PH 222-2A Spring 2015

Interference
Lecture 26-27

Chapter 35
(Halliday/Resnick/Walker, Fundamentals of Physics 9th edition)
Chapter 35

Interference

The concept of optical interference is critical to understanding many natural phenomena, ranging from color shifting in butterfly wings to intensity patterns formed by small apertures. These phenomena cannot be explained using simple geometrical optics, and are based on the wave nature of light.

In this chapter we explore the wave nature of light and examine several key optical interference phenomena.
Huygen’s Principle: All points on a wavefront serve as point sources of spherical secondary wavelets. After time $t$, the new position of the wavefront will be that of a surface tangent to these secondary wavelets.
Law of Refraction

$$t = \frac{\lambda_1}{v_1} = \frac{\lambda_2}{v_2} \rightarrow \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}$$

$$\sin \theta_1 = \frac{\lambda_1}{hc} \quad \text{(for triangle } hce)$$

$$\sin \theta_2 = \frac{\lambda_2}{hc} \quad \text{(for triangle } hcg)$$

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}$$

Index of Refraction: $$n = \frac{c}{v}$$

$$n_1 = \frac{c}{v_1} \quad \text{and} \quad n_2 = \frac{c}{v_2}$$

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1}$$

Law of Refraction: $$n_1 \sin \theta_1 = n_2 \sin \theta_2$$
Wavelength and Index of Refraction

\[ \frac{\lambda_n}{\lambda} = \frac{v}{c} \rightarrow \lambda_n = \lambda \frac{v}{c} \rightarrow \lambda_n = \frac{\lambda}{n} \]

\[ f_n = \frac{v}{\lambda_n} = \frac{c/n}{\lambda/n} = \frac{c}{\lambda} = f \]

The frequency of light in a medium is the same as it is in vacuum.

Since wavelengths in \( n_1 \) and \( n_2 \) are different, the two beams may no longer be in phase.

Number of wavelengths in \( n_1 \):

\[ N_1 = \frac{L}{\lambda_{n_1}} = \frac{L}{\lambda/n_1} = \frac{Ln_1}{\lambda} \]

Number of wavelengths in \( n_2 \):

\[ N_2 = \frac{L}{\lambda_{n_2}} = \frac{L}{\lambda/n_2} = \frac{Ln_2}{\lambda} \]

Assuming \( n_2 > n_1 \):

\[ N_2 - N_1 = \frac{Ln_2}{\lambda} - \frac{Ln_2}{\lambda} = \frac{L}{\lambda} (n_2 - n_1) \]

\[ N_2 - N_1 = 1/2 \text{ wavelength} \rightarrow \text{destructive interference} \]
The geometrical explanation of rainbows given in Ch. 34 is incomplete. Interference, constructive for some colors at certain angles, destructive for other colors at the same angles, is an important component of rainbows.
Diffraction

For plane waves entering a single slit, the waves emerging from the slit start spreading out, diffracting.

Fig. 35-7
Young’s Experiment

For waves entering two slits, the emerging waves interfere and form an interference (diffraction) pattern.

Fig. 35-8
Locating Fringes

The phase difference between two waves can change if the waves travel paths of different lengths.

What appears at each point on the screen is determined by the path length difference $\Delta L$ of the rays reaching that point.

Path Length Difference: $\Delta L = d \sin \theta$

**Fig. 35-10**
Locating Fringes

If $\Delta L = d \sin \theta = (\text{integer})(\lambda) \rightarrow$ bright fringe

**Maxima-bright fringes:**

$d \sin \theta = m\lambda$ \hspace{2mm} for \hspace{2mm} $m = 0, 1, 2, \ldots$

If $\Delta L = d \sin \theta = (\text{odd number})(\lambda / 2) \rightarrow$ dark fringe

**Minima-dark fringes:** \hspace{2mm} $d \sin \theta = (m + \frac{1}{2})\lambda$ \hspace{2mm} for \hspace{2mm} $m = 0, 1, 2, \ldots$

$m = 2$ bright fringe at: $\theta = \sin^{-1}\left(\frac{2\lambda}{d}\right)$ \hspace{2mm} $m = 1$ dark fringe at: $\theta = \sin^{-1}\left(\frac{1.5\lambda}{d}\right)$
Coherence

Two sources can produce an interference that is stable over time, if their light has a *phase relationship* that does not change with time: $E(t)=E_0\cos(\omega t+\phi)$.

