

The summation condition $i < j$ is imposed so that, as in the case of three point charges, each pair of charges is counted only once.
Step 1: Bring in $q_1$:
$W_1 = 0$
(no other charges around)

Step 2: Bring in $q_2$:
$W_2 = q_2V(2)$

$V(2) = \frac{q_1}{4\pi\varepsilon_0 r_{12}} \rightarrow W_2 = \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$

Step 3: Bring in $q_3$:
$W_3 = q_3V(3)$

$V(3) = \frac{1}{4\pi\varepsilon_0} \left( \frac{q_1}{r_{13}} + \frac{q_2}{r_{23}} \right) \rightarrow$

$W_3 = \frac{1}{4\pi\varepsilon_0} \left( \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right)$

$W = W_1 + W_2 + W_3$

$W = \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}} + \frac{q_2 q_3}{4\pi\varepsilon_0 r_{23}} + \frac{q_1 q_3}{4\pi\varepsilon_0 r_{13}}$
This expression for $U$ can be rewritten as

$$U = \frac{1}{2} V_{\text{other}}(1) q_1 + \frac{1}{2} V_{\text{other}}(2) q_2 + \frac{1}{2} V_{\text{other}}(3) q_3$$

which is just the potential at charge $q_1$ produced by charges $q_2$ and $q_3$. Similarly, $V_{\text{other}}(2)$ is the potential at charge $q_2$ produced by charges $q_1$ and $q_3$, and $V_{\text{other}}(3)$ is the potential at $q_3$ caused by $q_1$ and $q_2$. Since this expression for $U$ can be applied to any number of charges, the electric potential energy for $n$ point charges can be written as:

$$U = \frac{1}{2} V_{\text{other}}(1) q_1 + \frac{1}{2} V_{\text{other}}(2) q_2 + \ldots + \frac{1}{2} V_{\text{other}}(n) q_n$$

To see how useful this expression can be, let us apply it to evaluate the potential energy of a single charged conductor with total charge $Q$ whose surface is at a potential $V$. We can imagine the total charge $Q$ to be made up of a large number of very small charges $q$, so that $V_{\text{other}} = V$ for each of the many small charges $q$ contributing to the total charge $Q$. Then the total electric energy is just given by:

$$U = \frac{1}{2} V q_1 + \frac{1}{2} V q_2 + \frac{1}{2} V q_3 + \ldots$$

$$= \frac{1}{2} V(q_1 + q_2 + q_3 + \ldots) = \frac{1}{2} V Q$$

That is, the electric energy of a charged conductor is one-half the product of the charge times the electric potential. If we have a large number of charged conductors with charges $Q_1$, $Q_2$, $Q_3$, \ldots and respective potentials $V_1$, $V_2$, $V_3$, \ldots, the total electric energy is given directly by:

$$U = \frac{1}{2} Q_1 V_1 + \frac{1}{2} Q_2 V_2 + \frac{1}{2} Q_3 V_3 + \ldots$$
Energy of a system of charged conductors

Conducting bodies have constant potentials: \( V_1 = \text{const}; \ V_2 = \text{const} \ldots \)

\[
U_i = \frac{1}{2} \Delta \varphi_i \ V_{\text{const}} + \frac{1}{2} \Delta \varphi_1 \ V_{\text{const}} + \ldots = \\
= \frac{1}{2} V (\Delta \varphi_1, \Delta \varphi_2, \ldots) = \frac{1}{2} \varphi_i V_i \\
U_{\text{net}} = \frac{1}{2} \varphi_i V_i + \frac{1}{2} \varphi_2 V_2 + \frac{1}{2} \varphi_3 V_3 + \ldots
\]
Problem: Suppose that at one instant the electrons and the nucleus of a helium atom occupy the positions shown in Fig. 2. At this instant, the electrons are at a distance of $0.2 \times 10^{-10}$ m from the nucleus. What is the electric potential energy of this arrangement? Treat the electrons and the nucleus as point charges.

