ALL QUESTIONS ARE WORTH 20 POINTS. WORK OUT FIVE PROBLEMS.

NOTE: Clearly write out solutions and answers (circle the answers) by section for each part (a., b., c., etc.)

Important Formulas:

1. Motion along a straight line with a constant acceleration

- $v_{\text{aver. speed}} = \frac{\text{[dist. taken]}}{\text{[time trav.]}} = \frac{S}{t}$;
- $v_{\text{aver. vel.}} = \frac{\Delta x}{\Delta t}$;
- $v_{\text{ins}} = \frac{dx}{\Delta t}$;
- $a_{\text{aver.}} = \frac{\Delta v_{\text{aver. vel.}}}{\Delta t}$;
- $v = v_o + at$; $x = \frac{1}{2}(v_o + v)t$; $x = v_o t + \frac{1}{2}at^2$; $v^2 = v_o^2 + 2ax$ (if $x_o=0$ at $t_o=0$)

2. Free fall motion (with positive direction $\uparrow$)

- $g = 9.80 \text{ m/s}^2$;
- $y = v_{\text{aver.}} t$;
- $v_{\text{aver.}} = \frac{(v + v_o)}{2}$;
- $v = v_o - gt$; $y = v_o t - \frac{1}{2}gt^2$; $v^2 = v_o^2 - 2gy$ (if $y_o=0$ at $t_o=0$)

3. Integration in Motion Analysis (non-constant acceleration)

- $v_1 = v_o + \int_{t_o}^{t} a \, dt$;
- $x_1 = x_o + \int_{t_o}^{t} v \, dt$

4. Motion in a plane

- $v_x = v_o \cos \theta$;
- $v_y = v_o \sin \theta$;
- $x = v_{ox} t + \frac{1}{2}a_x t^2$; $y = v_{oy} t + \frac{1}{2}a_y t^2$; $v_x = v_{ox} + at$; $v_y = v_{oy} + at$

5. Projectile motion (with positive direction $\uparrow$)

- $v_x = v_{ox} = v_o \cos \theta$;
- $x = v_{ox} t$;
- $x_{\text{max}} = \left(2 v_o^2 \sin \theta \cos \theta / g\right)$; $v_{\text{in}} = v_{\text{fin}}$;
- $v_y = v_{oy} - gt = v_o \sin \theta - gt$;
- $y = v_{oy} t - \frac{1}{2}gt^2$

6. Uniform circular Motion

- $a = v^2/r$;
- $T = 2\pi/v$
7. Relative motion

\[ \vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA} \]
\[ \vec{a}_{PA} = \vec{a}_{PB} \]

8. Component method of vector addition

\[ \mathbf{A} = \mathbf{A}_1 + \mathbf{A}_2 ; \quad \mathbf{A}_x = \mathbf{A}_{x1} + \mathbf{A}_{x2} \text{ and } \mathbf{A}_y = \mathbf{A}_{y1} + \mathbf{A}_{y2} ; \quad A = \sqrt{A_x^2 + A_y^2} ; \quad \theta = \tan^{-1} \left| \frac{A_y}{A_x} \right| ; \]

The scalar product \( \mathbf{A} \cdot \mathbf{B} = ab \cos \phi \)

\[ \mathbf{a} \cdot \mathbf{b} = (a_x \mathbf{i} + a_y \mathbf{j} + a_z \mathbf{k}) \cdot (b_x \mathbf{i} + b_y \mathbf{j} + b_z \mathbf{k}) \]

\[ \mathbf{a} \cdot \mathbf{b} = a_x b_x + a_y b_y + a_z b_z \]

The vector product \( \mathbf{a} \times \mathbf{b} = (a_x \mathbf{i} + a_y \mathbf{j} + a_z \mathbf{k}) \times (b_x \mathbf{i} + b_y \mathbf{j} + b_z \mathbf{k}) \)

\[ \mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \mathbf{i} \begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix} - \mathbf{j} \begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix} + \mathbf{k} \begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix} = \]

\[ = (a_y b_z - b_y a_z) \mathbf{i} + (a_z b_x - b_z a_x) \mathbf{j} + (a_x b_y - b_x a_y) \mathbf{k} \]
1. At time \( t=0 \) a car has a velocity of 16 m/s. It slows down with an acceleration given by \(-0.50t\), in m/s\(^2\) for \( t \) in seconds. At what instant of time it will stop?

