ALL QUESTIONS ARE WORTH 30 POINTS. WORK OUT FIVE PROBLEMS.

NOTE: Clearly write out solutions and answers (circle the answers) by section for each part (a., b., c., etc.)

Important Formulas:

**Chapter 21**

1. \( \vec{F} = G \frac{m_1 m_2}{r^2} \vec{r} \) \text{ Newton’s Law of Gravitation}

2. \( \vec{F} = \frac{1}{4\pi \varepsilon_0} \frac{q_1 q_2}{r^2} \vec{r} = k \frac{q_1 q_2}{r^2} \vec{r} \) \text{ Coulomb’s Law}

3. \( F = \frac{1}{4\pi \varepsilon_0} \frac{|q_1| |q_2|}{r^2} = k \frac{|q_1| |q_2|}{r^2} \), \( \varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2 \), \( k = 8.99 \times 10^9 \text{ Nm}^2/\text{C}^2 \)

4. \( 1C = (1A)(1s) \)

5. \( i = \frac{dq}{dt} \) \text{ Electrical Current}

**Chapter 22**

6. \( \vec{E} = \frac{\vec{F}}{q_0} \) \text{ Electric field, } q_0 \text{ is a positive test charge; } F \text{ is electrostatic force that acts on the test charge}

7. \( \vec{F} = q \vec{E} \), Electrostatic force that acts on the particle with charge \( q \).

8. \( \vec{E} = \frac{1}{4\pi \varepsilon_0} \frac{q}{r^2} \vec{r} = k \frac{q}{r^2} \vec{r} \), \text{ Electric field due to Point charge}

9. \( E = \frac{1}{2\pi \varepsilon_0} \frac{q d}{z^3} = \frac{1}{2\pi \varepsilon_0} \frac{p}{z^3} \), \text{ Electric field on axis } z \text{ due to an electric dipole.}

\( p = qd \) \text{ is an electric dipole moment with direction from negative to positive ends of a dipole.}
1. \[ E = \frac{1}{4\pi \varepsilon_0} \cdot \frac{q z}{(z^2 + R^2)^{3/2}}, \text{ Electric field due to charged ring} \]

2. \[ E = \frac{1}{4\pi \varepsilon_0} \cdot \frac{q}{z^2}, \text{ Electric field due to charged ring at large distance } (z>>>R) \]

3. \[ E = \frac{\sigma}{2\varepsilon_0} \left( 1 - \frac{z}{\sqrt{z^2 + R^2}} \right), \text{ Electric field due to charged disk} \]

4. \[ E = \frac{\sigma}{2\varepsilon_0}, \text{ Electric field due to infinite sheet} \]

5. \( \vec{r} = \vec{p} \times \vec{E} = p E \sin(\theta), \text{ Torque on a dipole in electric field} \)

6. \( U = -\vec{p} \cdot \vec{E} = -p E \cos(\theta), \text{ Potential energy of a dipole in electric field.} \)

**Chapter 23**

7. \( \Phi = \sum \vec{E} \cdot \Delta \vec{A}, \text{ electric flux through a surface} \)

8. \( \Phi = \iint \vec{E} \cdot d\vec{A}, \text{ Electric flux through a Gaussian surface.} \)

9. \( \varepsilon_0 \Phi = q_{enc}, \text{ Gauss' Law} \)

10. \( \varepsilon_0 \iint \vec{E} \cdot d\vec{A} = q_{enc}, \text{ Gauss' Law} \)

11. \( E = \frac{\sigma}{\varepsilon_0}, \text{ Conducting surface} \)

12. \( E = \frac{\lambda}{2\pi \varepsilon_0 r}, \text{ Line of charge} \)

13. \( E = \frac{\sigma}{\varepsilon_0}, \text{ Electric field between two conducting plates} \)

14. \( E = \frac{1}{4\pi \varepsilon_0} \cdot \frac{q}{r^2}, \text{ Spherical shell field at } r\geq R \)

\[ E=0, \text{ Spherical shell field at } r<R \]
1. \( E = \left( \frac{q}{4\pi \varepsilon_0 R^2} \right) r \), Uniform spherical charge distribution at \( r \leq R \)

2. \( E = \frac{1}{4\pi \varepsilon_0} \cdot \frac{q}{r^2} \), Uniform spherical charge distribution at \( r > R \)

Chapter 24

3. \( \Delta U = U_f - U_i = -W \), Change in electric potential energy due to work done by electrostatic force.

4. \( U = -W_\infty \), Potential energy; \( W_\infty \) is work done by electrostatic force during particle move from infinity.