**Coherent sources:** Phase $\phi$ must be well defined and constant. When waves from coherent sources meet, stable interference can occur. Sunlight is coherent over a short length and time range. Since laser light is produced by cooperative behavior of atoms, it is coherent of long length and time ranges.

**Incoherent sources:** $\phi$ jitters randomly in time, no stable interference occurs,
Intensity in Double-Slit Interference

\[ E_1 = E_0 \sin \omega t \quad \text{and} \quad E_2 = E_0 \sin(\omega t + \phi) \]

\[ I = 4I_0 \cos^2 \frac{1}{2} \phi \]

\[ \phi = \frac{2\pi d}{\lambda} \sin \theta \]

Maxima when: \( \frac{1}{2} \phi = m\pi \) for \( m = 0,1,2,\ldots \) \( \rightarrow \phi = 2m\pi = \frac{2\pi d}{\lambda} \sin \theta \)

\( \rightarrow d \sin \theta = m\lambda \) for \( m = 0,1,2,\ldots \) (maxima)

Minima when: \( \frac{1}{2} \phi = (m + \frac{1}{2})\pi \) \( \rightarrow d \sin \theta = (m + \frac{1}{2})\lambda \) for \( m = 0,1,2,\ldots \) (minima)

\[ I_{\text{avg}} = 2I_0 \]

*Fig. 35-12*
Proof of Eqs. 35-22 and 35-23

**Eq. 35-22** \[ E(t) = E_0 \sin \omega t + E_0 \sin (\omega t + \phi) = ? \]

\[ E = 2 \left( E_0 \cos \beta \right) = 2E_0 \cos \frac{1}{2} \phi \]

\[ E^2 = 4E_0^2 \cos^2 \frac{1}{2} \phi \]

\[ \frac{I}{I_0} = \frac{E^2}{E_0^2} = 4 \cos^2 \frac{1}{2} \phi \rightarrow I = 4I_0 \cos^2 \frac{1}{2} \phi \]

**Eq. 35-23**

\[ \left( \frac{\text{phase difference}}{\text{path length difference}} \right) = \left( \frac{\text{path length difference}}{\text{phase difference}} \right) \]

\[ \frac{2\pi}{\lambda} = \frac{2\pi}{\lambda} \left( \frac{\text{path length difference}}{\text{phase difference}} \right) \]

\[ \phi = \frac{2\pi}{\lambda} \left( d \sin \theta \right) \]

If \( S_b \) is \( \frac{1}{2} \lambda \), then \( \phi \) is \( \pi \)

If \( S_b \) is \( \lambda \), then \( \phi \) is \( 2\pi \)
Combining More Than Two Waves

In general, we may want to combine more than two waves. For example, there may be more than two slits.

Procedure:

1. Construct a series of phasors representing the waves to be combined. Draw them end to end, maintaining proper phase relationships between adjacent phasors.

2. Construct the sum of this array. The length of this vector sum gives the amplitude of the resulting phasor. The angle between the vector sum and the first phasor is the phase of the resultant with respect to the first. The projection of this vector sum phasor on the vertical axis gives the time variation of the resultant wave.
Interference from Thin Films

Fig. 35-15

\[ \phi_{12} = ? \]

\[ \theta \approx 0 \]

Fig. 35-15
Reflection Phase Shifts

Fig. 35-16

Reflection
Off lower index
0
Reflection Phase Shift
0.5 wavelength

Reflection
Off higher index

\[ n_1 > n_2 \]

\[ n_1 < n_2 \]
Equations for Thin-Film Interference

Three effects can contribute to the phase difference between $r_1$ and $r_2$.

1. Differences in reflection conditions.
2. Difference in path length traveled.
3. Differences in the media in which the waves travel. One must use the wavelength in each medium ($\lambda / n$) to calculate the phase.
Thin film interference

3. General description of constructive and destructive interference.

Constructive: \[ 2t + \Delta = \lambda_f, 2\lambda_f, 3\lambda_f, \ldots \]

An observer would see a uniform bright film

Destructive: \[ 2t + \Delta = \frac{\lambda_f}{2}, \frac{3\lambda_f}{2}, \frac{5\lambda_f}{2}, \ldots \]

4. For a particular gasoline film floating on a puddle of water

Constructive: \[ 2t = \frac{m\lambda_f}{2} \]

Destructive: \[ 2t = \lambda_f, 2\lambda_f, 3\lambda_f, \ldots \]

where 2t is extra distance traveled by wave 2

1. Because of reflection and refraction, two light waves, represented by rays 1 and 2, enter the eye when light shines on a film of gasoline floating on a thick layer of water.