\[
v_1 = v_o + \int_{t_o}^{t_1} adt = 16 + \int_{0}^{t_1} (-0.5t)dt = 16 + \left(-\frac{t_1^2}{4} + \frac{0^2}{4}\right) = 16 - \frac{t_1^2}{4} = 0
\]

\[t_1 = 8.0s\]
2. At a stop light, a truck traveling at 15 m/s passes a car as it starts from rest. The truck travels at constant velocity and the car accelerates at 3 m/s². How much time does the car take to catch up to the truck?

When the car catches up with the truck they both will have the same displacement $x$ with respect to the origin point (stop light), and it will take them the same time $t$ to reach this point.

For the truck moving with constant speed $v = 15m / s$; $x = vt$

For the car moving with constant acceleration $a$ from rest $x = 0 + \frac{at^2}{2}$

Hence, $vt = \frac{at^2}{2}$ and $t = \frac{2v}{a} = \frac{2 \times (15m / s)}{(3m / s^2)} = 10s$
3. At time $t=0$ s, a puck is sliding on a horizontal table with a velocity $3.00 \text{ m/s}$, $65.0^\circ$ above the $+x$ axis. As the puck slides, a constant acceleration acts on it that has the following components: $a_x=-0.460 \text{ m/s}^2$ and $a_y=-0.980 \text{ m/s}^2$. What is the velocity of the puck at time $t=1.50 \text{ s}$?

**x direction**

<table>
<thead>
<tr>
<th></th>
<th>$v_{ox}$</th>
<th>$v_x$</th>
<th>$a_x$</th>
<th>$t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>?</td>
<td>$3.00\cos 65.0=1.268 \text{ m/s}$</td>
<td>?</td>
<td>-$0.460 \text{ m/s}^2$</td>
<td>1.5 s</td>
</tr>
</tbody>
</table>

$v_x = v_{ox} + a_x t = 1.268 \text{ m/s} - (0.460 \text{ m/s}^2) \times (1.5 \text{s}) = 0.578 \text{ m/s}$

**y direction**

<table>
<thead>
<tr>
<th></th>
<th>$v_{oy}$</th>
<th>$v_y$</th>
<th>$a_y$</th>
<th>$t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>?</td>
<td>$3.00\sin 65.0=2.720 \text{ m/s}$</td>
<td>?</td>
<td>-$0.980 \text{ m/s}^2$</td>
<td>1.5 s</td>
</tr>
</tbody>
</table>

$v_y = v_{oy} + a_y t = 2.720 \text{ m/s} - (0.980 \text{ m/s}^2) \times (1.5 \text{s}) = 1.250 \text{ m/s}$

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{0.578^2 + 1.250^2} = 1.38 \text{ m/s}$

$\phi = \arctan \frac{v_y}{v_x} = \arctan \frac{1.250}{0.578} = 65.2^\circ$
4. In the diagram, \( \vec{A} \) has magnitude 12 m and \( \vec{B} \) has magnitude 8 m.

Find the resultant of the vectors \( \vec{A} \) and \( \vec{B} \). Express in
(a) component notation,
(b) graphical form,
(c) magnitude-angle form

(a) Component notation
\[ A_x = A \cos 45 = 8.5 \text{ m}; \quad A_y = A \sin 45 = 8.5 \text{ m}; \quad \vec{A} = 8.5\hat{i} + 8.5\hat{j} \]
\[ B_x = B \cos 60 = 4 \text{ m}; \quad B_y = -B \sin 60 = -7 \text{ m}; \quad \vec{B} = 4\hat{i} - 7\hat{j} \]
\[ \vec{r} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} = 12.5\hat{i} + 1.5\hat{j} = 1 \times 10^1\hat{i} + 2\hat{j} \]

(c) Magnitude-angle form
\[ r = \sqrt{12.5^2 + 1.5^2} = 12.6 = 1 \times 10^1 \text{ m} \]
\[ \theta = \arctan \frac{r_y}{r_x} = \arctan \frac{1.5}{12.5} = 7^\circ \]
As shown in Fig., a ball is thrown from the top of one building toward a tall building 50 m away. The initial velocity of the ball is 20 m/s at 40° above the horizontal. How far below its original level will the ball strike the opposite wall?

