ALL QUESTIONS ARE WORTH 37.5 POINTS. WORK OUT FOUR PROBLEMS.

NOTE: Clearly write out solutions and answers (circle the answers) by section for each part (a., b., c., etc.)

Important Formulas:

**Chapter 21**

1. \( \vec{F} = G \frac{m_1 m_2}{r^2} \vec{r} \)  Newton’s Law of Gravitation

2. \( \vec{F} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} \vec{r} \)  Coulomb’s Law

3. \( F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{|q_1| |q_2|}{r^2} = k \frac{|q_1| |q_2|}{r^2}, \varepsilon_0=8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2, k=8.99 \times 10^9 \text{ Nm}^2/\text{C}^2 \)

4. \( 1\text{C}=(1\text{A})(1\text{s}) \)

5. \( i = \frac{dq}{dt} \)  Electrical Current

**Chapter 22**

6. \( \vec{E} = \frac{\vec{F}}{q_0} \)  Electric field, \( q_0 \) is a positive test charge; \( F \) is electrostatic force that acts on the test charge

7. \( \vec{F} = q \vec{E} \), Electrostatic force that acts on the particle with charge \( q \).

8. \( \vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \vec{r} = k \frac{q}{r^2} \vec{r}, \) Electric field due to Point charge

9. \( E = \frac{1}{2\pi\varepsilon_0} \cdot \frac{qd}{z^3} = \frac{1}{2\pi\varepsilon_0} \cdot \frac{p}{z^3}, \) Electric field on axis \( z \) due to an electric dipole.

\( p=qd \) is an electric dipole moment with direction from negative to positive ends of a dipole.
1. \( E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qz}{(z^2 + R^2)^{3/2}} \), Electric field due to charged ring

2. \( E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{z^2} \), Electric field due to charged ring at large distance (\( z \gg R \))

3. \( E = \frac{\sigma}{2\varepsilon_0} \left( 1 - \frac{z}{\sqrt{z^2 + R^2}} \right) \), Electric field due to charged disk

4. \( E = \frac{\sigma}{2\varepsilon_0} \), Electric field due to infinite sheet

5. \( \tau = \vec{p} \times \vec{E} = pE\sin(\theta) \), Torque on a dipole in electric field

6. \( U = -\vec{p} \cdot \vec{E} = -pE\cos(\theta) \), Potential energy of a dipole in electric field.

Chapter 23

7. \( \Phi = \sum \vec{E} \cdot \Delta \vec{A} \), electric flux through a surface

8. \( \Phi = \oint \vec{E} \cdot d\vec{A} \), Electric flux through a Gaussian surface.

9. \( \varepsilon_0 \Phi = q_{\text{enc}} \), Gauss’ Law

10. \( \varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{\text{enc}} \), Gauss’ Law

11. \( E = \frac{\sigma}{\varepsilon_0} \), Conducting surface

12. \( E = \frac{\lambda}{2\pi\varepsilon_0 r} \), Line of charge

13. \( E = \frac{\sigma}{\varepsilon_0} \), Electric field between two conducting plates

14. \( E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \), Spherical shell field at \( r \geq R \)

\( E = 0 \), Spherical shell field at \( r < R \)
1. \[ E = \left( \frac{q}{4\pi \varepsilon_0 R^3} \right) r , \] Uniform spherical charge distribution at \( r \leq R \)

2. \[ E = \frac{1}{4\pi \varepsilon_0} \cdot \frac{q}{r^2} , \] Uniform spherical charge distribution at \( r > R \)

**Chapter 24**

3. \[ \Delta U = U_f - U_i = -W , \] Change in electric potential energy due to work done by electrostatic force.

4. \[ U = -W_\infty , \] Potential energy; \( W_\infty \) is work done by electrostatic force during particle move from infinity.

5. \[ V = \frac{U}{q} = -\frac{W_\infty}{q} , \] Electric potential

6. \[ \Delta V = V_f - V_i = \frac{U_f}{q} - \frac{U_i}{q} = \frac{\Delta U}{q} = -\frac{W}{q} , \] Electric potential difference

7. \[ V = -\int \vec{E} \cdot d\vec{s} , \] Potential at any point \( f \) in the electric field relative to the zero potential at point \( i \).

8. \[ V = \frac{1}{4\pi \varepsilon_0} \cdot \frac{q}{r} , \] Potential due to point charge.

9. \[ V = \sum_i^n V_i = \frac{1}{4\pi \varepsilon_0} \sum_i^n \frac{q_i}{r_i} , \] Potential due to group of \( n \) point charges.

10. \[ V = \int dV = \frac{1}{4\pi \varepsilon_0} \int \frac{dq}{r} , \] Potential due to continuous charge distribution.

11. \[ V = \frac{\lambda}{4\pi \varepsilon_0} \ln \left( \frac{L + (L^2 + d^2)^{1/2}}{d} \right) , \] Potential due to line of charge

12. \[ V = \frac{\sigma}{2\varepsilon_0} \left( \sqrt{z^2 + R^2} - z \right) , \] Potential due to charged disk
1. \[ V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p\cos(\theta)}{r^2}, \] Potential due to electric dipole.

