ALL QUESTIONS ARE WORTH 20 POINTS. WORK OUT FIVE PROBLEMS.
NOTE: Clearly write out solutions and answers (circle the answers) by section for each part (a., b., c., etc.)

Chapter 21

1. \( \vec{F} = G \frac{m_1 m_2}{r^2} \vec{r} \)  \( \text{Newton's Law of Gravitation} \)

2. \( \vec{F} = \frac{1}{4 \pi \varepsilon_0} \cdot \frac{q_1 q_2}{r^2} \vec{r} = k \frac{q_1 q_2}{r^2} \vec{r} \)  \( \text{Coulomb's Law} \)

3. \( F = \frac{1}{4 \pi \varepsilon_0} \cdot \frac{|q_1||q_2|}{r^2} = k \frac{|q_1||q_2|}{r^2}, \varepsilon_0=8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2, k=8.99 \times 10^9 \text{ Nm}^2/\text{C}^2 \)

4. \( 1 \text{C}=(1\text{A})(1\text{s}) \)

5. \( i = \frac{dq}{dt} \)  \( \text{Electrical Current} \)

Chapter 22

6. \( \vec{E} = \frac{\vec{F}}{q_0} \)  \( \text{Electric field, } q_0 \text{ is a positive test charge; } F \text{ is electrostatic force that acts on the test charge} \)

7. \( \vec{F} = q \vec{E} \)  \( \text{Electrostatic force that acts on the particle with charge } q \).

8. \( \vec{E} = \frac{1}{4 \pi \varepsilon_0} \cdot \frac{q}{r^2} \vec{r} = k \frac{q}{r^2} \vec{r} \)  \( \text{Electric field due to Point charge} \)

9. \( E = \frac{1}{2 \pi \varepsilon_0} \cdot \frac{q d}{z^3} = \frac{1}{2 \pi \varepsilon_0} \cdot \frac{p}{z^3}, \text{Electric field on axis } z \text{ due to an electric dipole.} \)

\( p=qd \) is an electric dipole moment with direction from negative to positive ends of a dipole.
1. \[ E = \frac{1}{4 \pi \varepsilon_0} \cdot \frac{q z}{(z^2 + R^2)^{3/2}}, \text{ Electric field due to charged ring} \]

2. \[ E = \frac{1}{4 \pi \varepsilon_0} \cdot \frac{q}{z^2}, \text{ Electric field due to charged ring at large distance } (z>R) \]

3. \[ E = \frac{\sigma}{2 \varepsilon_0} \cdot \left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right), \text{ Electric field due to charged disk} \]

4. \[ E = \frac{\sigma}{2 \varepsilon_0}, \text{ Electric field due to infinite sheet} \]

5. \[ \vec{F} = \vec{p} \times \vec{E} = p E \sin(\theta), \text{ Torque on a dipole in electric field} \]

6. \[ U = -\vec{p} \cdot \vec{E} = -p E \cos(\theta), \text{ Potential energy of a dipole in electric field.} \]

**Chapter 23**

7. \[ \Phi = \sum \vec{E} \cdot \Delta \vec{A}, \text{ electric flux through a surface} \]

8. \[ \Phi = \oint \vec{E} \cdot d\vec{A}, \text{ Electric flux through a Gaussian surface.} \]

9. \[ \varepsilon_0 \Phi = q_{\text{enc}}, \text{ Gauss’ Law} \]

10. \[ \varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{\text{enc}}, \text{ Gauss’ Law} \]

11. \[ E = \frac{\sigma}{\varepsilon_0}, \text{ Conducting surface} \]

12. \[ E = \frac{\lambda}{2 \pi \varepsilon_0 r}, \text{ Line of charge} \]

13. \[ E = \frac{\sigma}{\varepsilon_0}, \text{ Electric field between two conducting plates} \]

14. \[ E = \frac{1}{4 \pi \varepsilon_0} \cdot \frac{q}{r^2}, \text{ Spherical shell field at } r \geq R \]

\[ E=0, \text{ Spherical shell field at } r < R \]
1. \[ E = \left( \frac{q}{4 \pi \varepsilon_0 R^2} \right) r \], Uniform spherical charge distribution at \( r \leq R \)

2. \[ E = \frac{1}{4 \pi \varepsilon_0} \cdot \frac{q}{r^2} \], Uniform spherical charge distribution at \( r > R \)

**Chapter 24**

3. \[ \Delta U = U_f - U_i = -W \], Change in electric potential energy due to work done by electrostatic force.