### a) x direction

<table>
<thead>
<tr>
<th>x</th>
<th>$v_{0x}$</th>
<th>$v_x$</th>
<th>$a_x$</th>
<th>$t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 m</td>
<td>$v_{0x} \cdot \cos 40°$</td>
<td>15.3 m/s</td>
<td>0</td>
<td>?</td>
</tr>
</tbody>
</table>

\[ x = v_{0x} t \]

\[ t = \frac{x}{v_{0x}} = \frac{50}{15.3} = 3.26 \text{s} \]

### b) y direction

<table>
<thead>
<tr>
<th>y</th>
<th>$v_{0y}$</th>
<th>$v_y$</th>
<th>$a$</th>
<th>$t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>?</td>
<td>$v_{0y} \cdot \sin 40°$</td>
<td>?</td>
<td>-9.8 m/s²</td>
<td>3.26 s</td>
</tr>
</tbody>
</table>

\[ y = v_{0y} t + \frac{a t^2}{2} = (12.86 \text{ m/s})(3.26 \text{s}) - \frac{(9.8 \text{ m/s}^2)(3.26 \text{s})^2}{2} = -10.15 \text{ m} = \boxed{-10 \text{ m}} \]

10 m below the original level
6. A Ferris wheel with a radius of 8.0 m makes 1 revolution every 10 s. When a passenger is at the top, essentially a diameter above the ground, he releases a ball. How far from the point on the ground directly under the release point does the ball land?

Given:
\[ T = 10 \text{s}; \quad R = 8.0 \text{m}; \quad y = -16 \text{m}; \quad a_y = -9.8 \text{m/s}^2; \quad a_x = 0 \text{m/s}^2 \]

Find: \( x = ? \)

1) Magnitude of initial velocity of the ball equal to velocity of the radial point of the Ferris wheel
\[ v_o = \frac{2\pi R}{T} = \frac{2\pi(8.0 \text{m})}{(10 \text{s})} = 5.0 \text{m/s} \]

2) Direction of the initial velocity is tangent to the path. At the top of the wheel it is directed along positive direction of \( x \) axis. \( \Rightarrow v_{oy} = 0 \).

3) Consider motion of a ball along vertical \( y \) direction. Find time of flight \( t \)
\[ y = v_{oy}t + \frac{at^2}{2}; \quad t = \sqrt{\frac{2y}{a}} = \sqrt{\frac{2(-16 \text{m})}{(-9.8 \text{m/s}^2)}} = 1.8 \text{s} \]

4) Consider motion of a ball along \( x \) direction.
\[ x = v_{ox}t = v_o t = (5.0 \text{m/s}) \times (1.8 \text{s}) = 9.0 \text{m} \]
7. A motor boat can travel at 10 km/h in still water. A river flows at 5km/h west. A boater wishes to cross from the south bank to a point directly opposite on the north bank. At what angle must the boat be headed?

\[ \vec{v}_{bb} = 10 \text{ km/h} \]

\[ \vec{v}_{wb} = 5 \text{ km/h} \]

\[ \vec{v}_{bw} = ? \]

\( \vec{v}_{bb} \) – velocity of a boat with respect to the bank of the river

\( \vec{v}_{wb} \) – velocity of a water with respect to the bank of the river

\( \vec{v}_{bw} \) – velocity of a boat with respect to the water

According to the relative motion rule \( \vec{v}_{bb} = \vec{v}_{wb} + \vec{v}_{bw} \)

\[ \Rightarrow \theta = \arctan \left( \frac{\vec{v}_{wb}}{\vec{v}_{bw}} \right) = \arctan \left( \frac{5}{10} \right) = \frac{3 \times 10^1}{\circ} \]