Waves - II

Lectures 27-28

Chapter 17
(Halliday/Resnick/Walker, Fundamentals of Physics 8th edition)
Chapter 17  Waves II

In this chapter we will study sound waves and concentrate on the following topics:

- Speed of sound waves
- Relation between displacement and pressure amplitude
- Interference of sound waves
- Sound intensity and sound level
- Beats
- The Doppler effect
Sound

Wave propagating on a string – one dimensional wave. Can be represented graphically by a plot of the displacement of the string vs. the distance.

Sound waves emerging from a source (a loudspeaker, the horn of a car, a human mouth) spreads outward in all directions, filling the three-dimensional volume of air surrounding the source.

Wave fronts – Locations of the wave crests at a given instant of time – simplest graphical representation of three-dimensional waves.

The wave fronts of a sound wave emerging from the earpiece of a telephone headset.

A spherical wave. The wave fronts of this wave are concentric spherical surfaces.

\[ A_1 > A_2 > A_3 > A_4 \]

At a very large distance from the source the graphical wave fronts of a sound wave can be regarded as nearly flat.
Sound Waves In Air

A sound wave in air consists of alternating zones of low density and high density (or zones of low pressure and of high pressure.)

• Such zones are generated by the vibrating diaphragm of a loudspeaker which exerts successive pushes on the air that is in contact with it.
• The alternating zones of low and high density travel away from the source.
• The air as a whole does not travel (the air molecules oscillate back and forth.) Travel only density disturbances.
• Sound wave – longitudinal wave. Oscillation along the direction of propagation of the wave.
• The restoring force that drives these oscillations is the pressure of the air. Wherever the density of molecules is higher than normal, the pressure is higher than normal and pushed molecules apart.
• The frequency of the sound determines the pitch we hear (like color for light)

White noise – mixture of harmonic waves of all frequencies with equal strength.

Musical tones – mixture of just a few harmonic waves, fundamental and first few overtones.
Anatomy of a Human Ear

The ear converts the mechanical oscillations of a sound wave into electric nerve impulses. The ear is unmatched in its ability to accommodate a wide range of intensities of sound (dynamic range is about $10^{12}$).

Three main parts of the human ear:

The outer ear – auricle and the ear canal

Auricle – focus sound waves into the ear.

Ear canal – 2.7 cm long tube closed off at the inner end by the eardrum.

Ear canal guides sound waves toward the eardrum.

The middle ear is an air-filled chamber in the temporal bone of the skull.

Contains three small bones: the hammer, the anvil and the stirrup.

The inner ear – complex system of fluid-filled cavities in the temporal bone.

$20\text{Hz} - 20,000\text{Hz}$ – the range of frequencies audible for ear

$f>20,000\text{Hz}$ – ultrasound, propagate through liquids and solids – sonography ($10^6\text{Hz}$).
Sound waves are mechanical *longitudinal* waves that propagate in solids, liquids, and gases. Seismic waves used by oil explorers propagate in the earth’s crust. Sound waves generated by a sonar system propagate in the sea. An orchestra creates sound waves that propagate in the air.

The locus of the points of a sound wave that has the same displacement is called a “**wavefront**”. Lines perpendicular to the wavefronts are called “**rays**” and they point along the direction which the sound wave propagates. An example of a point source of sound waves is given in the figure. We assume that the surrounding medium is isotropic i.e. sound propagates with the same speed for all directions. In this case the sound wave spreads outwards uniformly and the wavefronts are spheres centered at the point source. The single arrows indicate the rays. The double arrows indicate the motion of the molecules of the medium in which sound propagates.
**Bulk modulus**

If we apply an overpressure $\Delta p$ on an object of volume $V$, this results in a change of volume $\Delta V$ as shown in the figure. The bulk modulus of the compressed material is defined as: 

$$B = -\frac{\Delta p}{\Delta V / V}$$

SI unit: the Pascal

**Note:** The negative sign denotes the decrease in volume when $\Delta p$ is positive.

**The speed of sound**

Using the above definition of the bulk modulus and combining it with Newton's second law one can show that the speed of sound in a homogeneous isotropic medium with bulk modulus $B$ and density $\rho$

is given by the equation:

$$v = \sqrt{\frac{B}{\rho}}$$

**Note 1:**

$$|\Delta V| = \frac{pV}{B}$$

Bulk modulus is smaller for more compressible media. Such media exhibit lower speed of sound.