4. \[ U = -W_\infty \], Potential energy; \( W_\infty \) is work done by electrostatic force during particle move from infinity.

5. \[ V = \frac{U}{q} = -\frac{W_\infty}{q} \], Electric potential

6. \[ \Delta V = V_f - V_i = \frac{U_f}{q} - \frac{U_i}{q} = \frac{\Delta U}{q} = -\frac{W}{q} \], Electric potential difference

7. \[ V = -\int \vec{E} \cdot d\vec{s} \], Potential at any point \( f \) in the electric field relative to the zero potential at point \( i \).

8. \[ V = \frac{1}{4 \pi \varepsilon_0} \cdot \frac{q}{r} \], Potential due to point charge.

9. \[ V = \sum_i^n V_i = \frac{1}{4 \pi \varepsilon_0} \sum_i^n \frac{q_i}{r_i} \], Potential due to group of \( n \) point charges.

10. \[ V = \int dV = \frac{1}{4 \pi \varepsilon_0} \int \frac{dq}{r} \], Potential due to continuous charge distribution.

11. \[ V = \frac{\lambda}{4 \pi \varepsilon_0} \ln \left( \frac{L + \left( L^2 + d^2 \right)^{1/2}}{d} \right) \], Potential due to line of charge

12. \[ V = \frac{\sigma}{2 \varepsilon_0} \left( \sqrt{z^2 + R^2} - z \right) \], Potential due to charged disk
1. \( V = \frac{1}{4 \pi \varepsilon_0} \cdot \frac{p \cos(\theta)}{r^2} \), Potential due to electric dipole.

2. \( \vec{E} = E_x \hat{i} + E_y \hat{j} + E_z \hat{k} \), Calculating electric field from the potential
\[ E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial z} \]

3. \( E = -\frac{\Delta V}{\Delta s} \), Calculating electric field from the potential in a simple case of uniform electric field.

4. \( U = \frac{1}{4 \pi \varepsilon_0} \cdot \frac{q_1 q_2}{r_{12}} + \frac{1}{4 \pi \varepsilon_0} \cdot \frac{q_1 q_3}{r_{13}} + \frac{1}{4 \pi \varepsilon_0} \cdot \frac{q_2 q_3}{r_{23}} \), Potential energy of system of three charges.

5. \( U = \frac{1}{4 \pi \varepsilon_0} \sum_{i<j}^{n} \frac{q_i q_j}{r_{ij}} \), Potential energy of system of \( n \) charges

6. \( U = \frac{1}{2} \sum_{i=1}^{n} V_{other}(i) \cdot q_i \)

7. Alternative expression for Potential energy of system of \( n \) charges
\[ V_{other}(i) = \frac{1}{4 \pi \varepsilon_0} \sum_{j \neq i}^{n} \frac{q_j}{r_{ji}} \]

**Chapter 25**

8. Capacitance is defined from \( q = CV \).

9. A parallel plate capacitor \( C = \frac{\varepsilon_0 A}{d} \)

10. A cylindrical capacitor \( C = 2 \pi \varepsilon_0 \frac{L}{\ln(b/a)} \)

11. A spherical capacitor \( C = 4 \pi \varepsilon_0 \frac{ab}{b-a} \)

12. Isolated sphere \( C = 4 \pi \varepsilon_0 R \)

13. \( n \) capacitors in parallel \( C_{eq} = \sum_{j=1}^{n} C_j \)

14. \( n \) capacitors in series \( \frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j} \)

15. The electric potential energy of a charged capacitor \( U = \frac{q^2}{2C} = \frac{1}{2}CV^2 \)
1. The energy density in vacuum $u = \frac{1}{2} \varepsilon_0 E^2$
2. Capacitance with a dielectric filling the space between the plates $C = k \varepsilon_0 A$
3. Gauss law with a dielectric $\oint k \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$

Chapter 26.

