ALL QUESTIONS ARE WORTH 20 POINTS. WORK OUT FIVE PROBLEMS.

NOTE: Clearly write out solutions and answers (circle the answers) by section for each part (a., b., c., etc.)

1. Motion along a straight line with a constant acceleration

\[ v_{\text{aver. speed}} = \frac{\text{[dist. taken]}}{\text{[time trav.]}} = \frac{S}{t}; \]
\[ v_{\text{aver. vel.}} = \frac{\Delta v}{\Delta t}; \]
\[ v_{\text{ins}} = \frac{\Delta x}{\Delta t}; \]
\[ a_{\text{aver.}} = \frac{\Delta v}{\Delta t}; \]
\[ v = v_o + at; \quad x = \frac{1}{2}(v_o + v)t; \quad v_{\text{avg.}} = \frac{v_o + v}{2}; \quad v^2 = v_o^2 + 2ax \text{ (if } x_0 = 0 \text{ at } t_0 = 0) \]

2. Free fall motion (with positive direction \( \uparrow \))

\[ g = 9.80 \text{ m/s}^2; \]
\[ y = v_{\text{aver.}} t; \]
\[ v_{\text{aver.}} = \frac{(v + v_o)}{2}; \]
\[ v = v_o - gt; \quad y = v_o t - \frac{1}{2} gt^2; \quad v^2 = v_o^2 - 2gy \text{ (if } y_o = 0 \text{ at } t_o = 0) \]

3. Motion in a plane

\[ v_x = v_o \cos \theta; \]
\[ v_y = v_o \sin \theta; \]
\[ x = v_{ox} t + \frac{1}{2} a_x t^2; \quad y = v_{oy} t + \frac{1}{2} a_y t^2; \quad v_x = v_{ax} + at; \quad v_y = v_{ay} + at; \]

4. Projectile motion (with positive direction \( \uparrow \))

\[ v_x = v_{ox} = v_o \cos \theta; \]
\[ x = v_{ox} t; \]
\[ x_{\text{max}} = \frac{(2 v_o^2 \sin \theta \cos \theta)}{g} = \frac{(v_o^2 \sin 2 \theta)}{g} \text{ for } y_{\text{in}} = y_{\text{fin}}; \]
\[ v_y = v_{oy} - gt = v_o \sin \theta - gt; \]
\[ y = v_{oy} t - \frac{1}{2} gt^2; \]

5. Uniform circular Motion

\[ a = \frac{v^2}{r}; \]
\[ T = 2\pi \frac{r}{v}; \]

6. Relative motion

\[ \vec{v}_{\text{relative}} = \vec{v}_A - \vec{v}_B + \vec{v}_B - \vec{v}_A \]
\[ \vec{a}_{\text{relative}} = \vec{a}_A - \vec{a}_B \]

5. Component method of vector addition
\[ A = A_1 + A_2 \; \; ; A_x = A_{x1} + A_{x2} \text{ and } A_y = A_{y1} + A_{y2} ; \; \; A = \sqrt{A_x^2 + A_y^2} ; \; \; \theta = \tan^{-1} \left| \frac{A_y}{A_x} \right| ; \]

The scalar product \( \vec{a} \cdot \vec{b} = ab \cos \phi \)

\[ \vec{a} \cdot \vec{b} = (a_x \hat{i} + a_y \hat{j} + a_z \hat{k}) \cdot (b_x \hat{i} + b_y \hat{j} + b_z \hat{k}) \]

\[ \vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z \]

The vector product \( \vec{a} \times \vec{b} = (a_x \hat{i} + a_y \hat{j} + a_z \hat{k}) \times (b_x \hat{i} + b_y \hat{j} + b_z \hat{k}) \)

\[ \vec{a} \times \vec{b} = -\vec{b} \times \vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \hat{i} \begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix} - \hat{j} \begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix} + \hat{k} \begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix} = \]

\[ = (a_y b_z - b_y a_z) \hat{i} + (a_z b_x - b_z a_x) \hat{j} + (a_x b_y - b_x a_y) \hat{k} \]
1. Starting at time $t = 0$, an object moves along a straight line. Its coordinate in meters is given by $x(t) = 75t - 1.0t^3$, where $t$ is in seconds. What is its acceleration when it momentarily stops?

1) $v = \frac{dx}{dt} = 75 - 3.0t^2$

2) Time at which the object will stop corresponds to $v = 0$

$\Rightarrow 75 - 3.0t^2 = 0; \Rightarrow t=5s$

3) $a = \frac{dv}{dt} = -6.0t$

4) $\Rightarrow$ for $t=5s$ $a = -30 \frac{m}{s^2}$
2. A skier starts from rest and slides 9 m down a slope in 3 s. In what time after starting will the skier acquire a velocity of 24 m/s? Assume constant acceleration.

\[
x = x_0 t + \frac{1}{2} a t^2
\]

\[
a = \frac{2x}{t^2} = \frac{2 \cdot (9 \text{ m})}{(3 \text{ s})^2} = 2 \text{ m/s}^2
\]

\[
v = v_0 + at
\]

\[
t = \frac{v}{a} = \frac{24 \text{ m/s}}{2 \text{ m/s}^2} = 12 \text{ s}
\]

