PH 221-1D Spring 2013

Oscillations

Lectures 35-37

Chapter 15
(Halliday/Resnick/Walker, Fundamentals of Physics 9th edition)
Chapter 15
Oscillations

In this chapter we will cover the following topics:

Displacement, velocity and acceleration of a simple harmonic oscillator

Energy of a simple harmonic oscillator

Examples of simple harmonic oscillators: spring-mass system, simple pendulum, physical pendulum, torsion pendulum

Damped harmonic oscillator

Forced oscillations/Resonance
Oscillations

Periodic motion – the motion of a particle or a system of particles is periodic, or cyclic, if it repeats again and again at regular intervals of time.

Example:
• The orbital motion of a planet
• The uniform rotational motion of a phonograph turntable
• Back and forth motion of a piston in an automobile engine
• Vibrations of a guitar string

Oscillation – back and forth or swinging periodic motion is called an oscillation

Simple Harmonic Motion

• Simple harmonic motion is a special kind of one dimensional periodic motion
• The particle moves back and forth along a straight line, repeating the same motion again and again

Simple harmonic motion – the particles position can be expressed as a cosine or a sine function of time.

Cosines and sines are called harmonic functions => we call motion of the particle harmonic.
When an object attached to a horizontal spring is moved from its equilibrium position and released, the restoring force $F = -kx$ leads to simple harmonic motion.

$x = 0$ is the equilibrium position of the object.

$A =$ amplitude – the maximum displacement from equilibrium

Position as a function of time has the shape of trigonometric sine or cosine function.
In fig. a we show snapshots of a simple oscillating system.

The motion is periodic i.e. it repeats in time. The time needed to complete one repetition is known as the period \((\text{symbol } T, \text{ units: s})\). The number of repetitions per unit time is called the frequency \((\text{symbol } f, \text{ unit hertz, Hz})\) \(f = \frac{1}{T}\).

The displacement of the particle is given by the equation: \(x(t) = x_m \cos(\omega t + \phi)\)

Fig. b is a plot of \(x(t)\) versus \(t\). The quantity \(x_m\) is called the amplitude of the motion. It gives the maximum possible displacement of the oscillating object. The quantity \(\omega\) is called the angular frequency of the oscillator. It is given by the equation:

\[\omega = 2\pi f = \frac{2\pi}{T}\]
The quantity \( \phi \) is called the phase angle of the oscillator. The value of \( \phi \) is determined from the displacement \( x(0) \) and the velocity \( v(0) \) at \( t = 0 \). In fig.a \( x(t) \) is plotted versus \( t \) for \( \phi = 0 \). \( x(t) = x_m \cos \omega t \)

Velocity of SHM

\[
v(t) = \frac{dx(t)}{dt} = \frac{d}{dt} \left[ x_m \cos (\omega t + \phi) \right] = -\omega x_m \sin (\omega t + \phi)
\]

The quantity \( \omega x_m \) is called the velocity amplitude \( v_m \). It expresses the maximum possible value of \( v(t) \).

In fig.b the velocity \( v(t) \) is plotted versus \( t \) for \( \phi = 0 \). \( v(t) = -\omega x_m \sin \omega t \)

Acceleration of SHM:

\[
a(t) = \frac{dv(t)}{dt} = \frac{d}{dt} \left[ -\omega x_m \sin (\omega t + \phi) \right] = -\omega^2 x_m \cos \omega t = -\omega^2 x
\]

The quantity \( \omega^2 x_m \) is called the acceleration amplitude \( a_m \). It expresses the maximum possible value of \( a(t) \). In fig.c the acceleration \( a(t) \) is plotted versus \( t \) for \( \phi = 0 \). \( a(t) = -\omega^2 x_m \cos \omega t \)
The Force Law for Simple Harmonic Motion

We saw that the acceleration of an object undergoing SHM is: \( a = -\omega^2 x \)

If we apply Newton's second law we get: \( F = ma = -m\omega^2 x = -\left(m\omega^2\right)x \)

Simple harmonic motion occurs when the force acting on an object is proportional to the displacement but opposite in sign. The force can be written as: \( F = -Cx \) where \( C \) is a constant. If we compare the two expressions for \( F \) we have:

\[ m\omega^2 = C \quad \text{and} \quad T = 2\pi \sqrt{\frac{m}{C}} \]

Consider the motion of a mass \( m \) attached to a spring of spring constant \( k \) that moves on a frictionless horizontal floor as shown in the figure.

