ALL QUESTIONS ARE WORTH 20 POINTS. WORK OUT FIVE PROBLEMS.

NOTE: Clearly write out solutions and answers (circle the answers) by section for each part (a., b., c., etc.)

Important Formulas:

1. **Motion along a straight line with a constant acceleration**
   \[ v_{\text{aver. speed}} = \frac{\text{[dist. taken]}}{\text{[time trav.]}} = \frac{S}{t}; \]
   \[ v_{\text{aver. vel.}} = \frac{\Delta x}{\Delta t}; \]
   \[ v_{\text{ins}} = \frac{dx}{dt}; \]
   \[ a_{\text{aver.}} = \frac{\Delta v_{\text{aver. vel.}}}{\Delta t}; \]
   \[ a = \frac{dv}{dt}; \]
   \[ v = v_0 + at; \quad x = \frac{1}{2}(v_0+v)t; \quad x = v_0 t + \frac{1}{2} at^2; \quad v^2 = v_0^2 + 2ax \text{ (if } x_0=0 \text{ at } t_0=0) \]

2. **Free fall motion (with positive direction ↑)**
   \[ g = 9.80 \text{ m/s}^2; \]
   \[ y = v_{\text{aver. t}}; \]
   \[ v_{\text{aver.}} = \frac{(v+v_0)}{2}; \]
   \[ v = v_0 - gt; \quad y = v_0 t - \frac{1}{2} g t^2; \quad v^2 = v_0^2 - 2gy \text{ (if } y_0=0 \text{ at } t_0=0) \]

3. **Motion in a plane**
   \[ v_x = v_0 \cos \theta; \]
   \[ v_y = v_0 \sin \theta; \]
   \[ x = v_{ox} t + \frac{1}{2} a_x t^2; \quad y = v_{oy} t + \frac{1}{2} a_y t^2; \quad v_x = v_{ox} + at; \quad v_y = v_{oy} + at; \]

4. **Projectile motion (with positive direction ↑)**
   \[ v_x = v_{ox} = v_0 \cos \theta; \]
   \[ x = v_{ox} t; \]
   \[ x_{\text{max}} = \left(2 v_0^2 \sin \theta \cos \theta / g\right) = \left(v_0^2 \sin 2 \theta / g\right) \text{ for } y_{\text{in}} = y_{\text{fin}}; \]
   \[ v_y = v_{oy} - gt = v_0 \sin \theta - gt; \]
   \[ y = v_{oy} t - \frac{1}{2} gt^2; \]

5. **Uniform circular Motion**
   \[ a = v^2/r, \]
   \[ T = 2\pi\sqrt{r/v} \]

6. **Relative motion**
   \[ \vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}; \]
   \[ \vec{a}_{PA} = \vec{a}_{PB}; \]

7. **Component method of vector addition**
\[ \mathbf{A} = \mathbf{A}_1 + \mathbf{A}_2 ; \mathbf{A}_x = \mathbf{A}_{x1} + \mathbf{A}_{x2} \text{ and } \mathbf{A}_y = \mathbf{A}_{y1} + \mathbf{A}_{y2}; \quad A = \sqrt{A_x^2 + A_y^2}; \quad \theta = \tan^{-1} \left| \frac{A_y}{A_x} \right|; \]

The scalar product \( \mathbf{A} \cdot \mathbf{B} = ab \cos \phi \)

\[
\mathbf{a} \cdot \mathbf{b} = (a_x \mathbf{i} + a_y \mathbf{j} + a_z \mathbf{k}) \cdot (b_x \mathbf{i} + b_y \mathbf{j} + b_z \mathbf{k})
\]

\[
\mathbf{a} \cdot \mathbf{b} = a_x b_x + a_y b_y + a_z b_z
\]