4. An electric current $i = \frac{d q}{d t}$
5. Current and current density relationship $i = \int \vec{J} \cdot d\vec{A}$
6. Current density relationship with a drift speed and carrier charge density $\vec{J} = (n e) \vec{v}_d$
7. Resistance of a conductor $R = \frac{V}{i}$
8. Resistivity and conductivity of materials $\rho = \frac{1}{\sigma} = \frac{E}{J}; E = \rho \vec{J}$
9. Resistance of a conductive wire $R = \rho \frac{L}{A}$
10. Change of resistivity with temperature $\rho - \rho_o = \rho_o \alpha (T - T_o)$
11. Ohm’s Law $R = \frac{V}{i}$ is independent of the applied potential difference $V$.
12. Resistivity of metal $\rho = \frac{m}{e^2 n \tau}$
13. Power $P = iV$
14. Resistive dissipation $P = i^2 R = \frac{V^2}{R}$

Chapter 27.

15. Emf $\varepsilon = \frac{d W}{d q}$
16. Loop rule: the algebraic sum of electromotive forces and potential changes across circuit elements in a complete traversal of any loop of a circuit must be zero.
17. Single loop circuits $i = \frac{\varepsilon}{R + r}$
18. Power $P = iV; P = i^2 R; P_{emf} = i\varepsilon$
19. Series resistances $R_{eq} = \sum_{j=1}^{n} R_j$
20. Parallel resistances $\frac{1}{R_{eq}} = \sum_{j=1}^{n} \frac{1}{R_j}$
1. **RC circuits. Charging the capacitor** \( q = C \varepsilon (1 - e^{-t/RC}) \); \( i = \frac{dq}{dt} = \left( \frac{\varepsilon}{R} \right) e^{-t/RC} \)

2. **Discharging the capacitor** \( q = q_o e^{-t/RC} \); \( i = \frac{dq}{dt} = -\left( \frac{q_o}{RC} \right) e^{-t/RC} \)
1. A 10.0 μF capacitor is charged so that the potential difference between its plates is 10.0 V. A 5.0 μF capacitor is similarly charged so that the potential difference between its plates is 5.0 V. The two charged capacitors are then connected to each other in parallel with positive plate connected to the positive plate and negative plate connected to negative plate. How much charge flows from one capacitor to the other when the capacitors are connected?

\[ \Delta V_1 = 10.0 \text{ V} \]
\[ C_1 = 10.0 \mu \text{F} \]
\[ \Delta V_2 = 5.0 \text{ V} \]
\[ C_2 = 5.0 \mu \text{F} \]

2) During connection capacitors in parallel their total charge is conserved.

- \[ Q_1 = C_1 \Delta V_1 = 10.0 \mu \text{F} \times 10.0 \text{ V} = 100 \mu \text{C} \]
- \[ Q_2 = C_2 \Delta V_2 = 5.0 \mu \text{F} \times 5.0 \text{ V} = 25 \mu \text{C} \]
- \[ Q = Q_1 + Q_2 = 125 \mu \text{C} \]

\[ \Delta V = \frac{Q}{Q_1 + Q_2} = \frac{125 \mu \text{C}}{15.0 \mu \text{F}} = 8.33 \text{ V} \]

- \[ Q_1' = C_1 \Delta V = 83.3 \mu \text{C} \]
- \[ Q_2' = C_2 \Delta V = 5.0 \mu \text{F} \times 8.33 \text{ V} = 41.7 \mu \text{C} \]

\[ \Delta Q = Q_1 - Q_1' = 100 \mu \text{C} - 83.3 \mu \text{C} = 16.7 \mu \text{C} \]

flows from capacitor 1 to capacitor 2.
2. Three capacitors are connected in Fig. Find
   (a) the charge on each of the capacitors
   (b) the potential difference across each of the capacitors

\[ C_1 = 1.0 \, \mu F \]

\[ C_2 = 1.0 \, \mu F \]

\[ C_3 = 1.0 \, \mu F \]

100 V

\[ \text{\( Q = C_3 \cdot V = 0.67 \times 100 = 67 \mu C \)} \]

\[ \Rightarrow Q_3 = 67 \mu C \]

\[ Q_1 = Q_2 = \frac{Q}{2} = 33 \mu C \]

\[ \frac{1}{C_3} = \frac{1}{2.0} + \frac{1}{1.0} = \frac{3}{2.0} \]

\[ C_E = \frac{2}{3} \mu F \]

\[ \Delta V_{AB} = \Delta V_1 = \Delta V_2 = \frac{Q}{2.0 \mu F} = \frac{67 \mu C}{2.0 \mu F} = 33 \, V \]

\[ \Delta V_{AC} = \Delta V_3 = \frac{Q}{1.0 \mu F} = \frac{67 \mu C}{1 \mu F} = 67 \, V \]
3. The current in Fig. is 1.0 A in the direction shown. For each of the following pairs of points, what is their potential difference, and which point is at the higher potential?