<table>
<thead>
<tr>
<th>(x)</th>
<th>(a)</th>
<th>(v)</th>
<th>(v_0)</th>
<th>(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>9 m</td>
<td>?</td>
<td>?</td>
<td>0</td>
<td>3 s</td>
</tr>
</tbody>
</table>

\[
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\[
a = \frac{2x}{t^2} = \frac{2 \cdot (9 \text{ m})}{(3 \text{ s})^2} = 2 \text{ m/s}^2
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\]
3. A baseball is thrown straight upward on the moon with a velocity of 35 m/s. The acceleration due to gravity on the moon is \( g = 1.60 \text{ m/s}^2 \). Compute:
(a) the maximum height reached by the ball?
(b) the time taken to reach that height?
(c) its velocity 30 s after it is thrown?
(d) the time when the ball's height is 100 m? (Hint: Use the quadratic formula)

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \text{ where } ax^2 + bx + c = 0
\]

\[
y = V_0 t + \frac{a t^2}{2}
\]

\[
-1.6 t^2 + 35 \cdot 2.35 \cdot t - 200 = 0
\]

\[
t_{1,2} = \frac{70 \pm \sqrt{70^2 - 4(-1.6)(-200)}}{-2 \cdot 1.6} = \frac{-70 \pm \sqrt{4900 - 1280}}{-3.2} = \frac{-70 \pm \sqrt{4} \cdot 1920}{-3.2}
\]

\[
t_1 = 41.5
\]

\[
t_2 = 3.15
\]

\[
y = \frac{V_0^2 + 2ay_0 - 2V_0^2}{2a} = \frac{-V_0^2}{2(-1.6)} = 3.8 \text{ m}
\]

\[
V = V_0 + at
\]

\[
t_4 = \frac{V - V_0}{a} = \frac{35 - 35}{-1.6} = 21.88 \text{ s}
\]

\[
y = V_0 + at = (35) + (-16) \cdot 30 = -13 \text{ m/s}
\]
4. A spaceship is observed traveling in the positive x direction with a speed of 150 m/s when it begins accelerating at a constant rate. The spaceship is observed 25 s later traveling with an instantaneous velocity of 1500 m/s at an angle of 55° above the +x axis.

(a) what is the magnitude of the acceleration of the spaceship during the 25 seconds?
(b) what is the magnitude of the displacement of the spaceship during the 25 seconds?

\[
\begin{align*}
\text{x direction} & \\
\begin{array}{|c|c|c|c|c|}
\hline
V_{0x} & V_x & a_x & t \\
\hline
? & 150 \text{ m/s} & 1500 \cdot \cos 55^\circ & ? & 25s \\
\hline
\end{array}
\end{align*}
\]

\[
\begin{align*}
\begin{array}{|c|c|c|c|c|}
\hline
V_{0y} & V_y & a_y & t \\
\hline
? & 1500 \cdot \sin 55^\circ & 1228.7 \text{ m/s} & ? & 25s \\
\hline
\end{array}
\end{align*}
\]

\[
V_x = V_{0x} + a_x \cdot t \\
a_x = \frac{V_x - V_{0x}}{t} = \frac{860.4 - 150}{25} = 28.4 \text{ m/s}^2
\]

\[
a = \sqrt{a_x^2 + a_y^2} = \sqrt{28.4^2 + 49.1^2} = 57 \text{ m/s}^2
\]

\[
\begin{align*}
x &= \frac{1}{2} (V_{0x} + V_x) \cdot t = \\
&= \frac{1}{2} (150 + 860.4) \cdot 25 = \\
&= 1.263 \times 10^4 \text{ m}
\end{align*}
\]

\[
\begin{align*}
y &= \frac{1}{2} (V_{0y} + V_y) \cdot t = \\
&= \frac{1}{2} (0 + 1228.7) \cdot 25 = \\
&= 1.5359 \times 10^4 \text{ m}
\end{align*}
\]

\[
\Delta z = \sqrt{x^2 + y^2} = \sqrt{(1.263 \times 10^4)^2 + (1.5359 \times 10^4)^2} = \\
= 2.0 \times 10^4 \text{ m}
\]
5. Let $\vec{R} = \vec{S} \times \vec{T}$ and $\theta$ is the angle between $\vec{S}$ and $\vec{T}$ when they are drawn with their tails at the same point. Which of the following is not true?