2. When the index of refraction is increasing (as at an air-gasoline interface), a wave suffers a phase change \( \Delta = 180^\circ = \frac{\lambda_{film}}{2} \) in reflection. When the index of refraction decreases, no phase change occurs.

!!! The wavelength that is important for thin film interference is the wavelength within the film \( \lambda_{film} \).
6. What is the least thickness of a soap film \( (d = ?) \) which will appear black (destructive interference) when viewed with sodium light \( (\lambda = 589.3 \text{ nm}) \) reflected practically perpendicular to the film? The refractive index for soap solution is \( n = 1.38 \).

\[ \lambda = 589.3 \text{ nm} \]

\[ \text{Air} \quad \text{c} \quad a \quad b \quad \text{d} \quad \text{Air} \]

\[ n_1 > n_2 \]

Ray \( b \) has an extra path length of \( 2d \).

In addition, there is a phase change of 180°.

- Between the beams \( a \) and \( b \), or \( \frac{1}{2} \) between the beams \( a \) and \( b' \), doesn't have phase change with beam \( b' \) doesn't have phase change with beam \( a \).

With respect to the incidence one, beam \( a \) has 180° phase change - because beam \( a \) has 180° phase change - because beam \( a \) is directed from the medium with \( n_1 \) to the medium \( n_2 \) \((n_1 < n_2)\) and is reflected.

The formula for constructive interference is:

\[ 2d = \frac{\lambda_s}{2}, \frac{3\lambda_s}{2}, \frac{5\lambda_s}{2}, ... \]

Where \( \lambda_s = \frac{\lambda_0}{n} = \frac{589.3 \text{ nm}}{1.38} = 427 \text{ nm} \)

\( \lambda_s \) for destructive interference is:

\[ 2d = 0, \lambda_s, 2\lambda_s, 3\lambda_s, ... \]

The thinnest possible film other than 0 is when \( 2d = \lambda_s \),

\[ \Rightarrow d = \frac{\lambda_s}{2} = \frac{427 \text{ nm}}{2} = 214 \text{ nm} \]
Film Thickness Much Less Than $\lambda$

If $L$ is much less than $\lambda$, for example $L < 0.1\lambda$, then phase difference due to the path difference $2L$ can be neglected.

Phase difference between $r_1$ and $r_2$ will always be $\frac{1}{2}$ wavelength $\rightarrow$ destructive interference $\rightarrow$ film will appear dark when viewed from illuminated side.
2. A Newton's ring apparatus is illuminated by light with two wavelength components. One of the wavelengths is 546 nm. The spherical surface has a radius of 1 m. If the eleventh bright ring of the 546 nm fringe system coincides with the tenth ring of the other, what is the second wavelength?

Interference condition

\[ 2 n_f t_m + \Delta r = m \lambda \] - constructive

where \( \Delta r = \frac{\lambda}{2} \) for air film.

\[ \Rightarrow 2 n_f t_m = m \lambda - \frac{\lambda}{2} = \lambda (m - \frac{1}{2}) \]

\( n_f = 1 \)

\[ \Rightarrow t_m = \frac{1}{2} (m - \frac{1}{2}) \]

\( \Rightarrow \) according to given data

\[ t_{11} (\lambda_1) = \frac{1}{2} (11 - \frac{1}{2}) \]
\[ t_{10} (\lambda_2) = \frac{1}{2} (10 - \frac{1}{2}) \]

\[ \Rightarrow \lambda_1 = \frac{1}{2} (10,5) = \frac{1}{2} (9,5) \]

\[ \Rightarrow \lambda_2 = \frac{10,5 \lambda_1}{9,5} = \frac{10,5 \times (546 \text{ nm})}{9,5} = 603 \text{ nm} \]
The Michelson Interferometer

If $D_A - D_F = d$ the extra distance traversed by the beam $A$, includes distance $d$ twice, once before and once after reflection.

If, in addition, $2d = m\lambda$, the two beams interfere constructively. They will do so repeatedly for every $\lambda/2$ translation of adjustable mirror.

Precise distance measurements can be made with the Michelson interferometer by moving the mirror and counting the interference fringes which move by a reference point. The distance $d$ associated with $m$ fringes is $d = m\lambda/2$. 