\[ U = \frac{1}{4\pi\varepsilon_0} \left( \frac{e_1 e_2}{r_{1,2}} + \frac{e_2 e_3}{r_{2,3}} + \frac{e_1 e_3}{r_{1,3}} \right) = \]

\[ = \frac{1}{4\pi\varepsilon_0} \left( \frac{2e^2}{0.2 \times 10^{-10}} + \frac{-2e^2}{0.2 \times 10^{-10}} + \frac{e^2}{0.4 \times 10^{-10}} \right) = \]

\[ = (9 \times 10^9) \left[ \frac{-4 (1.6 \times 10^{-19})^2}{0.2 \times 10^{-10}} + \frac{1.6 \times 10^{-19}}{0.4 \times 10^{-10}} \right] = \]

\[ = -4.0 \times 10^{-17} \text{ J} = \frac{-4.0 \times 10^{-17} \text{ J}}{1.6 \times 10^{-19} \text{ eV}} = -250 \text{ eV} \]
**Problem.** Four equal particles of positive charges of equal masses are initially held at the four corners of a square of side \( l \). If these particles are released simultaneously, what will be their speeds when they have separated by a very large distance?

\[
\frac{1}{4} \pi \varepsilon_0 \left( q_2/L + q_3/\sqrt{2}L + q_4/L \right) = \text{const.}
\]

\( q = q_1 = q_2 = q_3 = q_4 \)

1. \( U = \frac{1}{2} q \varepsilon_0 \left( \frac{q}{2} + \frac{q}{\sqrt{2}L} + \frac{q}{L} \right) \times 4 = \frac{q^2}{2 \pi \varepsilon_0} \left( \frac{1}{2} + \frac{1}{\sqrt{2}L} \right) \)

2. \( KE \) of all 4 particles = \( U \)

\( KE = q \left( \frac{1}{2} m v^2 \right) \) where \( U = \text{speed of each} \)

\( 2m v_2^2 = \frac{q^2}{2 \pi \varepsilon_0} \left( 2 + \frac{1}{\sqrt{2}L} \right) \)

\( v_2 = \sqrt{\frac{q^2}{4 \pi \varepsilon_0 m L} \left( 2 + \frac{1}{\sqrt{2}L} \right)} \)
Potential of an Isolated Conductor

We shall prove that all the points on a conductor (either on the surface or inside) have the same potential.

A conductor is an equipotential surface.

Consider two points $A$ and $B$ on or inside a conductor. The potential difference $V_B - V_A$ between these two points is given by the equation:

$$V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{S}.$$  

We already know that the electrostatic field $\vec{E}$ inside a conductor is zero. Thus the integral above vanishes, and $V_B = V_A$ for any two points on or inside the conductor.
Isolated Conductor in an External Electric Field

We already know that the surface of a conductor is an equipotential surface. We also know that the electric field lines are perpendicular to the equipotential surfaces.

From these two statements it follows that the electric field vector \( \vec{E} \) is perpendicular to the conductor surface, as shown in the figure. All the charges of the conductor reside on the surface and arrange themselves in such a way so that the net electric field inside the conductor \( E_{\text{in}} = 0 \).

The electric field just outside the conductor is: \( E_{\text{out}} = \frac{\sigma}{\varepsilon_0} \).
1. All the charges reside on the conductor surface.
2. The electric field inside the conductor is zero: \( E_{\text{in}} = 0 \).
3. The electric field just outside the conductor is: \( E_{\text{out}} = \frac{\sigma}{\varepsilon_0} \).
4. The electric field just outside the conductor is perpendicular to the conductor surface.
5. All the points on the surface and inside the conductor have the same potential. The conductor is an equipotential surface.
Electric Field and Electric Potential for a Spherical Conductor of Radius $R$ and Charge $q$

For $r < R$, \[ V = \frac{1}{4\pi\varepsilon_0} \frac{q}{R}. \]

For $r > R$, \[ V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}. \]

For $r < R$, \[ E = \frac{1}{4\pi\varepsilon_0} \frac{q}{R^2}. \]

For $r > R$, \[ E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}. \]

**Note:** Outside the spherical conductor, the electric field and the electric potential are identical to that of a point charge equal to the net conductor charge and placed at the center of the sphere.