The net force \( F \) on \( m \) is given by Hooke's law: \( F = -kx \). If we compare this equation with the expression \( F = -Cx \) we identify the constant \( C \) with the spring constant \( k \).

We can then calculate the angular frequency \( \omega \) and the period \( T \).

\[ \omega = \sqrt{\frac{C}{m}} = \sqrt{\frac{k}{m}} \quad \text{and} \quad T = 2\pi \sqrt{\frac{m}{C}} = 2\pi \sqrt{\frac{m}{k}} \]
Problem: When an object of mass \( m_1 \) is hung on a vertical spring and set into vertical simple harmonic motion, its frequency is 12.0 Hz. When another object of mass \( m_2 \) is hung on the spring along with \( m_1 \), the frequency of the motion is 4.00 Hz. Find the ratio \( m_2/m_1 \) of the masses.

\[
\begin{align*}
 f_1 &= \frac{1}{2\pi} \sqrt{\frac{k}{m_1}}, \\
 \frac{f_1}{f_2} &= \sqrt{\frac{m_1 + m_2}{m_1}} = 3.00 \\
 (\sqrt{\frac{m_1 + m_2}{m_1}})^2 &= (3.00)^2 \Rightarrow \frac{m_1 + m_2}{m_1} = 9.00, \quad m_1 + m_2 = 9.80 m, \\
 m_2 &= 8.00 m, \\
 \frac{m_2}{m_1} &= 8.00
\end{align*}
\]
Three springs with force constants $k_1 = 10.0$ N/m, $k_2 = 12.5$ N/m, and $k_3 = 15.0$ N/m are connected in parallel to a mass of $0.500$ kg. The mass is then pulled to the right and released. Find the period of the motion.

\[
F_{tot} = F_1 + F_2 + F_3 = k_1 x + k_2 x + k_3 x = (k_1 + k_2 + k_3) x = k_{\text{equivalent}} x
\]

\[
T = 2\pi \sqrt{\frac{m}{k_e}} = 2\pi \sqrt{\frac{m}{k_1 + k_2 + k_3}} = 2\pi \sqrt{\frac{0.500\text{kg}}{10.0 + 12.5 + 15}} = 0.7265 \text{s}
\]
A mass of 0.300 kg is placed on a vertical spring and the spring stretches by 10.0 cm. It is then pulled down an additional 5.00 cm and then released. Find:

a) \( K \);  
b) \( \omega \);  
c) frequency;  
d) \( T \);  
e) max velocity;  
f) \( a_{\text{max}} \);  
g) \( F_{\text{max}} \) (max restoring force);  
h) \( V \) for \( x = 2.00 \) cm;  
i) The equations for displacement, velocity and acceleration at any time

\[
F = mg = 0; \quad F = mg; \quad K = \frac{F}{x} = \frac{0.300 \text{ kg} \times 9.80 \text{ m/s}^2}{0.100 \text{ m}} = 29.4 \text{ N/m}
\]

\[
\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{29.4}{0.300}} = 9.90 \text{ rad/s}
\]

\[
f = \frac{\omega}{2\pi} = \frac{9.90 \text{ rad/s}}{2\pi \text{ rad/s}} = 1.58 \text{ Hz}
\]

\[
T = \frac{1}{f} = \frac{1}{1.58 \text{ Hz}} = 0.633 \text{ s}
\]

\[
V_{\text{max}} = \omega A = (9.90 \text{ rad/s}) \times (5.00 \times 10^{-2} \text{ m}) = 0.495 \text{ m/s}
\]

\[
a_{\text{max}} = \omega^2 A = (9.90 \text{ rad/s})^2 \times (5.00 \times 10^{-2} \text{ m})^2 = 4.90 \text{ m/s}^2
\]

\[
F_{\text{max}} = K x_{\text{max}} = KA = (29.4 \text{ N/m}) \times (5.00 \times 10^{-2} \text{ m})^2 = 1.47 \text{ N}
\]
\[ h) \quad v = \pm \omega \sqrt{A^2 - x^2} = \pm (9.90 \text{ m/s}) \sqrt{\frac{5.00 \times 10^{-2}}{5.00 \times 10^{-2}}} = \pm 0.454 \text{ m/s} \]

where \( v \) is positive when moving up and negative when moving down.