The vector product \( \mathbf{a} \times \mathbf{b} = (a_x \mathbf{i} + a_y \mathbf{j} + a_z \mathbf{k}) \times (b_x \mathbf{i} + b_y \mathbf{j} + b_z \mathbf{k}) \)

\[
\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a} = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
a_x & a_y & a_z \\
b_x & b_y & b_z
\end{vmatrix} = \mathbf{i} \left| \begin{array}{cc}
a_y & a_z \\
b_y & b_z
\end{array} \right| - \mathbf{j} \left| \begin{array}{cc}
a_x & a_z \\
b_x & b_z
\end{array} \right| + \mathbf{k} \left| \begin{array}{cc}
a_x & a_y \\
b_x & b_y
\end{array} \right| = (a_y b_z - a_z b_y) \mathbf{i} + (a_z b_x - a_x b_z) \mathbf{j} + (a_x b_y - a_y b_x) \mathbf{k}
\]

1. Second Newton’s Law  
   \( m \mathbf{a} = F_{\text{net}}; \)

2. Kinetic friction \( f_k = \mu_k N; \)

3. Static friction \( f_s = \mu_s N; \)

4. Universal Law of Gravitation: \( F = G \frac{m M}{r^2}; \) \( G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2; \)

5. Drag coefficient \( D = \frac{1}{2} \rho C A v^2; \)

6. Terminal speed \( v_t = \sqrt{\frac{2 m g}{C \rho A}}; \)

7. Centripetal force: \( F_c = m v^2 / r; \)

8. Speed of the satellite in a circular orbit: \( v^2 = G M_\odot / r; \)

9. The work done by a constant force acting on an object: \( W = F d \cos \phi = \mathbf{F} \cdot \mathbf{d}; \)

10. Kinetic energy: \( K = \frac{1}{2} m v^2; \)

11. Total mechanical energy: \( E = K + U; \)

12. The work-energy theorem: \( W = K_f - K_i; \) \( W_{mc} = \Delta K + \Delta U = E_f - E_o; \)

13. The principle of conservation of mechanical energy: when \( W_{mc} = 0, E_f = E_o; \)

14. Work done by the gravitational force: \( W_g = m g d \cos \phi; \)
1. **Work done in Lifting and Lowering the object:**

\[ \Delta K = K_f - K_i = W_a + W_g \quad \text{if} \quad K_f = K_i \quad W_a = -W_g \]

2. **Spring Force:** \( F_s = -kx \) (Hooke's law)

3. **Work done by a spring force:** \( W_s = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2 \); if \( x_i = 0 \) and \( x_f = x \); \( W_s = -\frac{1}{2}kx^2 \)

4. **Work done by a variable force:** \( W = \int_{x_i}^{x_f} F(x)\,dx \)

5. **Power:** \( P_{avg} = \frac{W}{\Delta t} \); \( P = \frac{dW}{dt} \); \( P = F \cos \phi = \vec{F} \cdot \vec{v} \)

6. **Potential energy:** \( \Delta U = -W \); \( \Delta U = -\int_{x_i}^{x_f} F(x)\,dx \)

7. **Gravitational Potential Energy:**

\[ \Delta U = mg(y_f - y_i) = mg\Delta y \quad \text{if} \quad y_i = 0 \quad \text{and} \quad U_i = 0 \quad U(y) = mg\ y \]

8. **Elastic potential Energy:** \( U(x) = \frac{1}{2}kx^2 \)

9. **Potential energy curves:** \( F(x) = -\frac{dU(x)}{dx} \); \( K(x) = E_{ mec} - U(x) \)

10. **Work done on a system by an external force:**

    Friction is not involved \( W = \Delta E_{mec} = \Delta K + \Delta U \)

    When kinetic friction force acts within the system \( W = \Delta E_{mec} + \Delta E_{sk} \)

    \( \Delta E_{sk} = \int f_k \,dt \)

11. **Conservation of energy:**

    \( W = \Delta E = \Delta E_{mec} + \Delta E_{sk} + \Delta E_{int} \)

    for isolated system \( (W = 0) \quad \Delta E_{mec} + \Delta E_{sk} + \Delta E_{int} = 0 \)