2. \( \vec{E} = E_x \hat{i} + E_y \hat{j} + E_z \hat{k} \), Calculating electric field from the potential

\[ E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial z} \]

3. \( E = -\frac{\Delta V}{\Delta s} \), Calculating electric field from the potential in a simple case of uniform electric field.

4. \( U = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1q_2}{r_{12}} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1q_3}{r_{13}} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2q_3}{r_{23}} \), Potential energy of system of three charges.

5. \( U = \frac{1}{4\pi\varepsilon_0} \sum_{i,j=1 \atop i \neq j}^{n} \frac{q_iq_j}{r_{ij}} \), Potential energy of system of \( n \) charges

6. \( U = \frac{1}{2} \sum_{i=1}^{n} V_{other}(i)q_i \)

7. Alternative expression for Potential energy of system of \( n \) charges

\[ V_{other}(i) = \frac{1}{4\pi\varepsilon_0} \sum_{j=1 \atop j \neq i}^{n} \frac{q_j}{r_{ji}} \]
1. A small sphere of mass $1.0 \times 10^{-6}$ kg carries a total charge of $2.0 \times 10^{-8}$ C. The sphere hangs from a silk thread between two large parallel conducting plates. The excess charge on each plate is equal in magnitude, but opposite in sign. If the thread makes an angle of $30^\circ$ with the positive plate as shown, what is the magnitude of the charge density on each plate?

\[
y : \quad T \cos 30^\circ - mg = 0 \\
x : \quad T \sin 30^\circ - Eq = 0
\]

\[
\begin{align*}
\text{Eq} & = \frac{mg}{\sin 30^\circ} \\
\text{Eq} & = 2mg
\end{align*}
\]

\[
\Rightarrow \quad E = \frac{mg \tan 30^\circ}{g}
\]

\[
\frac{\sigma}{\varepsilon_0} = \frac{1}{2}
\]

we also know that \( E = \frac{\sigma}{\varepsilon_0} \)

\[
\sigma = \frac{mg \varepsilon_0 \tan 30^\circ}{g} = \frac{(1.0 \times 10^{-6}) (9.8 \text{ m/s}^2) (8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2) \\
(2.0 \times 10^{-8} \text{ C})}{(2.5 \times 10^{-9} \text{ C} / \text{m}^2)}
\]

\[
= (2.5 \times 10^{-9} \text{ C} / \text{m}^2)
\]
2. Electric charge is uniformly distributed over each of three large, parallel sheets of paper. The charges per unit area on the sheets are $+10^{-6}\text{C/m}^2$, $+10^{-6}\text{C/m}^2$, $-10^{-6}\text{C/m}^2$, respectively. Find the strength of the electric field $\mathbf{E}$ above the sheets, below the sheets, and in the space between the sheets. Find the direction of $\mathbf{E}$ at each place.

From a single sheet the electric field is

\[
E = \frac{1}{2 \varepsilon_0} \frac{10^{-6} \left( \frac{\text{C}}{\text{m}^2} \right)}{2 \times 8.85 \times 10^{-12} \left( \frac{\text{C}^2}{\text{N} \cdot \text{m}^2} \right)} = 5.6 \times 10^4 \frac{\text{N}}{\text{C}}
\]

\[
E_A = (5.6 \times 10^4 \frac{\text{N}}{\text{C}}) (\text{+1}+\text{l}+\text{l}) = +5.6 \times 10^4 \frac{\text{N}}{\text{C}}
\]

\[
E_B = (5.6 \times 10^4 \frac{\text{N}}{\text{C}}) (\text{-1}+\text{l}+\text{l}) = -5.6 \times 10^4 \frac{\text{N}}{\text{C}}
\]

\[
E_c = (5.6 \times 10^4 \frac{\text{N}}{\text{C}}) (\text{-1}-\text{l}-\text{l}) = -1.17 \times 10^5 \frac{\text{N}}{\text{C}}
\]

\[
E_D = (5.6 \times 10^4 \frac{\text{N}}{\text{C}}) (\text{-1}-\text{l}+\text{l}) = -5.6 \times 10^4 \frac{\text{N}}{\text{C}}
\]
3. Two point charges are held at the corners of a rectangle as shown in the figure. The lengths of sides of the rectangle are 0.050 m and 0.150 m. Assume that the electric potential is defined to be zero at infinity. What is the potential difference, \( V_B - V_A \), between corners A and B?