\[ i) \quad x = A \cos \omega t = (5.00 \times 10^{-2} \text{ m}) \cos [(9.90 \text{ m/s}) t] \]

\[ v = -\omega A \sin \omega t = -(9.90 \text{ m/s}) (5.00 \times 10^{-2} \text{ m}) \sin [(9.90 \text{ m/s}) t] = -0.495 \sin 9.90 t \]

\[ a = -\omega^2 A \cos \omega t = -(9.90 \text{ m/s}^2) (5.00 \times 10^{-2} \text{ m}) \cos [(9.90 \text{ m/s}) t] = -(4.90 \text{ m/s}^2) \cos [(9.90 \text{ m/s}) t] \]
A 2 kg block is at rest on a frictionless horizontal table. It is attached to a massless ideal spring for which $k=200\text{N/m}$. A 500 g block rests on top of the larger block. The coefficient of static friction between the two blocks is 0.45. What is the maximum amplitude SHM the system can undergo without having the smaller block slip on the larger one?

1) At turning point of maximum elongation of the spring, the acceleration of the SHM has maximum value $a = \omega^2 A$

2) FBD of 500 g mass

3) FBD of 2 kg mass

4) FBD of 2 kg block

$N - mg = 0$
$N = mg$

$F_{\text{max}} = \mu_s N = \mu_s mg$

$\Rightarrow \mu_s mg = M \omega^2 A$

5) For SHM $\omega = \sqrt{\frac{K}{M+m}}$

$\Rightarrow A = \frac{\mu_s g (M+m)}{K} = 0.45 \cdot \left(9.8 \text{m/s}^2\right) \left(\frac{2 \text{kg} + 0.5 \text{kg}}{200 \text{N/m}}\right) = 1.70 \text{m}$
Energy in Simple Harmonic Motion

The mechanical energy $E$ of a SHM is the sum of its potential and kinetic energies $U$ and $K$.

Potential energy $U = \frac{1}{2} kx^2 = \frac{1}{2} kx_m^2 \cos^2(\omega t + \phi)$

Kinetic energy $K = \frac{1}{2} mv^2 = \frac{1}{2} m\omega^2 x_m^2 \sin^2(\omega t + \phi) = \frac{1}{2} m \frac{k}{m} x_m^2 \sin^2(\omega t + \phi)$

Mechanical energy $E = U + K = \frac{1}{2} kx_m^2 \left[ \cos^2(\omega t + \phi) + \sin^2(\omega t + \phi) \right] = \frac{1}{2} kx_m^2$

In the figure we plot the potential energy $U$ (green line), the kinetic energy $K$ (red line) and the mechanical energy $E$ (black line) versus time $t$. While $U$ and $K$ vary with time, the energy $E$ is a constant. The energy of the oscillating object transfers back and forth between potential and kinetic energy, while the sum of the two remains constant.
Problem 35. A block of mass $M=5.4$ kg, at rest on a horizontal frictionless table, is attached to a rigid support by a string of constant $k=6000$ N/m. A bullet of mass $m=9.5$ g and velocity $v$ of magnitude 630 m/s strikes and is embedded in the block. Assuming the compression of the spring is negligible until the bullet is embedded, determine (a) the speed of the block immediately after the collision, and (b) the amplitude of the resulting simple harmonic motion.

The problem consists of two distinct parts: the completely inelastic collision (which is assumed to occur instantaneously, the bullet embedding itself in the block before the block moves through significant distance) followed by simple harmonic motion (of mass $m+M$ attached to a spring of spring constant $k$).

(a) Momentum conservation readily yields $v' = \frac{mv}{m+M}$. With $m = 9.5$ g, $M = 5.4$ kg and $v = 630$ m/s, we obtain $v' = 1.1$ m/s.

(b) Since $v'$ occurs at the equilibrium position, then $v' = v_m$ for the simple harmonic motion. The relation $v_m = \omega x_m$ can be used to solve for $x_m$, or we can pursue the alternate (though related) approach of energy conservation. Here we choose the latter:

$$\frac{1}{2} (m+M) (v')^2 = \frac{1}{2} k x_m^2 \quad \Rightarrow \quad \frac{1}{2} (m+M) \frac{m^2 v^2}{(m+M)^2} = \frac{1}{2} k x_m^2$$

which simplifies to

$$x_m = \frac{mv}{\sqrt{k(m+M)}} = \frac{(9.5 \times 10^{-3} \text{kg})(630 \text{ m/s})}{\sqrt{(6000 \text{ N/m})(9.5 \times 10^{-3} \text{kg} + 5.4 \text{kg})}} = 3.3 \times 10^{-2} \text{ m.}$$
An Angular Simple Harmonic Oscillator; Torsion Pendulum