5. \( V = \frac{U}{q} = -\frac{W_\infty}{q} \), Electric potential

6. \( \Delta V = V_f - V_i = \frac{U_f}{q} - \frac{U_i}{q} = \frac{\Delta U}{q} = -\frac{W}{q} \), Electric potential difference

7. \( V = -\int f \vec{E} \cdot d\vec{s} \), Potential at any point \( f \) in the electric field relative to the zero potential at point \( i \).

8. \( V = \frac{1}{4\pi \varepsilon_0} \cdot \frac{q}{r} \), Potential due to point charge.

9. \( V = \sum_i^n V_i = \frac{1}{4\pi \varepsilon_0} \sum_i^n \frac{q_i}{r_i} \), Potential due to group of \( n \) point charges.

10. \( V = \int dV = \frac{1}{4\pi \varepsilon_0} \int \frac{dq}{r} \), Potential due to continuous charge distribution.

11. \( V = \frac{\lambda}{4\pi \varepsilon_0} \ln \left( \frac{L + \left( L^2 + d^2 \right)^{1/2}}{d} \right) \), Potential due to line of charge

12. \( V = \frac{\sigma}{2\varepsilon_0} \left( \sqrt{z^2 + R^2} - z \right) \), Potential due to charged disk
1. \[ V = \frac{1}{4 \pi \varepsilon_0} \cdot \frac{p \cdot C \cdot \cos(\theta)}{r^2}, \] Potential due to electric dipole.

2. \[ \mathbf{E} = E_x \hat{i} + E_y \hat{j} + E_z \hat{k}, \] Calculating electric field from the potential
\[ E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial z} \]

3. \[ E = -\frac{\Delta V}{\Delta s}, \] Calculating electric field from the potential in a simple case of uniform electric field.

4. \[ U = \frac{1}{4 \pi \varepsilon_0} \cdot \frac{q_1 q_2}{r_{12}} + \frac{1}{4 \pi \varepsilon_0} \cdot \frac{q_1 q_3}{r_{13}} + \frac{1}{4 \pi \varepsilon_0} \cdot \frac{q_2 q_3}{r_{23}}, \] Potential energy of system of three charges.

5. \[ U = \frac{1}{4 \pi \varepsilon_0} \sum_{i \neq j}^{n} \frac{q_i q_j}{r_{ij}}, \] Potential energy of system of \( n \) charges

6. \[ U = \frac{1}{2} \sum_{i=1}^{n} V_{other}(i) q_i \]

7. \[ V_{other}(i) = \frac{1}{4 \pi \varepsilon_0} \sum_{j \neq i}^{n} \frac{q_j}{r_{ji}} \]

**Chapter 25**

8. Capacitance is defined from \( q = C V \).

9. A parallel plate capacitor \( C = \frac{\varepsilon_0 A}{d} \).

10. A cylindrical capacitor \( C = 2 \pi \varepsilon_0 \frac{L}{\ln(b/a)} \).

11. A spherical capacitor \( C = 4 \pi \varepsilon_0 \frac{ab}{b-a} \).

12. Isolated sphere \( C = 4 \pi \varepsilon_0 R \).

13. \( n \) capacitors in parallel \( C_{eq} = \sum_{j=1}^{n} C_j \)

14. \( n \) capacitors in series \( C_{eq} = \sum_{j=1}^{n} \frac{1}{C_j} \)

15. The electric potential energy of a charged capacitor \( U = \frac{q^2}{2C} = \frac{1}{2} CV^2 \)
1. The energy density in vacuum $u = \frac{1}{2} \varepsilon_0 E^2$

2. Capacitance with a dielectric filling the space between the plates $C = k C_{air}$

3. Gauss law with a dielectric $\mathbf{\int} k\mathbf{E} \cdot d\mathbf{A} = \frac{q}{\varepsilon_0}$

Chapter 26.

4. An electric current $i = \frac{d q}{d t}$

5. Current and current density relationship $i = \mathbf{\int} \mathbf{J} \cdot d\mathbf{A}$

6. Current density relationship with a drift speed and carrier charge density $\mathbf{J} = (n e) \mathbf{v}_d$

7. Resistance of a conductor $R = \frac{V}{i}$

8. Resistivity and conductivity of materials $\rho = \frac{1}{\sigma} = \frac{E}{J} \Rightarrow E = \rho J$

9. Resistance of a conductive wire $R = \rho \frac{L}{A}$

10. Change of resistivity with temperature $\rho - \rho_o = \rho_o \alpha (T - T_o)$

11. Ohm’s Law $R = \frac{V}{i}$ is independent of the applied potential difference $V$.

12. Resistivity of metal $\rho = \frac{m}{e^2 n \tau}$

13. Power $P = iV$

14. Resistive dissipation $P = i^2 R = \frac{V^2}{R}$

Chapter 27.

