MA 587 (Advanced Probability), Dr. Chernov — Midterm test
10 problems. — Tue, Mar 13, 2012

1. Five boys and five girls are seated at a round (circular) table at a party. Find the number of ways this can be done if boys and girls must be seated alternately. Answer: $5! \cdot 4! = 2880$

2. How many rearrangements of the word PROBABILISTICAL are there? Answer: $\frac{15!}{2!\,2!\,2!\,3!}$

3. A survey of a group's viewing habits over the last year revealed the following information

- 30% watched gymnastics
- 28% watched baseball
- 36% watched soccer
- 10% watched gymnastics and baseball
- 14% watched baseball and soccer
- 12% watched gymnastics and soccer
- 4% watched all three sports.

Find the probability of the group that watched none of the three sports during the last year. Answer: 38%

4. Each time a hurricane arrives, a new home has a 0.5 probability of experiencing damage. The occurrences of damage in different hurricanes are independent. Calculate the mode of the number of hurricanes it takes for the home to experience damage from three hurricanes. Answer: 4 and 5 (two modes)

5. As part of an air-pollution survey, an inspector decided to examine the exhaust of eight of a company's 75 trucks. If 15 of the company's trucks emit excessive amounts of pollutants, what is the probability that exactly three of them will be included in the inspector's sample? Answer: $\frac{C_{15,3}\,C_{75-15,8-3}}{C_{75,8}} = 0.147$

6. A random variable X has cumulative distribution function

$$F(x) = \begin{cases} 0 & \text{for } x < -1 \\ 0.1 & \text{for } -1 < x < 0 \\ 0.2 + 0.2\sqrt{x} & \text{for } 0 < x < 0.25 \\ 0.3 & \text{for } 0.25 < x < 0.5 \\ 0.2(3 + x) & \text{for } 0.5 < x < 1 \\ 1 & \text{for } x > 1 \end{cases}$$

(a) What is the probability that $X = -1$? Answer: 0.1
(b) What is the probability that $X = 0$? Answer: 0.1
(c) What is the probability that $X = 1$? Answer: 0.2
(d) What is the probability that $|X| < 0.6$? Answer: 0.62
(Bonus) Identify all points where $F(x)$ is discontinuous and find the value of F at each such point. Answers: points are -1, 0, 0.5, and 1; values of F are 0.1, 0.2, 0.7, and 1.0, respectively.

7. An insurer's annual weather-related loss, X, is a random variable with density function

$$f(x) = \begin{cases} x^{-3/2} & \text{for } x > 4 \\ 0 & \text{elsewhere} \end{cases}$$

Calculate the difference between the 25th and 75th percentiles of X. Answer: 56.89

8. Suppose that your journey time from home to campus is normally distributed with mean equal to 50 minutes and standard deviation equal to 10 minutes. What is the latest time that you should leave home if you want to be over 98% sure of arriving in time for a class at noon? Answer: 10:49

9. A fisherman whose average time for catching a fish is 40 minutes wants to bring home exactly 2 fishes. What is the probability he will need between 50 and 100 minutes to catch them? Answer: 0.357

10. Consider the following hypothetical situation. Grade data indicates that on the average 25% of the students in senior engineering classes have received A grades. There is variation among classes, however, and the proportion X must be considered a random variable. From past data we have measured a standard deviation of 10%. We would like to model the proportion X of A grades with a Beta distribution.
(a) Find the density function of X.
(b) Find the probability that more than 40% of the students had an A. Answer: $a = 4.4375$ and $b = 13.3125$. Other formulas are messy... Full credit was given just for correct values of a and b.