ALL QUESTIONS ARE WORTH 20 POINTS. WORK OUT FIVE PROBLEMS.

NOTE: Clearly write out solutions and answers (circle the answers) by section for each part (a., b., c., etc.)

Important Formulas:

1. **Motion along a straight line with a constant acceleration**
   \[ v_{\text{aver. speed}} = \frac{\text{dist. taken}}{\text{time traveled}} = \frac{S}{t}; \]
   \[ v_{\text{aver. vel.}} = \frac{\Delta x}{\Delta t}; \]
   \[ v_{\text{ins.}} = \frac{dx}{dt}; \]
   \[ a_{\text{aver.}} = \frac{\Delta v_{\text{aver. vel.}}}{\Delta t}; \]
   \[ a = \frac{dv}{dt}; \]
   \[ v = v_0 + at; \quad x = \frac{1}{2}(v_0 + v)t; \quad v_{\text{aver. vel.}} = \frac{v + v_0}{2}; \quad v^2 = v_0^2 + 2ax \text{ (if } x_0 = 0 \text{ at } t_0 = 0) \]

2. **Free fall motion (with positive direction ↑)**
   \[ g = 9.80 \text{ m/s}^2; \]
   \[ v = v_{\text{aver.}} t \]
   \[ y = \frac{(v + v_0)}{2}; \]
   \[ v = v_0 - gt; \quad y = v_0 t - \frac{1}{2} gt^2; \quad v^2 = v_0^2 - 2gy \text{ (if } y_o = 0 \text{ at } t_0 = 0) \]

3. **Integration in Motion Analysis (non-constant acceleration)**
   \[ v_1 = v_o + \int v_o \, dt \]
   \[ x_1 = x_o + \int v_o \, dt \]

4. **Motion in a plane**
   \[ v_x = v_0 \cos \theta; \]
   \[ v_y = v_0 \sin \theta; \]
   \[ x = v_{ox} t + \frac{1}{2} a_x t^2; \quad y = v_{oy} t + \frac{1}{2} a_y t^2; \quad v_x = v_{ox} + at; \quad v_y = v_{oy} + at; \]

5. ** Projectile motion (with positive direction ↑)**
   \[ v_x = v_{ox} = v_0 \cos \theta; \]
   \[ x = v_{ox} t; \]
   \[ x_{\text{max}} = (2 v_0^2 \sin \theta \cos \theta)/g = (v_0^2 \sin 2 \theta)/g \text{ for } y_{\text{in}} = y_{\text{fin}}; \]
   \[ v_y = v_{oy} - gt = v_0 \sin \theta - gt; \]
   \[ y = v_{oy} t - \frac{1}{2} gt^2; \]

6. **Uniform circular Motion**
   \[ a = \frac{v^2}{r}; \]
   \[ T = 2\pi r/v \]
1. Relative motion
\[ \vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA} \]
\[ \vec{a}_{PA} = \vec{a}_{PB} \]

2. Component method of vector addition
\[ \mathbf{A} = \mathbf{A}_1 + \mathbf{A}_2 ; \mathbf{A}_x = \mathbf{A}_{x1} + \mathbf{A}_{x2} \text{ and } \mathbf{A}_y = \mathbf{A}_{y1} + \mathbf{A}_{y2} ; \quad A = \sqrt{A_x^2 + A_y^2} ; \quad \theta = \tan^{-1} \left| \frac{A_y}{A_x} \right| ; \]

The scalar product \[ \mathbf{a} \cdot \mathbf{b} = ab \cos \phi \]
\[ \mathbf{a} \cdot \mathbf{b} = (a_x \hat{i} + a_y \hat{j} + a_z \hat{k}) \cdot (b_x \hat{i} + b_y \hat{j} + b_z \hat{k}) \]
\[ \mathbf{a} \cdot \mathbf{b} = a_x b_x + a_y b_y + a_z b_z \]
The vector product \[ \mathbf{a} \times \mathbf{b} = (a_x \hat{i} + a_y \hat{j} + a_z \hat{k}) \times (b_x \hat{i} + b_y \hat{j} + b_z \hat{k}) \]
\[ \mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix} \mathbf{a} - \begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix} \mathbf{b} + \begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix} \mathbf{c} = (a_y b_z - a_z b_y) \hat{i} + (a_z b_x - a_x b_z) \hat{j} + (a_x b_y - a_y b_x) \hat{k} \]

3. Second Newton’s Law: \( ma = F_{net} \)

4. Kinetic friction \( f_k = \mu kN \)

5. Static friction \( f_s = \mu sN \)

6. Universal Law of Gravitation: \( F = GMm/r^2 ; G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2 \)

7. Drag coefficient \( D = \frac{1}{2} C \rho A v^2 \)

8. Terminal speed \( v = \sqrt{\frac{2 \frac{m g}{C \rho A}}{}} \)

9. Centripetal force: \( F_c = m \frac{v^2}{r} \)

10. Speed of the satellite in a circular orbit: \( v^2 = GMm/r \)

11. The work done by a constant force acting on an object: \( W = \mathbf{F} \cdot \mathbf{d} = \mathbf{F} \cdot \mathbf{a} \)