In the figure we show another type of oscillating system. It consists of a disc of rotational inertia $I$ suspended from a wire that twists as $m$ rotates by an angle $\theta$. The wire exerts on the disc a restoring torque $\tau = -\kappa \theta$. This is the angular form of Hooke's law. The constant $\kappa$ is called the torsion constant of the wire.

\[ \tau = -\kappa \theta \]

If we compare the expression $\tau = -\kappa \theta$ for the torque with the force equation $F = -Cx$ we realize that we identify the constant $C$ with the torsion constant $\kappa$. We can thus readily determine the angular frequency $\omega$ and the period $T$ of the oscillation.

\[ \omega = \sqrt{\frac{C}{I}} = \sqrt{\frac{\kappa}{I}} \quad \quad T = 2\pi \sqrt{\frac{I}{C}} = 2\pi \sqrt{\frac{I}{\kappa}} \]

We note that $I$ is the rotational inertia of the disc about an axis that coincides with the wire. The angle $\theta$ is given by the equation:

\[ \theta(t) = \theta_m \cos(\omega t + \phi) \]
The Simple Pendulum

A simple pendulum consists of a particle of mass $m$ suspended by a string of length $L$ from a pivot point. If the mass is disturbed from its equilibrium position the net force acting on it is such that the system executes simple harmonic motion. There are two forces acting on $m$: The gravitational force and the tension from the string. The net torque of these forces is:

$$\tau = -r \perp F_g = -Lmg \sin \theta$$

Here $\theta$ is the angle that the thread makes with the vertical. If $\theta \ll 1$ (say less than 5 °) then we can make the following approximation: $\sin \theta \approx \theta$ where $\theta$ is expressed in radians. With this approximation the torque $\tau$ is:

$$\tau \approx -(Lmg)\theta$$

If we compare the expression for $\tau$ with the force equation $F = -Cx$ we realize that we identify the constant $C$ with the term $Lmg$. We can thus readily determine the angular frequency $\omega$ and the period $T$ of the oscillation.

$$\omega = \sqrt{\frac{C}{I}} = \sqrt{\frac{mgL}{I}}; \quad T = 2\pi \sqrt{\frac{I}{C}} = 2\pi \sqrt{\frac{I}{mgL}}$$
In the **small angle approximation** we assumed that $\theta << 1$ and used the approximation: $\sin \theta \approx \theta$. We are now going to decide what is a “small” angle i.e. up to what angle $\theta$ is the approximation reasonably accurate?

<table>
<thead>
<tr>
<th>$\theta$ (degrees)</th>
<th>$\theta$ (radians)</th>
<th>$\sin \theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.087</td>
<td>0.087</td>
</tr>
<tr>
<td>10</td>
<td>0.174</td>
<td>0.174</td>
</tr>
<tr>
<td>15</td>
<td>0.262</td>
<td>0.259 (1% off)</td>
</tr>
<tr>
<td>20</td>
<td>0.349</td>
<td>0.342 (2% off)</td>
</tr>
</tbody>
</table>

**Conclusion:** If we keep $\theta < 10^\circ$ we make less than 1% error
The rotational inertia $I$ about the pivot point is equal to $mL^2$

Thus $T = 2\pi \sqrt{\frac{I}{mgL}} = 2\pi \sqrt{\frac{mL^2}{mgL}}$

Physical Pendulum

A physical pendulum is an extended rigid body that is suspended from a fixed point $O$ and oscillates under the influence of gravity. The net torque $\tau = -mgh \sin \theta$. Here $h$ is the distance between point $O$ and the center of mass $C$ of the suspended body. If we make the small angle approximation $\theta \ll 1$, we have: $\tau \approx -(mgh)\theta$. If we compare the torques with the force equation $F = -Cx$ we realize that we identify the constant $C$ with the term $hmg$. We can thus readily determine period $T$ of the oscillation.

$T = 2\pi \sqrt{\frac{I}{mgh}}$ Here $I$ is the rotational inertia about an axis through $O$. 
Example 1: The pendulum can be used as a very simple device to measure the acceleration of gravity at a particular location.

- measure the length “l” of the pendulum and then set the pendulum into motion
- measure “T” by a clock

\[
T = 2\pi \sqrt{\frac{I}{C}} = 2\pi \sqrt{\frac{I}{mgh}} = 2\pi \sqrt{\frac{1/3mL^2}{mgh}} = 2\pi \sqrt{\frac{L^2}{3g(L/2)}} = 2\pi \sqrt{\frac{2L}{3g}}
\]

\[
⇒ g = \frac{8\pi^2 L}{3T^2}
\]
Example: The Length of a Pendulum. A student is in an empty room. He has a piece of rope, a small bob, and a clock. Find the volume of the room.