12. **Power:** \( P_{avg} = \frac{\Delta E}{\Delta t} \); \( P = \frac{dE}{dt} \)

13. **Center of mass:** \( \vec{r}_{com} = \frac{1}{M} \sum_{i=1}^{n} m_i \vec{r}_i \)

14. **Newtons' Second Law for a system of particles:** \( \vec{F}_{net} = M \ddot{a}_{com} \)
1. **Linear Momentum and Newton’s Second law for a system of particles:** \( \vec{P} = M \vec{v}_{com} \) and \( \vec{F}_{net} = \frac{d\vec{P}}{dt} \)

2. **Collision and impulse:** 
   \[
   \vec{J} = \int_{t_i}^{t_f} \vec{F}(t) \, dt; \quad J = F_{avg} \Delta t; \quad \text{when a stream of bodies with mass } m \text{ and speed } v, \text{ collides with a body whose position is fixed} \]
   \[ F_{avg} = - \frac{n}{\Delta t} \Delta p = - \frac{n}{\Delta t} m \Delta v = - \frac{\Delta m}{\Delta t} \Delta v \]

   **Impulse-Linear Momentum Theorem:** \( \vec{p}_f - \vec{p}_i = \vec{J} \)

3. **Law of Conservation of Linear momentum:** \( \vec{p}_i = \vec{p}_f \) for closed, isolated system

4. **Inelastic collision in one dimension:** \( \vec{p}_{1f} + \vec{p}_{2f} = \vec{p}_{1i} + \vec{p}_{2i} \)

5. **Motion of the Center of Mass:** The center of mass of a closed, isolated system of two colliding bodies is not affected by a collision.

6. **Elastic Collision in One Dimension:** 
   \[ v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}; \quad v_{2f} = \frac{2 m_1}{m_1 + m_2} v_{1i} \]

7. **Collision in Two Dimensions:** 
   \[ p_{1ix} + p_{2ix} = p_{1fx} + p_{2fx}; \quad p_{1iy} + p_{2iy} = p_{1fy} + p_{2fy} \]
   \[ R v_{rel} = M a \] (first rocket equation)

8. **Variable-mass system:** 
   \[ v_f - v_i = v_{rel} \ln \frac{M_i}{M_f} \] (second rocket equation)

9. **Angular Position:** \( \theta = \frac{S}{r} \) (radian measure)

10. **Angular Displacement:** \( \Delta \theta = \theta_2 - \theta_1 \) (positive for counterclockwise rotation)

11. **Angular velocity and speed:** \( \omega_{avg} = \frac{\Delta \theta}{\Delta t}; \quad \omega = \frac{d \theta}{dt} \) (positive for counterclockwise rotation)

12. **Angular acceleration:** \( \alpha_{avg} = \frac{\Delta \omega}{\Delta t}; \quad \alpha = \frac{d \omega}{dt} \)
1. **Angular Acceleration**: 
\[ \theta - \theta_0 = \frac{1}{2} (\omega_0 + \omega) t \]
\[ \omega = \omega_0 + \alpha t \]

\[ \theta - \theta_0 = \frac{1}{2} (\omega_0 + \omega) t \]
\[ \omega = \omega_0 + \alpha t \]
\[ \omega^2 = \omega_0^2 + 2 \alpha (\theta - \theta_0) \]
\[ \theta - \theta_0 = \omega t - \frac{1}{2} \omega t^2 \]

2. **Linear and Angular Variables Related**:
\[ s = \theta r ; \quad v = \omega r ; \quad a_r = \alpha r ; \quad a_r = \omega^2 r ; \quad T = \frac{2 \pi r}{\nu} = \frac{2 \pi}{\omega} \]