15. Emf $\varepsilon = \frac{d W}{d q}$

16. Loop rule: the algebraic sum of electromotive forces and potential changes across circuit elements in a complete traversal of any loop of a circuit must be zero

17. Single loop circuits $i = \frac{\varepsilon}{R + r}$

18. Power $P = iV \Rightarrow P = i^2 R \Rightarrow P_{emf} = i\varepsilon$

19. Series resistances $R_{eq} = \sum_{j=1}^{k} R_j$

20. Parallel resistances $\frac{1}{R_{eq}} = \sum_{j=1}^{k} \frac{1}{R_j}$
1. **RC circuits.** Charging the capacitor \( q = C \varepsilon (1 - e^{-t/RC}) \); \( i = \frac{dq}{dt} = \left( \frac{\varepsilon}{R} \right) e^{-t/RC} \)

2. **Discharging the capacitor** \( q = q_o e^{-t/RC} \); \( i = \frac{dq}{dt} = -\left( \frac{q_o}{RC} \right) e^{-t/RC} \)

### Chapter 28. Magnetic fields

3. **Magnetic force exerted on a point charge by a magnetic field:** \( \vec{F}_B = q \vec{v} \times \vec{B} \)

4. **The number density** \( n \) **of charge carriers** (Hall Effect): \( n = \frac{Bi}{Ve} \)

5. **Circular orbit in magnetic field:** \( r = \frac{mv}{|q|B} \)

6. **Magnetic force on a straight current carrying wire of length** \( L \): \( \vec{F}_B = i \vec{L} \times \vec{B} \)

7. **The force acting on a current element in a magnetic field:** \( d\vec{F}_B = i d\vec{L} \times \vec{B} \)

8. **Magnetic dipole moment of current loop:** \( \mu = [\text{current}] \times [\text{area}] \)

9. **Torque on a current carrying coil:** \( \vec{\tau} = \vec{\mu} \times \vec{B} \)

10. **Orientation energy of a magnetic dipole:** \( U(\theta) = -\vec{\mu} \cdot \vec{B} \)

11. **The work done on the dipole by the agent is:** \( W_a = \Delta U = U_f - U_i \)

12. **Permeability constant:** \( \mu_o = 1.26 \times 10^{-6} \text{ Ns}^2/\text{C}^2 = 1.26 \times 10^{-6} \text{ H/m}; \)

13. **Permittivity constant:** \( \varepsilon_o = 8.85 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2) = 8.85 \times 10^{-12} \text{ F/m}; \)
1. A 10.0 \mu F capacitor is charged so that the potential difference between its plates is 10.0 V. A 5.0 \mu F capacitor is similarly charged so that the potential difference between its plates is 5.0 V. The two charged capacitors are then connected to each other in parallel with positive plate connected to the positive plate and negative plate connected to negative plate. How much charge flows from one capacitor to the other when the capacitors are connected?

\[ \Delta V_1 = 10.0 \text{V} \]
\[ C_1 = 10.0 \mu F \]
\[ \Delta V_2 = 5.0 \text{V} \]
\[ C_2 = 5.0 \mu F \]

2. During connection, capacitors in parallel have total charge conserved.

\[ Q_1 = C_1 \Delta V_1 = 10.0 \mu F \times 10.0 \text{V} = 100 \mu C \]
\[ Q_2 = C_2 \Delta V_2 = 5.0 \mu F \times 5.0 \text{V} = 25 \mu C \]
\[ Q = Q_1 + Q_2 = 125 \mu C \]

\[ \Delta V = \frac{Q}{Q_1 + Q_2} = \frac{125 \mu C}{15.0 \mu F} = 8.33 \text{V} \]

\[ Q_1' = C_1 \Delta V = 83.3 \mu C \]
\[ Q_2' = C_2 \Delta V = 5.0 \mu F \times 8.33 \text{V} = 41.7 \mu C \]

\[ \Delta Q = Q_1 - Q_1' = 100 \mu C - 83.3 \mu C = 16.7 \mu C \]

flows from capacitor 1 to capacitor 2.
2. Three capacitors are connected in Fig. Find
   (a) the charge on each of the capacitors
   (b) the potential difference across each of the capacitors

(a)

\[ Q = C_E \cdot V = 0.67 \times 100 = 67 \mu C \]

\[ \Rightarrow Q_3 = 67 \mu C \]

\[ Q_1 = Q_2 = \frac{Q}{2} = 33 \mu C \]

(b) \[ \Delta V_{AB} = \Delta V_1 = \Delta V_2 = \frac{Q}{2.0 \mu F} = \frac{67 \mu C}{2.0 \mu F} = 33V \]

\[ \Delta V_{BC} = \Delta V_3 = \frac{Q}{1.0 \mu F} = \frac{67 \mu C}{1 \mu F} = 67V \]
3. The current in Fig. is 1.0 A in the direction shown. For each of the following pairs of points, what is their potential difference, and which point is at the higher potential?

