

On system realizations of Herglotz-Nevanlinna functions

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In this overview talk we discuss realization problems for linear systems of Livšic type (L-systems)

$$\Theta = \begin{pmatrix} \mathbb{A} & K & J \\ \mathcal{H}_+ \subset \mathcal{H} \subset \mathcal{H}_- & & E \end{pmatrix}, \quad (1)$$

with an unbounded state-space operator \mathbb{A} . The main object of our study is a class of operator-valued Herglotz-Nevanlinna (H-N) functions that can be realized as a linear-fractional transformation of the transfer function

$$W_{\Theta}(z) = I - 2iK^*(\mathbb{A} - zI)^{-1}KJ \quad (2)$$

of a system of the form (1). We will show how direct and inverse problems for such type of systems are solved and provide a complete description of realizable H-N operator-functions in a finite-dimensional Hilbert space E . Various subclasses of the class of realizable H-N functions will be described in connection with specific properties of realizing L-systems. We will also talk about the general realization problem with non-canonical systems. As an important application, we will discuss the realization of certain scalar H-N functions by L-systems that are based on a non-self-adjoint Schrödinger operator in $L_2[a, +\infty)$.

This talk is based on joint work with Yury Arlinskiĭ and Eduard Tsekanovskii and will survey all the above mentioned developments and connections.