**Note 2:** Denser materials (higher $\rho$) have lower speed of sound.
The Speed of Sound

As in the case of a wave on a string, the speed of sound in air depends on the pressure and on the amount of mass.

\[ v = \sqrt{\frac{P_0}{\rho}} \]

$P_0$ - atmospheric pressure
$\rho$ - the atmospheric density.

Standard conditions:
$P_0 = 1.01 \times 10^5 \text{ N/m}^2$
$\rho = 1.29 \text{ kg/m}^3 \ (t = 0 \degree C)$

\[ v = \sqrt{1.40 \frac{P_0}{\rho}} = \sqrt{1.40 \times \frac{1.01 \times 10^5 \text{ N/m}^2}{1.29 \text{ kg/m}^3}} = 331 \text{ m/s} \]

The speed of sound in liquids and in solids is considerably higher than in air.

- Water: 1,531 \text{ m/s}
- Iron: 5,130 \text{ m/s}
- Glass: 5,000 \text{ m/s}
\[ u = \sqrt{\frac{\rho}{\rho_0}} = \sqrt{\frac{P \cdot V}{P_0 \cdot V}} = \sqrt{\frac{NKT}{M}} \]

**ideal gas law**

\[ PV = NKT \quad \text{Temperature} \]

\[ M = m_a \cdot N \]

\[ \Rightarrow u = \sqrt{\frac{NKT}{m_a \cdot N}} = \sqrt{\frac{KT}{m_a}} \]

\[ f = \frac{C_p}{C_v} \quad \text{ratio of the specific heat capacity at constant pressure to the specific heat capacity at constant volume} \]
Example: Thunder, lightning and a rule of thumb

There is a rule of thumb for estimating how far away a thunderstorm is. You can estimate your distance from a bolt of lightning by counting the seconds between seeing the flash and hearing the thunder and then dividing by 3 to obtain the distance in km. Why does this rule work?

\[ d = \text{Veight} \times \text{tVeight} \quad \text{since} \quad \text{Veight} = 3.0 \times 10^8 \text{ m/s} \quad \text{light travels distance} \quad d \quad \text{rapidly and tVeight} \approx 0 \]

\[ d = \text{Vsoun} \times \text{tSoun} = (0.331 \text{ km/s}) \times (\text{tSoun}, \text{s}) = \]
\[ = (0.331 \text{ km/s} \times 3) \times (\frac{\text{tSoun}, \text{s}}{3}) = (\frac{\text{tSoun}, \text{km}}{3}) \]
The Speed of Sound

Problem: Both krypton (Kr) and neon (Ne) can be approximated at monatomic ideal gases. The atomic mass of krypton is 83.8 u, while that of neon is 20.2 u. A loudspeaker produces a sound whose wavelength in krypton is 1.25 m. If the loudspeaker were used to produce sound of the same frequency in neon at the same temperature, what would be the wavelength?

\[
\text{\textbf{Solution}}
\]

According to equation of wave propagation, 
\[
\lambda = \frac{v}{f} = \frac{\lambda}{f}
\]

Since frequency is the same in each gas we have 
\[
f_{\text{Kr}} = f_{\text{Ne}} \quad \text{or} \quad \frac{v_{\text{Kr}}}{\lambda_{\text{Kr}}} = \frac{v_{\text{Ne}}}{\lambda_{\text{Ne}}}
\]

\[
\Rightarrow \lambda_{\text{Ne}} = \lambda_{\text{Kr}} \left( \frac{v_{\text{Ne}}}{v_{\text{Kr}}} \right)
\]

