PH 222-2C Fall 2012

CURRENT AND RESISTANCE

Lecture 9

Chapter 26
(Halliday/Resnick/Walker, Fundamentals of Physics 8th edition)
Chapter 26
Current and Resistance

In this chapter we will introduce the following new concepts:

- Electric current (symbol \( i \))
- Electric current density vector (symbol \( \mathbf{J} \))
- Drift speed (symbol \( v_d \))
- Resistance (symbol \( R \)) and resistivity (symbol \( \rho \)) of a conductor
- Ohmic and non-Ohmic conductors

We will also cover the following topics:

- Ohm’s law
- Power in electric circuits
Electric Current
Consider the conductor shown in fig. a. All the points inside the conductor and on its surface are at the same potential. The free electrons inside the conductor move in random directions and thus there is no net charge transport. We now make a break in the conductor and insert a battery as shown in fig. b. Points $A$ and $B$ are now at potentials $V_A$ and $V_B$, respectively ($V_A - V_B = V$, the voltage of the battery). The situation is not static any more, but charges move inside the conductor so that there is a net charge flow in a particular direction. We define this net flow of electric charge as "electric current."
Consider the conductor shown in the figure. It is connected to a battery (not shown) and thus charges move through the conductor. Consider one of the cross sections through the conductor (aa' or bb' or cc').

The electric current $i$ is defined as $i = \frac{dq}{dt}$.

Current = rate at which charge flows

Current SI Unit: C/s, known as the "ampere"

**Current Direction.** An electric current is represented by an arrow, which has the same direction as the charge velocity. For historical reasons we use the following convention:

A current arrow is drawn in the direction in which positive charge carriers would move, even if the actual charge carriers are negative and move in the opposite direction.
Current Density

Current density is a vector that is defined as follows:

Its magnitude is \( J = \frac{i}{A} \)

SI unit for \( J \) : \( A / m^2 \)

The direction of \( \vec{J} \) is the same as that of the current.

The current through a conductor of cross-sectional area \( A \) is given by the equation \( i = JA \)

if the current density is constant.

If \( \vec{J} \) is not constant, then \( i = \int \vec{J} \cdot d\vec{A} \).

We note that even though the current density is a vector, the electric current is not. This is illustrated in the figure to the left. An incoming current \( i_0 \) branches at point \( a \) into two currents, \( i_1 \) and \( i_2 \).

Current \( i_0 = i_1 + i_2 \). This equation expresses the conservation of charge at point \( a \). Please note that we have not used vector addition.
Drift Speed

When a current flows through a conductor the electric field causes the charges to move with a constant drift speed $v_d$. This drift speed is superimposed on the random motion of the charges.

\[
\begin{align*}
J &= nev_d \\
\vec{J} &= n\vec{e}v_d
\end{align*}
\]

Consider the conductor of cross-sectional area $A$ shown in the figure. We assume that the current in the conductor consists of positive charges. The total charge $q$ within a length $L$ is given by $q = (nAL)e$, where $n$ is a number of carriers per unit volume. This charge moves through area $A$ in a time $t = \frac{L}{v_d}$. The current is $i = \frac{q}{t} = \frac{nALe}{L/v_d} = nAv_d e$.

The current density is $J = \frac{i}{A} = \frac{nAv_d e}{A} = nv_d e$.

In vector form: $\vec{J} = ne\vec{v}_d$. 
If we apply a voltage $V$ across a conductor (see figure) a current $i$ will flow through the conductor.

We define the conductor resistance as the ratio $R = \frac{V}{i}$.

**Resistance**

A conductor across which we apply a voltage $V = 1$ volt and results in a current $i = 1$ ampere is defined as having resistance of 1 Ω.

**Q:** Why not use the symbol "Ω" instead of "Ω"?

**A:** Suppose we had a 1000 Ω resistor. We would then write: 1000 Ω, which can easily be mistakenly read as 10000 Ω.

A conductor whose function is to provide a specified resistance is known as a "resistor."

The symbol is given to the left.
Unlike the electrostatic case, the electric field in the conductor of the figure is not zero. We define as resistivity $\rho$ of the conductor the ratio $\rho = \frac{E}{J}$.

In vector form: $\vec{E} = \rho \vec{J}$.

**SI unit for $\rho$:** $\frac{V/m}{A/m^2} = \frac{V}{A} m = \Omega \cdot m$

The conductivity $\sigma$ is defined as $\sigma = \frac{1}{\rho}$.

Using $\rho$, the previous equation takes the form: $\vec{J} = \sigma \vec{E}$.

Consider the conductor shown in the figure above. The electric field inside the conductor is $E = \frac{V}{L}$. The current density is $J = \frac{i}{A}$. We substitute $E$ and $J$ into equation $\rho = \frac{E}{J}$ and get: $\rho = \frac{V/L}{i/A} = \frac{V}{i} \frac{A}{L} = R \frac{A}{L} \rightarrow R = \rho \frac{L}{A}$. 