1. From the piece of rope and a bob we make a simple pendulum
2. We set pendulum into motion
3. We measure period “T” by a clock
4. We calculate the length of the pendulum (rope)

\[ l = \frac{T^2}{4\pi^2} \]

\[ T = 2\pi \sqrt{\frac{l}{g}} \]

5. With a help of the rope of the known length we measure the dimensions of the room a x b x h and its volume \( V = a \times b \times h \)
Problem 44. A physical pendulum consists of a meter stick that is pivoted at a small hole drilled through the stick a distance \( d \) from the 50 cm mark. The period of oscillation is 2.5 s. Find \( d \)?

\[
T = 2\pi \sqrt{\frac{I}{mgd}}
\]

We use Eq. 15-29 and the parallel-axis theorem \( I = I_{\text{cm}} + mh^2 \) where \( h = d \), the unknown. For a meter stick of mass \( m \), the rotational inertia about its center of mass is \( I_{\text{cm}} = mL^2/12 \) where \( L = 1.0 \) m. Thus, for \( T = 2.5 \) s, we obtain

\[
T = 2\pi \sqrt{\frac{mL^2/12 + md^2}{mgd}} = 2\pi \sqrt{\frac{L^2}{12gd}} + \frac{d}{g}.
\]

Squaring both sides and solving for \( d \) leads to the quadratic formula:

\[
d = \frac{g \left( T/2\pi \right)^2 \pm \sqrt{g^2 \left( T/2\pi \right)^4 - L^2/3}}{2}.
\]

Choosing the plus sign leads to an impossible value for \( d \) (\( d = 1.5 > L \)). If we choose the minus sign, we obtain a physically meaningful result: \( d = 0.056 \) m.
Simple Harmonic Motion and Uniform Circular Motion

Consider an object moving on a circular path of radius $x_m$ with a uniform speed $v$. If we project the position of the moving particle at point $P'$ on the x-axis we get point $P$.

The coordinate of $P$ is: $x(t) = x_m \cos(\omega t + \phi)$.

While point $P'$ executes unifrom circular motion its projection $P$ moves along the x-axis with simple harmonic motion.

The speed $v$ of point $P'$ is equal to $\omega x_m$. The direction of the velocity vector is along the tangent to the circular path. If we project the velocity $\vec{v}$ on the x-axis we get: $v(t) = -\omega x_m \sin(\omega t + \phi)$

The acceleration $\vec{a}$ points along the center $O$. If we project $\vec{a}$ along the x-axis we get: $a(t) = -\omega^2 x_m \cos(\omega t + \phi)$

Conclusion: Whether we look at the displacement, the velocity, or the acceleration, the projection of uniform circular motion on the x-axis is SHM.
Damped Simple Harmonic Motion

When the amplitude of an oscillating object is reduced due to the presence of an external force the motion is said to be damped. An example is given in the figure. A mass $m$ attached to a spring of spring constant $k$ oscillates vertically. The oscillating mass is attached to a vane submerged in a liquid. The liquid exerts a damping force $\vec{F}_d$ whose magnitude is given by the equation: $F_d = -bv$

The negative sign indicates that $\vec{F}_d$ opposes the motion of the oscillating mass. The parameter $b$ is called the damping constant. The net force on $m$ is:

$$F_{net} = -kx - bv$$

From Newton's second law we have:

$$-kx - bv = ma$$

We substitute $v$ with $\frac{dx}{dt}$ and $a$ with $\frac{d^2x}{dt^2}$ and get

the following differential equation:

$$m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx = 0$$
Newton's second law for the damped harmonic oscillator:

\[ m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx = 0 \]

The solution has the form:

\[ x(t) = x_m e^{-bt/2m} \cos \left( \omega' t + \phi \right) \]

In the picture above we plot \( x(t) \) versus \( t \). We can regard the above solution as a cosine function with a time-dependent amplitude \( x_m e^{-bt/2m} \). The angular frequency \( \omega' \) of the damped harmonic oscillator is given by the equation:

\[ \omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} \]

For an undamped harmonic oscillator the energy \( E = \frac{1}{2} kx_m^2 \).

If the oscillator is damped its energy is not constant but decreases with time.