\[
V_B = \frac{1}{4\pi \varepsilon_0} \frac{q_1}{r_1} + \frac{1}{4\pi \varepsilon_0} \frac{q_2}{r_2} = \text{ where } r_1 = a, \quad r_2 = b
\]

\[
= \frac{1}{4\pi \times 8.85 \times 10^{-12}} \left( \frac{-5 \times 10^{-6}}{0.05} + \frac{2 \times 10^{-6}}{0.15} \right) = -7.79 \times 10^5 \text{ V}
\]

\[
V_A = \frac{1}{4\pi \varepsilon_0} \frac{q_1}{b} + \frac{1}{4\pi \varepsilon_0} \frac{q_2}{a} =
\]

\[
= \frac{1}{4\pi \times 8.85 \times 10^{-12}} \left( \frac{-5.0 \times 10^{-6}}{0.150} + \frac{2.0 \times 10^{-6}}{0.050} \right) = 5.99 \times 10^5 \text{ V}
\]

\[
V_B - V_A = -7.79 \times 10^5 - 5.99 \times 10^5 = -8.39 \times 10^5 \text{ V}
\]
4. In Fig we show two large metal plates connected to a 120 V battery. Assume the plates to be in vacuum and to be much larger than shown. Find:
(a) $E$ between the plates
(b) the force experienced by an electron between the plates
(c) the potential energy lost by an electron as it moves from plate B to plate A
(d) the speed of the electron released from plate B just before striking plate A.

$E = \frac{\Delta V}{d} = \frac{120V}{2 \times 10^{-2} m} = 6 \times 10^3 \text{ N/m}$

$F = |E| e = |6 \times 10^3 \text{ N/m}| \times (1.6 \times 10^{-19} \text{ C}) = 9.6 \times 10^{-16} \text{ N}$

$c) \quad |\Delta U| = |\Delta V| \cdot e = (120V \cdot 1.6 \times 10^{-19} \text{ C}) = 1.92 \times 10^{-17} \text{ J} = (2 \times 10^{-17} \text{ J})$

$d) \quad \text{Use law of conservation of energy.}$

$U_B = U_A + \frac{mv^2}{2}$

$\Delta U = \frac{mv^2}{2}$

$v = \sqrt{\frac{2 \Delta U}{m}} = \sqrt{\frac{2 \times 2 \times 10^{-17}}{9.1 \times 10^{-31}}} = (7 \times 10^6 \text{ m/s})$
5. Positive charge $Q$ is distributed uniformly throughout an insulating sphere of radius $R$, centered at the origin. A particle with positive charge $Q$ is placed at $x = 2R$ on the $x$ axis. What is the magnitude of the electric field at $x = R/2$ on the $x$ axis?

At $x=R/2$ charged insulating sphere generates electric field

$$E_s = \left( \frac{Q}{4\pi \varepsilon_o R^3} \right) \frac{R}{2} = \frac{Q}{8\pi \varepsilon_o R^2}$$
directed to the positive direction of $x$ axis

A particle with a positive charge $Q$ placed at a distance $3R/2$ from $x=R/2$ generates electric field directed to the negative direction of $x$ axis with a magnitude

$$E_p = \frac{Q}{4\pi \varepsilon_o \left( \frac{3R}{2} \right)^2} = \frac{Q}{9\pi \varepsilon_o R^2}$$

The resultant electric field at $x=R/2$ is directed to the positive direction of $x$ axis

and has magnitude of

$$E = \frac{Q}{8\pi \varepsilon_o R^2} - \frac{Q}{9\pi \varepsilon_o R^2} = \frac{Q}{72\pi \varepsilon_o R^2}$$
Two conducting spheres are far apart. The smaller sphere carries a total charge $Q$. The larger sphere has a radius that is twice that of the smaller and is neutral. After the two spheres are connected by a conducting wire, what are the charges on the smaller and larger spheres?

a) After connecting two spheres by a conducting wire some charge from sphere 1 will be transferred to sphere 2. This transfer will proceed until the potentials of sphere 2 will be equal to potential 1. This translates into

$$\frac{Q_1}{4\pi\varepsilon_o R} = \frac{Q_2}{4\pi\varepsilon_o 2R} \Rightarrow Q_1 = \frac{Q_2}{2} \quad (1)$$

b) Of course total charge should be conserved

$$Q_1 + Q_2 = Q \quad (2)$$

c) Substituting (1) into (2) gives

$$\frac{3Q_2}{2} = Q$$

$$\Rightarrow Q_2 = \frac{2Q}{3}; \quad Q_1 = \frac{Q}{3}$$
7. What is the magnitude of the electric field at the point \((3.00\hat{i} - 2.00\hat{j} + 4.00\hat{k})\) m if the electric potential is given by \(V = 2.00xyz^2\), where \(V\) is in volts and \(x, y,\) and \(z\) are in meters?

We apply

\[
E_x = -\frac{\partial V}{\partial x} = -2.00yz^2
\]

\[
E_y = -\frac{\partial V}{\partial y} = -2.00xz^2
\]

\[
E_z = -\frac{\partial V}{\partial z} = -4.00xyz
\]

which, at \((x, y, z) = (3.00 \text{ m}, -2.00 \text{ m}, 4.00 \text{ m})\), gives

\((E_x, E_y, E_z) = (64.0 \text{ V/m}, -96.0 \text{ V/m}, 96.0 \text{ V/m})\).

The magnitude of the field is therefore

\[
|\vec{E}| = \sqrt{E_x^2 + E_y^2 + E_z^2} = 150 \text{ V/m} = 150 \text{ N/C}.
\]