We know that \[v = \sqrt{\frac{\text{kT}}{m}}\]

\[
\Rightarrow \text{the ratio of the speeds} \quad \frac{v_{\text{Ne}}}{v_{\text{Kr}}} = \sqrt{\frac{\text{m}_{\text{Kr}}}{\text{m}_{\text{Ne}}}} = \sqrt{\frac{83.8 \times 10^{-3} \text{ kg/mole}}{20.2 \times 10^{-3} \text{ kg/mole}}} = \frac{1.39 \times 10^{-25} \text{ kg}}{1.35 \times 10^{-25} \text{ kg}} = \frac{1.39}{1.35} \approx 0.55 \text{ m}
\]

We need to calculate the mass of each type of atom in the gas

\[
\text{m}_{\text{Kr}} = \frac{\text{atomic mass, kg/mol}}{\text{Avogadro number, mol}} = \frac{83.8 \times 10^{-3} \text{ kg/mol}}{6.022 \times 10^{23} \text{ mol}} = 1.39 \times 10^{-25} \text{ kg}
\]

\[
\text{m}_{\text{Ne}} = \frac{20.2 \times 10^{-3}}{6.022 \times 10^{23}} = 3.35 \times 10^{-25} \text{ kg}
\]

\[
\Rightarrow \lambda_{\text{Ne}} = \lambda_{\text{Kr}} \sqrt{\frac{\text{m}_{\text{Kr}}}{\text{m}_{\text{Ne}}}} = (1.25 \text{ m}) \sqrt{\frac{1.39 \times 10^{-25} \text{ kg}}{3.35 \times 10^{-25} \text{ kg}}} = 0.55 \text{ m}
\]
Consider the tube filled with air shown in the figure. We generate a harmonic sound wave traveling to the right along the axis of the tube. One simple method is to place a speaker at the left end of the tube and drive it at a particular frequency. Consider an air element of thickness $\Delta x$ which is located at position $x$ before the sound wave is generated. This is known as the "equilibrium position" of the element. Under these conditions the pressure inside the tube is constant. In the presence of the sound wave the element oscillates about the equilibrium position. At the same time the pressure at the location of the element oscillates about its static value. The sound wave in the tube can be described using one of two parameters:

\[ s(x, t) = s_m \cos(kx - \omega t) \]

\[ \Delta p(x, t) = \Delta p_m \sin(kx - \omega t) \]
Traveling sound waves.

One such parameter is the distance \( s(x,t) \) of the element from its equilibrium position
\[ s(x,t) = s_m \cos(kx - \omega t) \]
The constant \( s_m \) is the displacement amplitude of the wave. The angular wavenumber \( k \) and the angular frequency \( \omega \) have the same meaning as in the case of the transverse waves studied in chapter 16.

The second possibility is to use the pressure variation \( \Delta p \) from the static value.
\[ \Delta p(x,t) = \Delta p_m \sin(kx - \omega t) \]
The constant \( \Delta p_m \) is the wave's pressure amplitude.

The two amplitudes are connected by the equation:
\[ \Delta p_m = (v \nu \omega) s_m \]

Note: The displacement and the pressure variation have a phase difference of 90°. As a result when one parameter has a maximum the other has a minimum and vice versa.
Interference

Consider two point sources of sound waves $S_1$ and $S_2$ shown in the figure. The two sources are in phase and emit sound waves of the same frequency. Waves from both sources arrive at point P whose distance from $S_1$ and $S_2$ is $L_1$ and $L_2$ respectively. The two waves interfere at point P.

At time $t$ the phase of sound wave 1 arriving from $S_1$ at point P is $\phi_1 = kL_1 - \omega t$
At time $t$ the phase of sound wave 2 arriving from $S_2$ at point P is $\phi_2 = kL_2 - \omega t$

In general the two waves at P have a phase difference

$$\phi = |\phi_2 - \phi_1| = |kL_2 - \omega t - (kL_1 - \omega t)| = k|L_2 - L_1| = \frac{2\pi}{\lambda}|L_2 - L_1|$$