3. **Rotational Kinetic Energy and Rotational Inertia**:
\[ K = \frac{1}{2} I \omega^2 ; \quad I = \sum m_i r_i^2 \text{ for body as a system of discrete particles;} \]
\[ I = \int r^2 dm \text{ for a body with continuously distributed mass.} \]

4. **The Parallel Axes Theorem**: \[ I = I_{com} + M h^2 \]

5. **Torque**: \[ \tau = r F = r_\perp F = r F \sin \phi \]

6. **Newton’s Second Law in Angular Form**: \[ \tau_{net} = I \alpha \]

7. **Work and Rotational Kinetic Energy**: \[ W = \int_\theta^\phi \tau d \theta ; \quad W = \tau (\theta_f - \theta_i) \text{ for } \tau = \text{const;} \]
\[ P = \frac{dW}{dt} \]
\[ \Delta K = K_f - K_i = \frac{1}{2} I \omega_f^2 - \frac{1}{2} I \omega_i^2 = W \text{ work energy theorem for rotating bodies} \]
\[ v_{com} = \omega R \]
\[ K = \frac{1}{2} I_{com} \omega^2 + \frac{1}{2} m v_{com}^2 \]

8. **Rolling Bodies**: \[ a_{com} = \alpha R \]
\[ a_{com} = \frac{g \sin \theta}{1 + \frac{I_{com}}{M R^2}} \text{ for rolling smoothly down the ramp} \]

9. **Torque as a Vector**: \[ \vec{\tau} = \vec{r} \times \vec{F} ; \quad \tau = r F \sin \phi = r F_\perp = r_\perp F \]
1. Angular Momentum of a particle: 
\[ \vec{L} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v}) \; ; \]
\[ l = r m v \sin \phi = r p_\perp = r m v_\perp = r_\perp p = r_\perp m v \]

2. Newton’s Second law in Angular Form: 
\[ \vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt} \]
\[ \vec{L} = \sum_{i=1}^{n} \vec{L}_i \]

3. Angular momentum of a system of particles: 
\[ \vec{\tau}_{\text{net \, ext}} = \frac{d\vec{L}}{dt} \]

4. Angular Momentum of a Rigid Body: 
\[ L = I \omega \]

5. Conservation of Angular Momentum: 
\[ \vec{L}_i = \vec{L}_f \; \text{(isolated system)} \]

6. Static equilibrium: 
\[ \vec{F}_{\text{net}} = 0; \; \vec{\tau}_{\text{net}} = 0 \]
if all the forces lie in xy plane \( F_{\text{net,x}} = 0; \; F_{\text{net,y}} = 0; \; \tau_{\text{net,z}} = 0 \)

7. Elastic Moduli: stress=modulus \times strain

8. Tension and Compression: 
\[ \frac{F}{A} = E \frac{\Delta L}{L}, \; E \text{ is the Young's modulus} \]

9. Shearing: 
\[ \frac{F}{A} = G \frac{\Delta L}{L}, \; G \text{ is the shear modulus} \]

10. Hydraulic Stress: 
\[ p = B \frac{\Delta V}{V}, \; B \text{ is the bulk modulus} \]
1. At the same instant that a 0.50-kg ball is dropped from 25m above Earth, a second ball, with a mass of 0.25 kg, is thrown straight upward from Earth's surface with an initial speed of 15m/s. They move along nearby lines and pass each other without colliding. What is the height above Earth's surface of the center of mass of the two-ball system at the end of 2.0 s?

(a) Find final y coordinate of the ball 1 after 2.0 s.

\[ y_{1f} = 25 + 0 \times t - \frac{gt^2}{2} = 25 - \frac{9.8 \times 2^2}{2} = 5.4 \text{ m} \]

(b) Find final y coordinate of the ball 2 after 2.0 s.

\[ y_{2f} = v_{20}t - \frac{gt^2}{2} = 15 \times 2 - \frac{9.8 \times 2^2}{2} = 10.4 \text{ m} \]

(c) Find y coordinate of the center of mass of two balls.

