MA 486/586-1G (Statistics), Dr Chernov — Midterm test #1

Show your work. — Fri, Feb 8, 2008

1. (20 pts) The following are quiz scores in a calculus class:

0 18 9 17 1 20 9 7 14 9 4 12

(a) Determine the mode, the median, the quartiles and the IQR.

Answers: mode = 9, median = 9, quartiles are 5.5 and 15.5, IQR = 10.

(b) Find the sample mean, the sample variance and the sample standard deviation.

Answers: $\bar{x} = 10$, $s^2 = 42$, $s = 6.48$.

2. (30 pts) A random sample of size $n = 30$ from $N(\mu, \sigma^2)$ yielded

$$\sum_{i=1}^{30} x_i = 60 \qquad \text{and} \qquad \sum_{i=1}^{30} x_i^2 = 236$$

(a) Compute $\bar{x}$ and s_x.

Answers: $\bar{x} = 2$, $s = 2$.

(b) Construct an 80% confidence interval for μ.

Answer: $\bar{x} \pm t_{0.1}(r)s/\sqrt{n} = 2 \pm 1.311 \times 2/\sqrt{30} = 2 \pm 0.48$.

(c) Construct a 90% confidence interval for σ using χ^2 percentiles from Table IV.

Answer: [1.65,2.56].

(d) Construct the shortest 90% confidence interval for σ.

Answer: [1.6,2.49].

(Bonus) Construct a 90% confidence interval for σ using normal approximation.

Answer: [1.67,2.65]

3. (25 pts) Let $x_1, \ldots, x_{15}$ be a sample from a normal random variable $N(\mu_X, \sigma^2)$ and $y_1, \ldots, y_{17}$ be a sample from a normal random variable $N(\mu_Y, \sigma^2)$. Their sample means are $\bar{x} = 6$ and $\bar{y} = 4$, and their sample variances are $s_x^2 = 16$ and $s_y^2 = 9$, respectively. Give a lower endpoint for a one-sided 95% confidence interval for the difference $\mu_X - \mu_Y$.

Answers: Note that σ's are equal! So

$$\mu_X - \mu_Y > 6 - 4 - 1.697 \times \sqrt{\frac{14 \cdot 16 + 16 \cdot 9}{30}} \times \sqrt{\frac{1}{15} + \frac{1}{17}} = -0.1$$

4. (25 pts) Let $x_1, \ldots, x_n$ be a random sample from the distribution with probability density function

$$f(x; \mu, \theta) = \sqrt{\theta/\pi}\; e^{-\theta(x-\mu)^2}$$

where μ and $\theta > 0$ are unknown parameters. Find the maximum likelihood estimates for μ and θ. (Note: both parameters are unknown, thus you need to take two partial derivatives of the likelihood function, one with respect to μ, and the other with respect to θ.)

Answer: the likelihood function is

$$L(\mu, \theta) = \left(\frac{\theta}{\pi}\right)^{n/2} \; e^{-\theta \sum (x_i - \mu)^2}.$$

The log-likelihood function is

$$\ln L(\mu, \theta) = \frac{n}{2} \ln \frac{\theta}{\pi} - \theta \sum (x_i - \mu)^2.$$

Its partial derivatives are set to zero:

$$\frac{\partial}{\partial \mu} \ln L(\mu, \theta) = 2\theta \sum (x_i - \mu) = 0$$

$$\frac{\partial}{\partial \theta} \ln L(\mu, \theta) = \frac{n}{2\theta} - \sum (x_i - \mu)^2 = 0$$

Solving these equations gives

$$\hat{\mu} = \frac{1}{n} \sum x_i = \bar{x},$$

$$\hat{\theta} = \frac{n}{2 \sum (x_i - \bar{x})^2}.$$