The quantity $|L_2 - L_1|$ is known as the "path length difference" $\Delta L$

between the two waves. Thus $\phi = \frac{2\pi}{\lambda} \Delta L$

Here $\lambda$ is the wavelength of the two waves.
Constructive interference.
The wave at P resulting from the interference of the two waves that arrive from S<sub>1</sub> and S<sub>2</sub> has a maximum amplitude when the phase difference $\phi = 2\pi m$

$$m = 0, 1, 2, \ldots \rightarrow \frac{2\pi}{\lambda} \Delta L = 2\pi m \rightarrow \Delta L = m\lambda$$

$\Delta L = 0, \lambda, 2\lambda, \ldots$

Destructive interference.
The wave at P resulting from the interference of the two waves that arrive from S<sub>1</sub> and S<sub>2</sub> has a minimum amplitude when the phase difference

$$\phi = \pi(2m + 1) \quad m = 0, 1, 2, \ldots \rightarrow \frac{2\pi}{\lambda} \Delta L = \pi(2m + 1) \rightarrow$$

$$\Delta L = \left(m + \frac{1}{2}\right)\lambda \quad \Delta L = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots$$

$\Delta L$ equal to an integral multiple of $\lambda \rightarrow$ constructive interference

$\Delta L$ equal to a half-integral multiple of $\lambda \rightarrow$ destructive interference
At an open-air concert on a hot day \( (T_c = 25^\circ C, V_s = 346.5 \text{ m/s}) \), a person sits at a location that 7.0 m and 9.1 m respectively from speakers at each side of the stage. A musician, warming up, plays a single 494 Hz tone. What does the spectator hear?

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**Given:**
- \( d_1 = 7.0 \text{ m} \)
- \( d_2 = 9.1 \text{ m} \)
- \( f = 494 \text{ Hz} \)
- \( V_s = 346.5 \text{ m/s} \)

**Find:** Whether there is interference and what type?

The wavelength of the sound waves is

\[
\lambda = \frac{V_s}{f} = \frac{346.5 \text{ m/s}}{494 \text{ Hz}} = 0.701 \text{ m}
\]

Thus \( d_1 = (7.0 \text{ m})(\frac{\lambda}{0.701 \text{ m}}) = 10\lambda \)

Express the path length

\( d_2 = (9.1 \text{ m})(\frac{\lambda}{0.701 \text{ m}}) = 13\lambda \)

The path difference

\[
P_D = d_2 - d_1 = 13\lambda - 10\lambda = 3\lambda
\]

This is an integral number of wavelengths, so constructive interference occurs and a loud sound is heard by spectators.

If the path difference had been \( 2.5\lambda \),

\( 5(\lambda) - 2\lambda \),

- destructive — no sound would be heard.

During a concert the sound wouldn’t be a single frequency tone. Spectators at certain locations might not hear a small portion of the sound (wouldn’t notice).
Constructive and Destructive Interference of Sound Waves

Assume that two loudspeakers in the figure are vibrating out of phase instead of in phase. (see example 2 from the text) The speed of sound is 343 m/s. What is the smallest frequency that will produce destructive interference at point C?

\[
\begin{align*}
\text{Path difference} & \quad P \cdot D = AC - BC = 4.00\text{m} - 2.40\text{m} = 1.60\text{m} \\
\Delta \phi & \quad = \frac{2\pi}{\lambda} (P \cdot D) = \frac{2\pi m}{\lambda} = \frac{2\pi m}{\lambda} = \frac{2\pi(1.60\text{m})}{\lambda} \\
\end{align*}
\]

\[
\begin{align*}
\Rightarrow \text{Since initially two speakers vibrated out of phase, destructive interference will occur if this difference in distance AC} - \text{BC is an integer number of wavelengths.} \\
\text{The smallest } f \text{ that will produce destructive interference corresponds to the longest wavelength that satisfies this condition. The longest wavelength that satisfies this condition is } \lambda = 1.60\text{m}. \\
\text{Using } v = \lambda f, \text{we find} \\
f = \frac{v}{\lambda} = \frac{343\text{m/s}}{1.60\text{m}} = 214.42 \text{Hz}.
\end{align*}
\]
Constructive and Destructive Interference of Sound Waves

Speakers A and B are vibrating in phase. They are directly facing each other, are 7.80 m apart, and are each playing a 73.0 Hz tone. The speed of sound is 343 m/s. On the line between the speakers there are three points where constructive interference occurs. What are the distances of these three points from speaker A?