\[ y_{com} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2} = \frac{(0.5)(5.4) + (0.25)(10.4)}{(0.5 + 0.25)} = 7.1 \text{ m} \]
2. Two skaters with masses of 100 kg and 60 kg, respectively, stand 10.0 m apart; each holds one end of a piece of rope. If they pull themselves along the rope until they meet, how far does each skater travel? (Neglect friction)

(a) Since two skaters represent an isolated system and initially they are at rest, their center of mass will be at rest and they will meet at the center of mass.

\[
x_{\text{com}} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{(100)(0) + (60)(10)}{100 + 60} = 3.75m = 3.8m
\]

(b) The 100 kg skater moves 3.75 m

(c) The 60 kg skater moves 10 - 3.75 = 6.25 m = 6.2 m
3. A 2 kg block of wood rests on a long tabletop. A 5 g bullet moving horizontally with a speed of 150 m/s is shot into the block and sticks in it. The block then slides 270 cm along the table and stops.

(a) find the speed of the block just after impact.
(b) find the friction force between block and table.

\[ m_1 v_{10} + m_2 v_{20} = (m_1 + m_2)v; \quad \Rightarrow v = \frac{m_1 v_{10} + m_2 v_{20}}{m_1 + m_2} = \frac{(0.005)(150) + (2)(0)}{(0.005 + 2)} = 0.374 = 4 \times 10^{-1} m / s \]

(b) Calculate work done by the net force on a block-bullet system during its motion along the table. There are 3 forces acting on our system: Weight, Normal reaction, and kinetic friction. Only kinetic friction force will perform non zero work. \[ W_{net} = -f_k S \]

(c) Motion of a block-bullet system after the collision can be described by a work-energy theorem

\[ W_{net} = K_f - K_i \Rightarrow -f_k S = 0 - \frac{(m_1 + m_2)v^2}{2} \Rightarrow f_k = \frac{(m_1 + m_2)v^2}{2S} = \frac{(0.005 + 2)(0.374)^2}{2(2.70)} = 5 \times 10^{-2} N \]
4. The impact of the head of a golf club on a golf ball can be approximately regarded as an elastic collision. The mass of the head of the golf club is 0.15 kg. The speed of the club before the collision is 46 m/s. The ball acquires a speed of 70 m/s after the collision. The golf club and a ball are in contact for about 0.5 ms.
   a) What must be the mass of the ball?
   b) What is the average force exerted by the club on the ball?

(a) For elastic golf club-ball collision \( v_{bf} = \frac{2m_c}{m_c + m_b} v_{ci} \Rightarrow \)

\[
m_b = \frac{2m_v v_{ci}}{v_{bf}} - m_c = \frac{2(0.15)(46)}{70} - 0.15 = 0.047 = 4.7 \times 10^{-2} \text{ kg}
\]

(b) \( F_{avg} = \frac{\Delta P}{\Delta t} = \frac{P_{bf} - P_{bi}}{\Delta t} = \frac{(0.047)(70) - 0}{0.0005} = 6600 N = 6.6 \times 10^2 N \)
5. A 45-N brick is suspended by a light string from a 2.0-kg pulley. The brick is released from rest and falls to the floor below as the pulley rotates through 5.0 rad. The pulley may be considered a solid disk of radius 1.5 m. What is the angular speed of the pulley?

\[ T = \frac{1}{2} MR^2 \]  \hspace{1cm} (2)

\[ T = m a = m \omega^2 R \]  \hspace{1cm} (3)

\[ \omega = \sqrt{2} \omega_0 + 2 \alpha \theta \]  \hspace{1cm} (4)

\[ \omega = \sqrt{2 \cdot 5.36 \cdot 5.0} = \frac{7.3 \text{ rad/s}}{5} \]
6. Joe is painting the floor of his basement using a paint roller. The roller has a mass of 2.4 kg and a radius of 3.8 cm. In rolling the roller across the floor Joe applies a force \( F = 16 \text{N} \) directed at an angle of 35\(^\circ\) as shown. Ignoring the mass of the roller handle, what is the magnitude of the angular acceleration of the roller?