12. Kinetic energy: \( K = \frac{1}{2} m v^2 \)

13. Total mechanical energy: \( E = K + U \)

14. The work-energy theorem: \( W = K_f - K_o ; W_{nc} = \Delta K + \Delta U = E_f - E_o \)

15. The principle of conservation of mechanical energy: when \( W_{nc} = 0 , E_f = E_o \)
1. **Work done by the gravitational force:** \( W_g = m g d \cos \phi \)

2. **Work done in Lifting and Lowering the object:**
   \[ \Delta K = K_f - K_i = W_a + W_g; \text{ if } K_f = K_i; W_a = -W_g \]

3. **Spring Force:** \( F_x = -kx \) (Hook's law)

4. **Work done by a spring force:** \( W_s = \frac{1}{2} k x_f^2 - \frac{1}{2} k x_i^2; \text{ if } x_i = 0 \text{ and } x_f = x; W_s = -\frac{1}{2} k x^2 \)

5. **Work done by a variable force:** \( W = \int_{x_i}^{x_f} F(x) \, dx \)

6. **Power:** \( P_{avg} = \frac{W}{\Delta t}; \quad P = \frac{dW}{dt}; \quad P = F \cos \phi = \vec{F} \cdot \vec{v} \)

7. **Potential energy:** \( \Delta U = -W; \Delta U = -\int_{x_i}^{x_f} F(x) \, dx \)

8. **Gravitational Potential Energy:**
   \[ \Delta U = m g (y_f - y_i) = m g \Delta y; \text{ if } y_i = 0 \text{ and } U_i = 0; \quad U(y) = m g y \]

9. **Elastic potential Energy:** \( U(x) = \frac{1}{2} k x^2 \)

10. **Potential energy curves:** \( F(x) = -\frac{dU(x)}{dx}; \quad K(x) = E_{mec} - U(x) \)

11. **Work done on a system by an external force:**
    - Friction is not involved \( W = \Delta E_{mec} = \Delta K + \Delta U \)
    - When kinetic friction force acts within the system \( W = \Delta E_{mec} + \Delta E_{kh} \)
    \[ \Delta E_{kh} = f_k d \]

12. **Conservation of energy:**
    \( W = \Delta E = \Delta E_{mec} + \Delta E_{kh} + \Delta E_{int} \)
    For isolated system \( (W = 0) \Delta E_{mec} + \Delta E_{kh} + \Delta E_{int} = 0 \)

13. **Power:** \( P_{avg} = \frac{\Delta E}{\Delta t}; \quad P = \frac{dE}{dt} \)
1. The force $F=490\text{N}$ is required to lift a load of unknown mass $M$ with the pulley system shown in Figure. Determine the mass of the load $M$ and mechanical advantage of the system. Neglect friction and the weights of the pulleys.

\[ 2T_1 - Mg = 0 \]
\[ M = \frac{2T_1}{g} \]
\[ 2F - T_1 = 0 \]
\[ T_1 = 2F \]

\[ M = \frac{4F}{g} = \frac{4 \times (490\text{N})}{(9.8 \text{ m/s}^2)} = 200 \text{ kg} \]

Mechanical advantage $= \frac{Mg}{F} = 4$
A block of mass $m = 5$ kg starting from rest slides (from the top) down an incline plane of length 15 m and angle $\theta = 37^\circ$. Coefficient of friction is $\mu_k = 0.30$.

a) Show free body diagram and calculate acceleration of the block.

b) Calculate the velocity of the block when it reaches the bottom of the incline.

c) Calculate work done by the friction force when the block reaches the bottom of the incline.

d) Calculate the velocity of the block when it reaches the bottom of the incline using work-energy theorem.
3. In Fig.,
(a) How large is a force $F$ is needed to give the blocks the acceleration of 3.0 m/s$^2$ if the coefficient of friction between blocks and the table is 0.20?
(b) How large a force does the 1.5 kg block then exert on the 2.0 kg block?

![Diagram of blocks and forces](image)

\[ F - f_{1k} - f_{2k} - f_{3k} = (m_1 + m_2 + m_3) \cdot a \]
\[ \Rightarrow F = (m_1 + m_2 + m_3) \cdot a + \mu_{12} m_1 g = (1.5 + 2.0 + 1.0)(3.0 + 0.2 \cdot 9.8) = 22.3 \text{ N} \]

\[ F - f_{1k} = m_1 a \]
\[ F_{21} = F - f_{1k} - m_1 a = 22.3 - 0.2 \times 1.5 \times 9.8 - 1.5 \times 3 = 15 \text{ N} \]
4. Find the acceleration of the car necessary to prevent a block with a mass $m_b$ from falling off. The coefficient of static friction between the car and the block is $\mu_s = 1.0$.

\[ y: f_s - m_b g = 0 \] 
\[ f_s = m_b g \]

\[ x: N = m_b a \]
\[ f_s = \mu_s N = \mu_s m_b g \]

\[ \Rightarrow \mu_s m_b a = m_b g \]

\[ a = \frac{g}{\mu_s} = \frac{9.8 \text{ m/s}^2}{1.0} = 9.8 \text{ m/s}^2 \]
5. A 700 kg sports car goes with a constant speed of 10 m/s over a
   (a) horizontal bridge
   (b) convex bridge with a radius of curvature R=35 m
   (c) concave bridge with a radius of curvature R=50 m
   What is the apparent weight of the car in the middle of the bridge in the three cases?