![Diagram showing the positions of constructive interference points.]

- The points of constructive interference will occur symmetrically about the center point. So there is also a point of constructive interference 1.55 m from speaker B.

The wavelength of the tone is

\[ \lambda = \frac{\text{speed of sound}}{\text{frequency}} = \frac{343 \text{ m/s}}{73.0 \text{ Hz}} = 4.70 \text{ m} \]

**Constructive interference occur when PD in reaching point P is an integer # of wavelengths.**

\[ PD = (L - x) - x = n \lambda \]

Solving for \( x \):

\[ x = \frac{L - n \lambda}{2} \]

- When \( n = 0 \), \( x = \frac{7.80}{2} = 3.90 \text{ m} \)
- When \( n = 1 \), \( x = \frac{(7.80 - 4.70)}{2} = 1.55 \text{ m} \)

These values correspond to positions of constructive interference that lie to the left of A or to the right of B.
**Intensity of a sound wave**

Consider a wave that is incident normally on a surface of area $A$. The wave transports energy. As a result, power $P$ (energy per unit time) passes through $A$. We define at the wave intensity $I$ the ratio $P/A$:

$$I = \frac{P}{A} \quad \text{SI units: } \text{W/m}^2$$

The intensity of a harmonic wave with displacement amplitude $s_m$ is given by:

$$I = \left( \frac{\rho v \omega^2}{2} \right) s_m^2. \quad \text{In terms of the pressure amplitude } I = \left( \frac{1}{2 \rho v} \right) \Delta p_m^2$$

Consider a point source $S$ emitting a power $P$ in the form of sound waves of a particular frequency. The surrounding medium is isotropic so the waves spread uniformly. The corresponding wavefronts are spheres that have $S$ as their center. The sound intensity at a distance $r$ from $S$ is:

$$I = \frac{P}{4\pi r^2}$$

The intensity of a sound wave for a point sources is proportional to $\frac{1}{r^2}$.
**The decibel**

The auditory sensation in humans is proportional to the logarithm of the sound intensity $I$. This allows the ear to perceive a wide range of sound intensities. The threshold of hearing $I_o$ is defined as the lowest sound intensity that can be detected by the human ear. $I_o = 10^{-12}$ W/m$^2$

The sound level $\beta$ is defined in such a way as to mimic the response of the human ear. $\beta = 10 \log \left( \frac{I}{I_o} \right)$  $\beta$ is expressed in decibels (dB)

We can invert the equation above and express $I$ in terms of $\beta$ as:

$I = I_o \times 10^{(\beta/10)}$

**Note 1:** For $I = I_o$ we have: $\beta = 10 \log 1 = 0$

**Note 2:** $\beta$ increases by 10 decibels every time $I$ increases by a factor of 10

For example $\beta = 40$ dB corresponds to $I = 10^4 I_o$
Example 1: Express the threshold of hearing (2.5 x 10^{-12} \text{ W/m}^2) and the threshold of pain (1 \text{ W/m}^2) in decibels.

\[
I_{\text{in dB}} = (10 \text{ dB}) \times \log \left(\frac{2.5 \times 10^{-12} \text{ W/m}^2}{1 \times 10^{-12} \text{ W/m}^2}\right) = (10 \text{ dB})(\log 2.5) = 4.0 \text{ dB}.
\]

\[
I_{0 \text{ dB}} = (10 \text{ dB}) \times \log \left(\frac{1 \text{ W/m}^2}{1 \times 10^{-12} \text{ W/m}^2}\right) = (10 \text{ dB}) \times \frac{\log (10^2)}{\log 1} = 120 \text{ dB}
\]

\[
\log a^b = b \log a
\]