A. $|\vec{R}| = |\vec{S}| |\vec{T}| \sin \theta$. It is true by definition.

B. $-\vec{R} = \vec{T} \times \vec{S}$ it is true since

\[
\vec{a} \times \vec{b} = \begin{vmatrix}
\hat{i} & \hat{j} & \hat{k} \\
a_x & a_y & a_z \\
b_x & b_y & b_z
\end{vmatrix} = \hat{i} \begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix} - \hat{j} \begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix} + \hat{k} \begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix} =
\]

\[
= (a_y b_z - b_y a_z)\hat{i} + (a_z b_x - b_z a_x)\hat{j} + (a_x b_y - b_x a_y)\hat{k}.
\]

\[
\vec{b} \times \vec{a} = \begin{vmatrix}
\hat{i} & \hat{j} & \hat{k} \\
b_x & b_y & b_z \\
a_x & a_y & a_z
\end{vmatrix} = \hat{i} \begin{vmatrix} b_y & b_z \\ a_y & a_z \end{vmatrix} - \hat{j} \begin{vmatrix} b_x & b_z \\ a_x & a_z \end{vmatrix} + \hat{k} \begin{vmatrix} b_x & b_y \\ a_x & a_y \end{vmatrix} =
\]

\[
= -(a_y b_z - b_y a_z)\hat{i} - (a_z b_x - b_z a_x)\hat{j} - (a_x b_y - b_x a_y)\hat{k} = -(\vec{b} \times \vec{a}).
\]

C. $\vec{R} \cdot \vec{S} = 0$. It is true since $\vec{R} \cdot \vec{S} = RS \cos 90 = 0$

D. $\vec{R} \cdot \vec{T} = 0$. It is true since $\vec{R} \cdot \vec{T} = RT \cos 90 = 0$

E. $\vec{S} \cdot \vec{T} = 0$. It is not true since $\cos \theta \neq 0$
6. According to an ancient Greek source, a stone throwing machine on one occasion achieved a range of 730 m. If this is true, (a) What must have been the minimal initial speed of the stone as it was ejected from the engine? (b) When ejected with this speed, how long would the stone have taken to reach its target?

Given:

\[ x = 730 \text{ m} \]
\[ \theta = 45^\circ \]

Find (a) \( v_o = ? \)
(b) \( t_f \)

Solution:

(a) \[
    x = \frac{v_o^2 \sin 2\theta}{g} = \frac{v_o^2}{g} \quad \Rightarrow \quad v_o = \sqrt{gx} = \sqrt{(730m) \cdot (9.8 m/s^2)} = 84.6 m/s
\]

(b) Consider y motion. Given: 1) Time of flight corresponds to \( y=0 \);
2) \( a=-9.8 \text{ m/s}^2 \)
3) \( v_{oy} = v_o \sin 45^\circ = 84.6 \sin 45 = 59.8 m/s \)

Use equation (3) to find time of flight

\[
    0 = v_{oy} t_f + \frac{1}{2} at_f^2 \quad \Rightarrow \quad t_f = \frac{-2v_{oy}}{a} = \frac{-2(59.8 m/s)}{(-9.8 m/s^2)} = 12.2 s
\]
A ball is thrown horizontally from the top of a 20-m high hill. It strikes the ground at an angle of 45°. With what speed was it thrown?

a) Consider motion along y direction. Choose the origin point at the top of the hill and y axis directed vertically upwards. The ball is in a free fall from the height of 20 m. Given: 1) \( v_{oy} = 0 \); 2) \( y = -20 m \); 3) \( a = -9.8 m/s^2 \). Find \( v_y = ? \)

Use equation 4. \( v_y^2 = v_{oy}^2 + 2ay \); \( \Rightarrow v_y = \sqrt{2ay} = \sqrt{-2(-9.8 m/s^2)(-20 m)} = 19.8 m/s \)

b) Since a ball strikes the ground at 45°, final x component of velocity equal final y component of velocity

\[ v = v_{ox} = v_x = 20 m/s \]
8. A girl wishes to swim across a river to a point directly opposite as shown. She can swim at 2m/s in still water and the river is flowing at 1m/s. At what angle $\theta$ with respect to the line joining the starting and finishing points should she swim?

\[ \vec{v}_{gb} - \text{velocity of a girl with respect to the bank of the river} \]

\[ \vec{v}_{wb} - \text{velocity of a water with respect to the bank of the river} \]

\[ \vec{v}_{gw} - \text{velocity of a girl with respect to the still water} \]

According to the relative motion rule $\vec{v}_{gb} = \vec{v}_{wb} + \vec{v}_{gw}$

\[ \Rightarrow \theta = \arctan\left(\frac{\vec{v}_{wb}}{\vec{v}_{gw}}\right) = \arctan(\frac{1}{2}) = 3\times10^{1}\degree \]
9. A girl jogs around a horizontal circle with a constant speed. She travels one fourth of a revolution, a distance of 25m along the circumference of the circle, in 5.0 s. What is the magnitude of her acceleration?

(a) Find speed of the uniform circular motion.

since quarter of a full revolution travel corresponds to 25 m, and time it takes equal to 5.0 s the speed of the motion is $25/5.0 = 5.0\text{m/s}$

(b) Find radius $r$ of a circular trajectory.

$$\frac{2\pi r}{4} = 25\text{m}, \Rightarrow r = 15.9\text{m}$$

(c) Find centripital acceleration.

$$a = \frac{v^2}{r} = \frac{(5\text{m/s})^2}{(15.9\text{m})} = 1.6\text{m/s}^2$$