**Resistivity**

\[
\begin{align*}
\vec{E} &= \rho \vec{J} \\
\vec{J} &= \sigma \vec{E}
\end{align*}
\]
Variation of Resistivity with Temperature

In the figure we plot the resistivity $\rho$ of copper as a function of temperature $T$.

The dependence of $\rho$ on $T$ is almost linear. Similar dependence is observed in many conductors.

The following empirical equation is used for many practical applications:

$$\rho - \rho_0 = \rho_0 \alpha (T - T_0)$$

The constant $\alpha$ is known as the "temperature coefficient of resistivity." The constant $T_0$ is a reference temperature usually taken to be room temperature ($T_0 = 293 \text{ K}$), and $\rho_0$ is the resistivity at $T_0$. For copper, $\rho_0 = 1.69 \times 10^{-8} \text{ } \Omega \cdot \text{m}.$

**Note:** Temperature enters the equation above as a difference $(T - T_0)$.

Thus either the Celsius or the Kelvin temperature scale can be used.
Ohm's Law. A resistor was defined as a conductor whose resistance does not change with the voltage $V$ applied across it. In fig. $b$ we plot the current $i$ through a resistor as a function of $V$. The plot (known as the "$i$ -$V$ curve") is a straight line that passes through the origin. Such a conductor is said to be "Ohmic" and it obeys Ohm's law, which states: The current $i$ through a conductor is proportional to the voltage $V$ applied across it. Not all conductors obey Ohm's law (these are known as "non-Ohmic"). An example is given in fig. $c$ where we plot $i$ versus $V$ for a semiconductor diode. The ratio $V/i$ (and thus the resistance $R$) is not constant. As a matter of fact, the diode does not conduct for negative voltage values.

Note: Ohm's "law" is in reality a definition of Ohmic conductors (defined as the conductors that obey Ohm's law).
In order to understand why some materials such as metals obey Ohm's law, we must look into the details of the conduction process at the atomic level. A schematic of an Ohmic conductor such as copper is shown in the figure. We assume that there are free electrons that move around in random directions with an effective speed $v_{\text{eff}} = 1.6 \times 10^6$ m/s. The free electrons suffer collisions with the stationary copper atoms.

A schematic of a free electron path is shown in the figure using the dashed gray line. The electron starts at point $A$ and ends at point $B$. We now assume that an electric field $\vec{E}$ is applied. The new electron path is indicated by the dashed green line. Under the action of the electric force the electron acquires a small drift speed $v_d$. The electron drifts to the right and ends at point $B'$. 
Consider the motion of one of the free electrons. We assume that the average time between collisions with the copper atoms is equal to \( \tau \). The electric field exerts a force \( \vec{F} = eE \) on the electron, resulting in an acceleration \( a = \frac{F}{m} = \frac{eE}{m} \). The drift speed is given by the equation \( v_d = a\tau = \frac{eE\tau}{m} \) (eq. 1).

We can also get \( v_d \) from the equation \( J = nev_d \rightarrow v_d = \frac{J}{ne} \) (eq. 2).

If we compare equations 1 and 2 we get: \( v_d = \frac{J}{ne} = \frac{eE\tau}{m} \rightarrow E = \left( \frac{m}{ne^2\tau} \right) J \).

If we compare the last equation with \( E = \rho J \) we conclude that \( \rho = \frac{m}{ne^2\tau} \). This is a statement of Ohm's law (the resistance of the conductor does not depend on voltage and thus \( E \)). This is because \( m, n, \) and \( e \) are constants. The time \( \tau \) can also be considered to be independent of \( E \) since the drift speed \( v_d \) is so much smaller than \( v_{\text{eff}} \).
Power in Electric Circuits

Consider the circuit shown in the figure. A battery of voltage $V$ is connected across the terminals $a$ and $b$ of a device. This can be a resistor, a motor, etc. The battery maintains a potential difference $V$ between the terminals $a$ and $b$ and thus a current $i$ flows in the circuit as shown in the figure. During the time interval $dt$ a charge $dq = idt$ moves between the terminals. We note that $V_a > V_b$.

The potential energy of the charge decreases by an amount $dU = Vdq = Viddt$. Using energy conservation we conclude that the lost energy has been transferred by the battery to the device and has been converted into some other form of energy. The rate at which energy is transferred to the device is known as "power" and it is equal to $P = \frac{dU}{dt} = \frac{Viddt}{dt} = Vi$. 

**SI unit for P:** $V \cdot A$  It is known as the "watt" (symbol W).
If the device connected to the battery is a resistor $R$ then the energy transferred by the battery is converted as **heat** that appears on $R$. If we combine the equation $P = iV$ with Ohm's law $i = \frac{V}{R}$, we get the following two equivalent expressions for the rate at which heat is dissipated on $R$:

\[ P = i^2 R \quad \text{and} \quad P = \frac{V^2}{R} \]