For comparison:
- Jet engine: \(-130 \text{ dB} - 10 \text{ W/m}^2\)
- Rock music: \(-115 \text{ dB} - 0.3 \text{ W/m}^2\)
- Rupture of eardrum: \(-160 \text{ dB} - 10^{-4} \text{ W/m}^2\)

Example 2: At a distance of 60m from a jet airline the intensity is 1 \text{ W/m}^2. I_{180} = ?

\[
I_2 = \frac{r_2^2}{r_1^2} I_1 = \frac{(60 \text{ m})^2}{(180 \text{ m})^2} \frac{1 \text{ W/m}^2}{1 \text{ W/m}^2} = 0.11 \text{ W/m}^2
\]
The intensity of sound near a loud rock band is 120dB. What is the intensity of sound near two such rock bands playing together?

\[ \beta_1 = 120 \text{dB} = 10 \log \frac{I_1}{I_0} \]

where \( I_1 \) - intensity of sound in W/m² of the first band

\[ I_0 = 1 \times 10^{-12} \text{ W/m²} \]

\[ \beta_2 = 10 \log \left( \frac{2I_1}{I_0} \right) = 10 \log 2 + 10 \log \left( \frac{I_1}{I_0} \right) = 10 \log 2 + \beta_1 = 3 + \beta_1 = 123 \text{dB} \]

\[ \log ab = \log a + \log b \]

\[ \log \frac{a}{b} = \log a - \log b \]
Problem (Decibels): Two identical rifles are shot at the same time and the sound intensity level is 800dB. What would be the sound intensity level if only one rifle were shot? (hint: the answer is not 400db)

- Let $\beta_2$ and $I_2$ denote the intensity level and the intensity when 2 rifles are shot.

- Let $\beta_1$ and $I_1$ - when 1 rifle is shot

\[
\beta_2 = 10 \log \left( \frac{I_2}{I_0} \right) = 10 \log \left( \frac{2I_1}{I_0} \right)
\]

where $I_0$ is the intensity of a single rifle.

\[
\Rightarrow \frac{2I_1}{I_0} = 10^{\frac{\beta_2}{10}}
\]

\[
I_1 = \left( \frac{I_0}{2} \right) 10^{\frac{\beta_2}{10}} = \frac{\left( 1x10^{-12} \text{ W/m}^2 \right)}{2} \cdot 10^{8.00} = 5.00 \times 10^{-5} \text{ W/m}^2
\]

\[
\Rightarrow \beta_1 = 10 \log \left( \frac{I_1}{I_0} \right) = 10 \log \left( \frac{5.00 \times 10^{-5} \text{ W/m}^2}{100 \times 10^{-12} \text{ W/m}^2} \right) = 77.0 \text{ dB}
\]
**Problem**: Two sound sources each emit sound power uniformly in all directions. There are no reflections. Both sources are located on the x axis, one at the origin and the other at x = +123m. The source of the origin emits four times more power than the other source. Where on the x axis is the intensity of each sound equal? Note there are 2 answers.

\[
I = \frac{P}{4\pi x^2}
\]

for uniformly radiating sources intensity (the power per unit area)

Point where the intensity \(I_1 = I_2\) is located at a distance \(x\) from the origin.

\[
\frac{P_1}{4\pi x^2} = \frac{P_2}{4\pi (123-x)^2}
\]

or \(\frac{P_1}{P_2} = \frac{(x)^2}{(123-x)^2} = 4\)

Use the fact that \(P_1 = 4P_2\)

**Positive Root**

\[
x = \frac{2}{123-x} = 2
\]

\[
x = 2(123-x)
\]

\[
x = 82.0\text{ m}
\]

**Negative Root**

\[
x = \frac{2}{123-x} = -2
\]

\[
x = 2(123-x)
\]

\[
x = 246\text{ m}
\]
Sound standing waves in pipes
Consider a pipe filled with air that is open at both ends.
Sound waves that have wavelengths that satisfy a particular relation with the length L of the pipe setup standing waves that have sustained amplitudes.