If the damping is small we can replace \( x_m \) with \( x_m e^{-bt/2m} \). By doing so we find that:

\[ E(t) \approx \frac{1}{2} kx_m^2 e^{-bt/m} \]

The mechanical energy decreases exponentially with time.
Problem 59. The amplitude of a lightly damped oscillator decreases by 3.0% during each cycle. What percentage of the mechanical energy of the oscillator is lost in each cycle?

Since the energy is proportional to the amplitude squared (see Eq. 15-21), we find the fractional change (assumed small) is

\[ E = U + K = \frac{1}{2} kx_m^2 \]

Thus, if we approximate the fractional change in \( x_m \) as \( dx_m/x_m \), then the above calculation shows that multiplying this by 2 should give the fractional energy change. Therefore, if \( x_m \) decreases by 3%, then \( E \) must decrease by 6.0%. 

Forced Oscillations and Resonance

If an oscillating system is disturbed and then allowed to oscillate freely the corresponding angular frequency $\omega$ is called the natural frequency. The same system can also be driven as shown in the figure by a moving support that oscillates at an arbitrary angular frequency $\omega_d$. Such a forced oscillator oscillates at the angular frequency $\omega_d$ of the driving force. The displacement is given by:

$$x(t) = x_m \cos(\omega't + \phi)$$

The oscillation amplitude $x_m$ varies with the driving frequency as shown in the lower figure. The amplitude is approximately greatest when $\omega_d = \omega$.

This condition is called resonance. All mechanical structures have one or more natural frequencies and if a structure is subjected to a strong external driving force whose frequency matches one of the natural frequencies, the resulting oscillations may damage the structure.
Problem 63. A 100 kg car carrying four 82 kg people travels over a "washboard" dirt road with corrugations 4.0 m apart. The car bounces with maximum amplitude when its speed is 16 km/h. When the car stops, and the people get out, how much does the car body rise on its suspension?

\[ \omega = \sqrt{\frac{k}{m}} \]

With \( M = 1000 \text{ kg} \) and \( m = 82 \text{ kg} \), we adapt Eq. 15-12 to this situation by writing

\[ \omega = \frac{2\pi}{T} = \sqrt{\frac{k}{M + 4m}}. \]

If \( d = 4.0 \text{ m} \) is the distance traveled (at constant car speed \( v \)) between impulses, then we may write \( T = d/v \), in which case the above equation may be solved for the spring constant:

\[ \frac{2\pi v}{d} = \sqrt{\frac{k}{M + 4m}} \Rightarrow k = (M + 4m) \left( \frac{2\pi v}{d} \right)^2. \]

Before the people got out, the equilibrium compression is \( x_i = (M + 4m)g/k \), and afterward it is \( x_f = Mg/k \). Therefore, with \( v = 16000/3600 = 4.44 \text{ m/s} \), we find the rise of the car body on its suspension is

\[ x_i - x_f = \frac{4mg}{k} = \frac{4mg}{M + 4m} \left( \frac{d}{2\pi v} \right)^2 = 0.050 \text{ m}. \]
Problem 26. Two springs are joined in series and connected to a block of mass 0.245 kg that is set oscillating over a frictionless floor. The springs each have spring constant k=6430 N/m. What is the frequency of oscillations?

We wish to find the effective spring constant for the combination of springs. We do this by finding the magnitude $F$ of the force exerted on the mass when the total elongation of the springs is $\Delta x$. Then $k_{\text{eff}} = F/\Delta x$. Suppose the left-hand spring is elongated by $\Delta x_\ell$ and the right-hand spring is elongated by $\Delta x_r$. The left-hand spring exerts a force of magnitude $k \Delta x_\ell$ on the right-hand spring and the right-hand spring exerts a force of magnitude $k \Delta x_r$ on the left-hand spring. By Newton’s third law these must be equal, so $\Delta x_\ell = \Delta x_r$. The two elongations must be the same and the total elongation is twice the elongation of either spring: $\Delta x = 2 \Delta x_\ell$. The left-hand spring exerts a force on the block and its magnitude is $F = k \Delta x_\ell$. Thus $k_{\text{eff}} = k \Delta x_\ell / 2 \Delta x_r = k / 2$. The block behaves as if it were subject to the force of a single spring, with spring constant $k/2$. To find the frequency of its motion replace $k_{\text{eff}}$ in $f = (1/2\pi)\sqrt{k_{\text{eff}}/m}$ with $k/2$ to obtain

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{2m}}.$$ 

With $m = 0.245$ kg and $k = 6430$ N/m, the frequency is $f = 18.2$ Hz.