(a)

\[ y: N - mg = 0 \]
\[ \Rightarrow N = mg \]
\[ |\Delta W| = |N| = mg = (700 \text{ kg})(9.8 \text{ } \text{m/s}^2) = 6860 \text{ kg} \cdot \text{m/s}^2 = 6.9 \times 10^3 \text{ N} \]

(b)

\[ y: mg - N = \frac{m v^2}{\ell} \]
\[ |\Delta W| = |N| = m\left(g - \frac{v^2}{\ell}\right) = (700 \text{ kg})(9.8 \text{ } \text{m/s}^2 - \left(\frac{10 \text{ m/s}}{35 \text{ m}}\right)^2) = 4860 \text{ N} = 4.9 \times 10^3 \text{ N} \]

(c)

\[ y: N - mg = \frac{m v^2}{\ell} \]
\[ |\Delta W| = |N| = m\left(g + \frac{v^2}{\ell}\right) = (700 \text{ kg})(9.8 \text{ } \text{m/s}^2 + \left(\frac{10 \text{ m/s}}{50 \text{ m}}\right)^2) = 8260 \text{ N} = 8.3 \times 10^3 \text{ N} \]
6. A man pushes an 80-N crate a distance of 5.0 m upward along a frictionless slope that makes an angle of 30° with the horizontal. The force he exerts is parallel to the slope. If the speed of the crate is constant, then what is the work done by the man.

\[ W_{\text{net}} = K_f - K_i = 0 \text{ since speed of the crate = const and change of KE = 0} \]

\[ W_{\text{net}} = W_F + W_g + W_N \text{ since } N \perp \text{ displacement} \]

\[ W_{\text{net}} = 0 \]

\[ W_F = mg \cdot d \cdot \cos(90° + \theta) = \]

\[ = -(80N)(5.0m) \cdot \cos 120° = \left| -2.0 \times 10^2 \right| \text{J} \]

\[ \theta = 30° \]

\[ \text{Given:} \]

\[ d = 5.0m \]

\[ mg = 80N \]

\[ \theta = 30° \]

\[ a = 0 \]

\[ v = \text{const} \]

\[ \text{Find } W \]

\[ \text{Use work-kinetic energy theorem} \]
A toy cork gun contains a spring whose spring constant is 10.0N/m. The spring is compressed 5.00 cm and then used to propel a 6.00-g cork. The cork, however, sticks to the spring for 1.00 cm beyond its unstretched length before separation occurs. What is the muzzle velocity of this cork?

- Assume friction is negligible
  \[ W_{\text{net}} = 0 \]
  \[ \Rightarrow \text{ according to work energy theorem} \]
  \[ W_{\text{net}} = ME_f - ME_i = (K_f + U_f) - (K_i + U_i) \]
- Initial State
  \[ K_i = 0 \]
  \[ U_i = \frac{K x_i^2}{2} \]
- Final State when spring is elongated with deformation 1 cm.
  \[ K_f = \frac{m v_f^2}{2} \]
  \[ U_f = \frac{K x_f^2}{2} \]
- \[ \frac{K x_i^2}{2} = \frac{K x_f^2}{2} + \frac{m v^2}{2} \]
  \[ v^2 = \sqrt{\frac{K (x_i^2 - x_f^2)}{m}} = \sqrt{\frac{(10 \text{ N/m}) \left( (5 \times 10^{-2} \text{ m})^2 - (1 \times 10^{-2} \text{ m})^2 \right)}{(6 \times 10^{-3} \text{ kg})}} = \]
  \[ = 2.0 \text{ m/s} \]
The potential energy of a 0.20-kg particle moving along the x axis is given by

\[ U(x) = (8.0 \text{J/m}^2)x^2 + (2.0 \text{J/m}^4)x^4 \]

If the total mechanical energy is 9.0J, what are the limits of motion?

The limits of motion are defined by turning points where \( K(x) = E_m - U(x) = 0 \)

\[ g = 8.0x^2 + 2.0x^4 \]

\[ x^2 = \frac{-8.0 \pm \sqrt{64.0 + 32.0}}{4.0} \]

\[ x^2 = \frac{-8.0 \pm 11.66}{4.0} \]

\[ x^2 = (-2.0 \pm 2.92) \text{m}^2 \]

\[ x_1^2 = 0.92 \text{m}^2 \]

\[ x_1 = -0.96 \text{ m} \]

\[ x_2 = +0.96 \text{ m} \]