MA 485-1D (Probability), Dr. Chernov — Midterm test #1
6 problems, each is worth 17 points. Show your work. — Mon, Sep 29, 2008

1. In a class of 20 students, the math teacher selects five students at random for a test in math. The next day, the science teacher selects three students randomly for a test in science. The teachers made their choices independently.

(a) What is the chance that no student is selected by both teachers?

Answer:

$$\frac{\binom{20}{5}\binom{15}{3}}{\binom{20}{5}\binom{20}{3}} = 0.399$$

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(b) What is the chance that exactly one (unlucky) student is selected by both teachers?

Answer:

$$\frac{\binom{20}{5} \cdot 5 \cdot \binom{15}{2}}{\binom{20}{5}\binom{20}{3}} = 0.461$$

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2. Three missiles are fired at a target and hit it independently, with probabilities 0.6, 0.8 and 0.75, respectively.

(a) What is the probability that the target is hit?

Answer: $\mathbb{P}(\text{hit}) = 1 - \mathbb{P}(\text{all miss}) = 1 - 0.4 \cdot 0.2 \cdot 0.25 = 0.98$.

What is the probability that exactly one missile hits the target?

Answer: $0.6 \cdot 0.2 \cdot 0.25 + 0.4 \cdot 0.8 \cdot 0.25 + 0.4 \cdot 0.2 \cdot 0.75 = 0.17$.

3. Diseases D_1 and D_2 cause symptom A with probabilities 0.4 and 0.8, respectively. Suppose 6% of a population have disease D_1 and 2% have disease D_2. Assume that the only possible causes of symptom A are D_1 and D_2, and that no one carries more than one of those two diseases.

(a) What percent of the population have symptom A?

Answer: Law of total probability: $0.06 \cdot 0.4 + 0.02 \cdot 0.8 = 0.04$.

(b) Let a randomly selected person from the population have symptom A. What is the chance he/she carries disease D_2?

Answer: Bayes formula: $\frac{0.02 \cdot 0.8}{0.06 \cdot 0.4 + 0.02 \cdot 0.8} = 0.4$.

4. A discrete random variable X takes the following values with the corresponding probabilities:

X	-2	-1	0	2	5
$\mathbb{P}$	0.1	?	0.15	0.2	0.3

Note that one probability is missing. Assuming that X takes no other values, find the missing probability.

Answer: 0.25.

Then compute the following:

(a) $\mathbb{P}(X$ is negative) = Answer: 0.35.

(b) $\mathbb{P}(X$ is odd) = Answer: 0.55.

(c) $\mathbb{P}(|X| = 2)$ = Answer: 0.3.

(d) conditional probability $\mathbb{P}\big(|X| = 2 \,/\, X < 0\big)$ = Answer: 2/7.

(e) Plot the probability function of X

5. Suppose only 1% of lottery tickets in a certain state win.

(a) Is it enough to buy 100 tickets to be totally sure that at least one of them wins? [Just say "yes" or "no".]

Answer: No.

(b) Is it enough to buy 300 tickets to be totally sure that at least one of them wins? [Just say "yes" or "no".] If no, how many tickets would be enough?

Answer: No. No number of tickets would be enough. (Or, you can say "buy more than 99% of the tickets in the lottery".)

In questions (c)–(d) below, use Poisson approximation:

(c) Suppose Jim buys 300 tickets. What is the chance that he wins anything?

Answer: $1 - e^{-3} = 0.950$.

(d) What is the chance that Jim has at least two winning tickets?

Answer: $1 - e^{-3} - 3e^{-3} = 0.801$.

6. A continuous random variable X has density function

$$f(x) = 4 - cx$$

for $1 \leq x \leq 2$ and 0 elsewhere. Do the following:

(a) Find the value of c.

Answer: $c = 2$.

(b) Find $\mathbb{P}(X > 2) =$ Answer: 0.

(c) Find $\mathbb{P}(X > 1) =$ Answer: 1.

(d) Find $\mathbb{P}(X = 1.5) =$ Answer: 0.

(e) Find $\mathbb{P}(X > 1.5) =$ Answer: 0.25.

(Bonus) Find the distribution function $F(x)$.

Answer: $F(x) = -x^2 + 4x - 3$ for $1 < x < 2$.