(a) 2nd Newton law x axis: \( F \sin \theta - f_s = ma_{com} \)

(b) 2nd Newton Law for rotation about com: \( \tau = f_s R = I \alpha \)

(c) recall the relationship between \( a_{com} \) and \( \alpha \): \( a_{com} = \alpha R \);

(d) substitute eq.(b) and (c) in (a): \( F \sin \theta - \frac{I \alpha}{R} = m \alpha R \);

\[
\Rightarrow \alpha = \frac{F \sin \theta}{mR + \frac{I}{R}} = \frac{F \sin \theta}{mR + \frac{1}{2}mR} = \frac{2F \sin \theta}{3mR} = \frac{2(16) \sin 35}{3(2.4)(0.038)} = \]

\[
= 67 \frac{\text{rad}}{s^2}
\]
7. A hoop \((I_h=MR^2)\), a uniform disk \((I_d=1/2MR^2)\), and a uniform sphere \((I_s=2/5MR^2)\), all with the same mass and outer radius, start with the same speed and roll without sliding up identical inclines. Rank the objects according to how high they go, least to greatest.

\((a)\) During rolling up the incline motion the mechanical energy of the hoop-earth, disk-earth, and sphere-earth systems is conserved since 1) frictional force does not transfer any energy to thermal energy because the objects do not slide; 2) normal force is perpendicular to the pass; 3) gravitational force is a conservative one. \(\Rightarrow\)

\[
\frac{1}{2} I \omega^2 + \frac{1}{2} Mv^2 + 0 = 0 + Mg h
\]

\((b)\) second term is the same for all our objects, \(\omega\) is also the same, \(\Rightarrow\) the larger I the larger maximum height \(h\) reached by the object.

\(\Rightarrow\) from least to greatest in terms of height attained: sphere, disk, hoop.
8. A traffic light hangs from a pole as shown. The uniform aluminum pole AB is 7.50 m long and has a mass of 12.0 kg. The mass of the traffic light is 21.5 kg. Determine (a) the tension in the horizontal massless cable CD, and (b) the vertical and horizontal components of the force exerted by the pivot A on the aluminum pole.

\[ T = 3.8 - \frac{Mg \cos 37^\circ}{2} - mg \cos 37^\circ = 0 \]

\[ T = \frac{(12.0 \text{ kg})(9.8 \text{ m/s}^2)(3.75 \text{ m}) \cos 37^\circ + (21.5 \text{ kg})(9.8 \text{ m/s}^2)(7.5 \text{ m}) \cos 37^\circ}{3.80 \text{ m}} \]

\[ T = 425 \text{ N} \]

b) \[ G_y - Mg - mg = 0; \quad G_y = 12.0 \cdot 9.8 + 21.5 \cdot 9.8 = 328 \text{ N} \]

\[ G_x - T = 0; \quad \Rightarrow G_x = T = 425 \text{ N} \]
9. A uniform seesaw of length 6 m has two youngsters of weights $w_1=700\text{N}$ and $w_2=400\text{ N}$ sitting on the ends. Find the proper location of the pivot for the seesaw to be just in balance, if
(a) the weight of the seesaw can be ignored
(b) the seesaw weighs $Mg=300\text{N}$

\[(a)\text{ Use 2 nd condition of equilibrium:}\]

$W_1x - W_2(6-x) = 0;$

$x = \frac{6W_2}{W_1 + W_2} = \frac{6(400)}{700 + 400} = 2.18m$

\[(b)\text{Use 2 nd condition of equilibrium:}\]

$W_1x - Mg(3-x) - W_2(6-x) = 0;$

$x = \frac{6W_2 + 3Mg}{W_1 + W_2 + Mg} = \frac{6(400) + 3(300)}{700 + 400 + 300} = 2.36m$