(a) A, B;  
(b) B, C;  
(c) C, D;  
(d) D, E;  
(e) C, E;  
(f) E, C.

Recall the following facts:

1. The current is the same (1.0 A) at all points in the circuit because the charge has no other place to flow.
2. Current always flows from high to low potential through a resistor.
3. The + terminal of the battery is always the high potential terminal.

Taking potential drop as negative, we have the following:

a) \( V_{AB} = V_B - V_A = +10 \times 1 = +10 \text{ V} \) (B is higher)

b) \( V_{BC} = V_C - V_B = -20 \text{ V} \) (B is higher)

c) \( V_{CD} = V_D - V_C = V_0 - (V_0 - 10 \times 3 - 10 \times 2) = 5 \text{ V} \); (D is higher)

d) \( V_{DE} = V_E - V_D = +10 \text{ V} \) (E is higher)

e) \( V_{CE} = V_E - V_C = (V_C + 20 - 10 \times 1 - 10 \times 4) - V_C = 20 - 1 - 4 = +15 \text{ V} \) (E is higher)

f) \( V_{EC} = V_C - V_E = (V_E - 10 \times 3 - 3 \times 10 - 10 \times 2) - V_E = -15 \text{ V} \) (E is higher)
4. For the circuit shown in Fig., determine:
(a) the net resistance of the combination
(b) the current that passes through the combination
(c) the potential difference and the current for 9Ω resistor

4) The 19Ω and 5Ω are in series. Their joint resistance is
\[ 19Ω + 5Ω = 24Ω \]

2) 8Ω is in parallel to 24Ω. Their combined resistance is given by
\[ \frac{1}{R_1} = \frac{1}{8Ω} + \frac{1}{24Ω} = \frac{3}{24Ω} = \frac{1}{8Ω} \]
\[ R_1 = 6Ω \]

3) 15Ω and 6Ω are in series. Their joint resistance is 21Ω.

4) 9Ω and 21Ω are in parallel.
\[ \frac{1}{R_2} = \frac{1}{9Ω} + \frac{1}{21Ω} = \frac{21Ω + 9Ω}{189Ω} = \frac{10Ω}{189Ω} \]
\[ R_2 = 18.9Ω \]

5) \[ I = \frac{17V}{8.3Ω} = 2A \]

6) \[ \Delta V_{AB} = I \times 6.3 = 12.6V = 1\times10^1V \]
\[ I_g = \frac{\Delta V_{AB}}{9} = \frac{12.6V}{9} = 1.4A = 1\times10^0A \]
5. (a) Find the current in the three resistors shown in Fig.
(b) Find the power delivered by the battery $E_i$

\[ E_1 = 10 \text{ V} \]
\[ E_2 = 5 \text{ V} \]
\[ E_3 = 4 \text{ V} \]
\[ R_1 = 17 \Omega \]
\[ R_2 = 12 \Omega \]
\[ R_3 = 20 \Omega \]

\[ (a) \quad E_1 - I_1 R_1 - E_2 - (I_1 - I_2) R_2 = 0 \]
\[ 10 - 17I_1 - 5 - 12I_1 + 12I_2 = 0 \]
\[ E_2 + E_3 - I_2 R_3 - (I_2 - I_3) R_3 = 0 \]
\[ 5 + 4 - 20I_2 - 12I_2 + 12I_3 = 0 \]
\[ 5 - 29I_1 + 12I_2 = 0 \]
\[ 9 - 32I_2 + 12I_3 = 0 \]
\[ 9 - \frac{232I_1 + 40 + 12I_3}{3} = 0 \]
\[ 27 - 232I_1 + 40 + 36I_3 = 0 \]
\[ 196I_1 = 67 \]
\[ I_1 = 0.34 \text{ A} \]

\[ I_2 = \frac{29I_1 - 5}{12} = 0.41 \text{ A} \]

\[ I_{R2} = I_2 - I_1 = 0.41 - 0.34 = 0.07 \text{ A} \]

\[ I_{R3} = I_2 = 0.41 \text{ A} \]

\[ (b) \quad P_{E_1} = E_1 \cdot I_1 = (10 \text{ V})(0.34 \text{ A}) = 3.4 \text{ W} \]
6. The current density magnitude in a certain circular wire is $J = (2.75 \times 10^{10} \text{ A/m}^4) r^2$, where $r$ is the radial distance out to the wire’s radius of 3.00 mm. The potential applied to the wire (end to end) is 60.0 V. How much energy is converted to thermal energy in 1.00 h?

