FRANKEL CONJECTURE AND SASAKI GEOMETRY

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Abstract. We classify simply connected compact Sasaki manifolds of dimension $2n+1$ with positive transverse bisectional curvature. In particular, the Kähler cone corresponding to such manifolds must be bi-holomorphic to $\mathbb{C}^{n+1}\backslash\{0\}$. As an application we recover the Mori-Siu-Yau theorem on the Frankel conjecture and extend it to certain orbifold version. The main idea is to deform such Sasaki manifolds to the standard round sphere in two steps, both fixing the complex structure on the Kähler cone. First, we deform the metric along the Sasaki-Ricci flow and obtain a limit Sasaki-Ricci soliton with positive transverse bisectional curvature. Then by varying the Reeb vector field along the negative gradient of the volume functional, we deform the Sasaki-Ricci soliton to a Sasaki-Einstein metric with positive transverse bisectional curvature, i.e. a round sphere. The second deformation is only possible when one treats simultaneously regular and irregular Sasaki manifolds, even if the manifold one starts with is regular(quasi-regular), i.e. Kahler manifolds (orbifolds).