- (a) A, B; Recall the following facts:
  1. The current is the same (1.0 A) at all points in this circuit because the charge has no other place to flow.
  2. Current always flows from high to low potential through a resistor.
  3. The + terminal of the battery is always the high potential terminal.

- (b) B, C;
- (c) C, D;
- (d) D, E;
- (e) C, E;
- (f) E, C.

\[ I = 1.0 \, \text{A} \]

\[ A \quad 1 \, \Omega \quad B \quad 20 \, \text{V} \quad C \]

\[ 4 \, \Omega \quad 10 \, \text{V} \quad E \quad 3 \, \Omega \]

\[
\begin{align*}
\text{a)} \quad V_{AB} &= V_B - V_A = +10 \times 1 = 10 \, \text{V} \quad \text{(B is higher)} \\
\text{b)} \quad V_{BC} &= V_C - V_B = -20 \, \text{V} \quad \text{B is higher} \\
\text{c)} \quad V_{CD} &= V_D - V_C = V_D - (V_C - 1.0 \times 3 - 1.0 \times 2) = 5 \, \text{V} \quad \text{D is higher} \\
\text{d)} \quad V_{DE} &= V_E - V_D = +10 \, \text{V} \quad \text{(E is higher)} \\
\text{e)} \quad V_{CE} &= V_E - V_C = (V_C + 20 - 1.0 \times 1 - 1.0 \times 4) - V_C = 20 - 1 - 4 = +15 \, \text{V} \quad \text{E is higher} \\
\text{f)} \quad V_{EC} &= V_C - V_E = (V_E - 10 \, \text{V} - 3 \times 1.0 - 1.0 \times 2) - V_E = -15 \, \text{V} \quad \text{E is higher}
\end{align*}
\]
4. For the circuit shown in Fig., determine:
(a) the net resistance of the combination
(b) the current that passes through the combination
(c) the potential difference and the current for 9Ω resistor

![Circuit Diagram]

1) The 19Ω and 5Ω resistors are in series. Their joint resistance is:
\[ R_{\text{total}} = R_1 + R_2 = 19Ω + 5Ω = 24Ω \]

2) 8Ω is in parallel to 24Ω. Their combined resistance is given by:
\[ \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{8Ω} + \frac{1}{24Ω} = \frac{3+1}{24Ω} = \frac{1}{6Ω} \]
\[ R = 6Ω \]

3) 15Ω and 6Ω resistors are in series. Their joint resistance is:
\[ R_{\text{total}} = R_1 + R_2 = 15Ω + 6Ω = 21Ω \]

4) 9Ω and 21Ω are in parallel:
\[ \frac{1}{R_2} = \frac{1}{9Ω} + \frac{1}{21Ω} = \frac{7+3}{63Ω} = \frac{10}{63Ω} \]
\[ R_2 = 6.3Ω \]

5) \[ I = \frac{17V}{8.3Ω} = 2.0A \]

6) \[ V_{AB} = I \cdot 6.3 = 12.6V = 1 \times 10^1V \]

7) \[ I_9 = \frac{\Delta V_{AB}}{9Ω} = \frac{12.6}{9Ω} = 1.4A = 1 \times 10^0A \]
5. (a) Find the current in the three resistors shown in Fig.
(b) Find the power delivered by the battery $E_1$

\[ R_1 = 17 \, \Omega \]
\[ E_1 = 4 \, V \]
\[ R_2 = 12 \, \Omega \]
\[ E_2 = 5 \, V \]
\[ R_3 = 20 \, \Omega \]

\[ E_1 = 10 \, V \]

\[ I_1 \]
\[ I_2 \]
\[ I_3 \]

\[ I_1, I_2, I_3 \]