Assuming the current is along the wire (not radial) we find the current from:

$$i = \int |\vec{J}| \, dA = \int_0^R kr^2 2\pi rdr = \frac{1}{2} k\pi R^4 = 3.50 \text{ A}$$

where $k = 2.75 \times 10^{10} \text{ A/m}^4$ and $R = 0.00300 \text{ m}$. The rate of thermal energy generation is found from $P = iV = 210 \text{ W}$. Assuming a steady rate, the thermal energy generated in 3600 s is $Q = P\Delta t = (210 \text{ J/s})(3600 \text{ s}) = 7.56 \times 10^5 \text{ J}$. 
7. A beam consisting of five types of ions labels A, B, C, D, and E enters a region that contains a uniform magnetic field as shown in Figure below. The field is perpendicular to the plane of the paper, but its precise direction is not given. All ions in the beam travel with the same speed. The table below gives the masses and charges of the ions. Note: 1 mass unit = 1.67x10^{-27} kg and e = 1.6x10^{-19} C.

<table>
<thead>
<tr>
<th>Ion</th>
<th>Mass</th>
<th>Charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2 units</td>
<td>+e</td>
</tr>
<tr>
<td>B</td>
<td>4 units</td>
<td>+e</td>
</tr>
<tr>
<td>C</td>
<td>6 units</td>
<td>+e</td>
</tr>
<tr>
<td>D</td>
<td>2 units</td>
<td>-e</td>
</tr>
<tr>
<td>E</td>
<td>4 units</td>
<td>-e</td>
</tr>
</tbody>
</table>

a) Which ion falls at position 2?
b) What is the direction of the magnetic field?
c) Determine the magnitude of the magnetic field if ion A travels in a semicircular path of radius 0.50 m at a speed of 5.0x10^6 m/s.

a) Since there are two negative charges and three positive trajectories 1, 2, and 3 represent motion of positive charges and 4 and 5 - negative.

Since \[ \mathbf{r} = \frac{m \mathbf{v}}{q \mathbf{B}} \] for constant \( \mathbf{v} \), \( r \propto B \)

\[ 2 \propto m \Rightarrow 3 \propto A \]

\[ 2 \propto B \]

\[ 1 \propto C \]

b) According to the right hand rule 1, for given direction of velocity and \( \mathbf{F}_m \) directed to the left, \( \mathbf{B} \) is directed in the plane of the screen.

c) \[ B = \frac{m \mathbf{v}}{q \mathbf{r}} = \frac{1.67 \times 10^{-27} \text{ kg} \times (5.0 \times 10^6 \text{ m/s})}{(1.6 \times 10^{-19} \text{ C}) \times (0.50 \text{ m})} = 0.21 \text{ T} \]
8. In Fig. is shown one quarter of a single circular loop of wire that carries a current of 14 A. Its radius is 5 cm. OP is a normal to the loop and it forms an angle $\theta = 60^\circ$ with the x axis. A uniform magnetic field, $B = 0.03$ T is directed in the +x direction. Find:

(a) the torque on the loop;
(b) the direction in which it will rotate.

\[ \begin{align*}
\text{Given:} & \\
N &= 1 \\
I &= 14 \text{A} \\
R &= 5 \text{cm} = 5 \times 10^{-2} \text{m} \\
\theta &= 60^\circ \\
B &= 0.03 \text{T} \\
\text{Find:} & \\
& \quad (a) \tau \\
& \quad (b) \text{Direction of rotation.}
\end{align*} \]

\[ \begin{align*}
\text{a)} \quad \tau &= NIA \sin \theta \\
&= NIA \frac{1}{14} (\frac{5}{2}) (5 \times 10^{-2})^2 \\
&= 0.11 \text{ A.m}^2 \\
&= (0.11)(0.03) \sin 60^\circ = 2.86 \times 10^{-3} \text{ N.m} \\
&= \left(3 \times 10^{-3}\right) \text{ N.m}
\end{align*} \]

\[ \text{b)} \text{ Direction of rotation clockwise such that OP should coincide with B.} \]