The simplest pattern can be set up in a pipe that is open at both ends as shown in fig.a. In such a pipe standing waves have an antinode (maximum) in the displacement amplitude. The amplitude of the standing wave is plotted as function of distance in fig.b. The pattern has a node at the pipe center since two adjacent antinodes are separated by an anode (minimum). The distance between two adjacent antinodes is $\lambda/2$.

Thus $L = \lambda / 2 \rightarrow \lambda = 2L$. Its frequency $f = \frac{v}{\lambda} = \frac{v}{2L}$.

The standing wave of fig.b is known as the "fundamental mode" or "first harmonic" of the tube.

Note: Antinodes in the displacement amplitude correspond to nodes in the pressure amplitude. This is because $s_m$ and $\Delta p_m$ are 90° out of phase.
Standing waves in tubes open at both ends

The next three standing wave patterns are shown in fig.a. The wavelength \( \lambda_n = \frac{2L}{n} \)
where \( n = 1, 2, 3, ... \) The integer \( n \) is known as the harmonic number

The corresponding frequencies \( f_n = \frac{nv}{2L} \)

Standing waves in tubes open at one end and closed at the other

The first four standing wave patterns are shown in fig.a. They have an antinode at the open end and an node at the closed end.

The wavelength \( \lambda_n = \frac{4L}{n} \) for \( n=1,3,5,7, ... \)
Longitudinal Standing Waves

Problem A person hums into the top of a well and finds that standing waves are established at frequencies of 42, 70.0 and 98 Hz. The frequency of 42 Hz is not necessarily the fundamental frequency. The speed of sound is 343 m/s. How deep is the well?

The well is open at the top and closed at the bottom, so it can be approximated as a column of air that is open at only one end. 

\[ f_n = n \left( \frac{V}{4L} \right) \quad n = 1, 3, 5, 7 \ldots \]

The depth \( L \) can be calculated as

\[ L = \frac{V}{4f_n} \]

We know that two of the natural frequencies are 42 and 70.0 Hz. The ratio of these two frequencies

\[ \frac{70.0 \text{ Hz}}{42.0 \text{ Hz}} = \frac{5}{3} \]

\[ \Rightarrow f_3 = 42 \text{ Hz} \quad f_5 = 70.0 \text{ Hz} \]

\[ L = \frac{3V}{4f_3} = \frac{3(343 \text{ m/s})}{4(42 \text{ Hz})} = 6.1 \text{ m} \]
Beats.

If we listen to two sound waves of equal amplitude and frequencies \( f_1 \) and \( f_2 \) \((f_1 > f_2 \text{ and } f_1 \approx f_2)\) we perceive them as a sound of frequency \( f_{av} = \frac{f_1 + f_2}{2} \). In addition we also perceive "beats" which are variations in the intensity of the sound with frequency \( f_{beat} = f_1 - f_2 \). The displacements of the two sound waves are given by the equations: \( s_1 = s_m \cos \omega_1 t \), and \( s_2 = s_m \cos \omega_2 t \). These are plotted in fig.a and fig.b.

Using the principle of superposition we can determine the resultant displacement as:

\[
s = s_1 + s_2 = s_m (\cos \omega_1 t + \cos \omega_2 t) = 2s_m \cos \left(\frac{\omega_1 - \omega_2}{2}\right) t \cos \left(\frac{\omega_1 + \omega_2}{2}\right) t
\]

\[
s = [2s_m \cos \omega't] \cos \omega t \quad \text{where} \quad \omega' = \frac{\omega_1 - \omega_2}{2} \quad \text{and} \quad \omega = \frac{\omega_1 + \omega_2}{2}
\]

Since \( \omega_1 \approx \omega_2 \rightarrow \omega \square \omega' \)
\[ s = \left[ 2s_m \cos \omega't \right] \cos \omega t \quad \text{where} \quad \omega' = \frac{\omega_1 - \omega_2}{2} \quad \text{and} \quad \omega = \frac{\omega_1 + \omega_2}{2} \]

The displacement \( s \) is plotted as function of time in the figure. We can regard it as a cosine function whose amplitude is equal to \( |2s_m \cos \omega't| \).