\[ E_1 - I_1 R_1 - E_2 - (I_1 - I_2) R_2 = 0 \]
\[ E_2 + E_3 - I_2 R_3 - (I_2 - I_3) R_2 = 0 \]

\[ 10 - 17 I_1 - 5 - 12 I_2 + 12 I_3 = 0 \]
\[ 5 + 4 - 20 I_2 - 12 I_2 + 12 I_3 = 0 \]

\[ 5 - 29 I_1 + 12 I_2 = 0 \]
\[ 9 - 32 I_2 + 12 I_3 = 0 \]
\[ 9 - \frac{232 I_1 + 40}{3} + 12 I_3 = 0 \]

\[ 27 - 232 I_1 + 40 + 36 I_3 = 0 \]

\[ 196 I_1 = 67 \]

\[ I_1 = 0.34 \, A \]

\[ I_2 = \frac{29 - 0.34 - 5}{12} = 0.41 \, A \]

\[ I_{E1} = I_1 = 0.34 \, A \]

\[ I_{R2} = I_2 - I_1 = 0.41 - 0.34 = 0.07 \, A = \frac{7 \times 10^{-2}}{A} \]

\[ I_{R3} = I_2 = 0.41 \, A \]

\[ (6) \quad P_{E1} = E_1 \cdot I_1 = (10 \, V) \cdot (0.34 \, A) = 3.4 \, W \]
6. Find the current in the resistors shown in Fig.

\[ I_2 + I_1 R_2 = 0 \]

\[ I_1 = \frac{\Sigma V_1 + \Sigma V_2}{R_2} = \frac{4 + 8}{5} = 2.4 \text{A} \]

\[ \Sigma V_3 + \Sigma V_1 - I_2 R_1 = 0 \]

\[ I_2 = \frac{\Sigma V_1 + \Sigma V_3}{R_1} = \frac{4 + 6}{7} = 1.4 \text{A} \]
7. The current density magnitude in a certain circular wire is \( J = (2.75 \times 10^{10} \text{ A/m}^4) r^2 \), where \( r \) is the radial distance out to the wire’s radius of 3.00 mm. The potential applied to the wire (end to end) is 60.0 V. How much energy is converted to thermal energy in 1.00 h?

Assuming the current is along the wire (not radial) we find the current from:

\[
i = \int |\vec{J}| \, dA = \int_0^R kr^2 2\pi r dr = \frac{1}{2} k\pi R^4 = 3.50 \text{ A}
\]

where \( k = 2.75 \times 10^{10} \text{ A/m}^4 \) and \( R = 0.00300 \text{ m} \). The rate of thermal energy generation is found from \( P = iV = 210 \text{ W} \). Assuming a steady rate, the thermal energy generated in 3600 s is \( Q = P\Delta t = (210 \text{ J/s})(3600 \text{ s}) = 7.56 \times 10^5 \text{ J} \).
8. A caterpillar of length 4.0 cm crawls in the direction of electron drift along a 5.2 mm diameter bare copper wire that carries a uniform current of 12 A.
(a) What is the potential difference between the two ends of the caterpillar?
(b) Is its tail positive or negative with respect to its head?
(c) How much time does the caterpillar take to crawl 1.0 cm if it crawls at the drift speed of the electrons in the wire? (The number of charge carriers per unit volume is 8.49x10^{28} m^{-3}.)

(a) The potential difference between the two ends of the caterpillar is

\[ V = iR = i\rho \frac{L}{A} = \frac{(12 \text{ A})(1.69 \times 10^{-8} \Omega \cdot \text{m})(4.0 \times 10^{-2} \text{ m})}{\pi(5.2 \times 10^{-3} \text{ m} / 2)^2} = 3.8 \times 10^{-4} \text{ V}. \]

(b) Since it moves in the direction of the electron drift which is against the direction of the current, its tail is negative compared to its head.

(c) The time of travel relates to the drift speed:

\[ t = \frac{L}{v_d} = \frac{lAne}{i} = \frac{\pi Ld^2 ne}{4i} = \frac{\pi(1.0 \times 10^{-2} \text{ m})(5.2 \times 10^{-3} \text{ m})^2 (8.47 \times 10^{28} / \text{m}^3)(1.60 \times 10^{-19} \text{ C})}{4(12 \text{ A})} \]

\[ = 238 \text{ s} = 3 \text{ min 58 s}. \]