The amplitude is time dependent but varies slowly with time. The amplitude exhibits a maximum whenever \( \cos \omega't \) is equal to either +1 or -1 which happens twice within one period of the \( \cos \omega't \) function.

Thus the angular frequency of the beats \( \omega_{\text{beat}} = 2\omega' = 2\left( \frac{\omega_1 - \omega_2}{2} \right) = \omega_1 - \omega_2 \)

The frequency of the beats \( f_{\text{beat}} = 2\pi \omega_{\text{beat}} = 2\pi \omega_1 - 2\pi \omega_2 = f_1 - f_2 \)
The Doppler Effect

A train approaching a siren. The train encounters more wave fronts per unit time than when stationary.

A train receding from the siren. The train encounters fewer wave fronts per unit time than when stationary.

\[ f = f + \text{additional \# of condensations} \]
\[ \text{additional \# of condensations in a time } t = \frac{(V_r \cdot t)}{\lambda} \]

in 1 sec additional \# = \( \frac{V_r}{\lambda} \)

\[ f = f + \frac{V_r}{\lambda} = f(1 + \frac{V_r}{V}) = f(1 + \frac{V_r}{V}) \]

A receiver on the train will detect a higher frequency when approaching a siren, and a lower frequency when receding.

\[ f' = \frac{V}{\lambda} \text{ (ref. fr. ground)}; \quad f' = \frac{V}{\lambda} \text{ (ref. fr. of the train)} \]
\[ f' = \frac{V'}{\lambda'} \quad \text{ and } \quad \frac{V'}{V'} = V + V_r \quad (V_r - speed of the train) \]

\[ f = f(1 \pm \frac{V_r}{V}) \quad \text{ for approaching receiver} \]
\[ f = f(1 \pm \frac{V_r}{V}) \quad \text{ for receding receiver} \]
The source (train) is in motion, the receiver is stationary.

A stationary receiver will detect a higher frequency when it is front of the train and a lower frequency when behind the train.

The wavelength ahead of the train is shorter $\lambda' = \lambda - \frac{V_E}{f}$ and behind the train is longer $\lambda' = \lambda + \frac{V_E}{f}$ when the train is stationary.

$$f' = \frac{V}{\lambda'} = \frac{V}{\lambda - \frac{V_E}{f}} = \frac{V}{\lambda + \frac{V_E}{f}} = f \left( \frac{1}{1 \pm \frac{V_E}{c}} \right)$$

for approaching emitter

$$f' = f \left( \frac{1}{1 \pm \frac{V_E}{c}} \right)$$

for receding emitter

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Problem: The trucks travel at the same speed. They are far apart on adjacent lanes and approach each other essentially head-on. One driver hears the horn of the other truck at a frequency that is 1.20 times the frequency he hears when the truck is stationary. The speed of sound is 343 m/s. At what speed is each truck moving?
The Doppler effect
Consider the source and the detector of sound waves shown in the figure. We assume that the frequency of the source is equal to $f$.

We take as the reference frame that surrounding air through which the sound waves propagate. If there is relative motion between the source and the detector then the detector perceives the frequency of the sound as $f' \neq f$. If the source or the detector move towards to each other $f' > f$. If on the other hand the source or the detector move away from each other $f' < f$. This is known as the "Doppler" effect. The frequency $f'$ is given by the equation: $f' = f \frac{v \pm v_D}{v \pm v_S}$. Here $v_S$ and $v_D$ are the speeds of the source and detector with respect to air, respectively.

When the motion of the detector or source is towards each other the sign of the speed must give an upward shift in frequency. If on the other hand the motion is away from each other the sign of the speed must give a downward shift in frequency. The four possible combinations are illustrated in the next page.
\[ f' = f \frac{v + v_D}{v - v_S} \quad f' > f \]

\[ f' = f \frac{v - v_D}{v + v_S} \quad f' < f \]

\[ f' = f \frac{v - v_D}{v + v_S} \]