PH 221-3A Fall 2009

Review for test 2
Lecture 13

Chapters 5-8
(Halliday/Resnick/Walker, Fundamentals of Physics 8th edition)
The potential Energy Curve

If we plot the potential energy $U$ versus $x$ for a force $F$ that acts along the $x$-axis we can get a wealth of information about the motion of a particle on which $F$ is acting. The first parameter that we can determine is the force $F(x)$ using the equation:

$$F(x) = -\frac{dU(x)}{dx}$$

An example is given in the figures below.

In fig. a we plot $U(x)$ versus $x$.

In fig. b we plot $F(x)$ versus $x$.

For example at $x_2$, $x_3$ and $x_4$ the slope of the $U(x)$ vs $x$ curve is zero, thus $F = 0$.

The slope $dU/dx$ between $x_3$ and $x_4$ is negative; Thus $F > 0$ for the this interval.

The slope $dU/dx$ between $x_2$ and $x_3$ is positive; Thus $F < 0$ for the same interval.
Turning Points:
The total mechanical energy is $E_{mec} = K(x) + U(x)$
This energy is constant (equal to $5 \text{ J}$ in the figure) and is thus represented by a horizontal line. We can solve this equation for $K(x)$ and get:

$$K(x) = E_{mec} - U(x)$$

At any point $x$ on the $x$-axis we can read the value of $U(x)$. Then we can solve the equation above and determine $K$

From the definition of $K = \frac{mv^2}{2}$ the kinetic energy cannot be negative.

This property of $K$ allows us to determine which regions of the $x$-axis motion is allowed. $K(x) = K(x) = E_{mec} - U(x)$

If $K > 0 \rightarrow E_{mech} - U(x) > 0 \rightarrow U(x) < E_{mec}$ Motion is allowed

If $K < 0 \rightarrow E_{mech} - U(x) < 0 \rightarrow U(x) > E_{mec}$ Motion is forbidden

The points at which: $E_{mec} = U(x)$ are known as turning points for the motion. For example $x_1$ is the turning point for the $U$ versus $x$ plot above. At the turning point $K = 0$
Given the $U(x)$ versus $x$ curve the turning points and the regions for which motion is allowed depends on the value of the mechanical energy $E_{mec}$.

In the picture to the left consider the situation when $E_{mec} = 4$ J (purple line). The turning points ($E_{mec} = U$) occur at $x_1$ and $x > x_5$. Motion is allowed for $x > x_1$. If we reduce $E_{mec}$ to 3 J or 1 J the turning points and regions of allowed motion change accordingly.

**Equilibrium Points:** A position at which the slope $dU/dx = 0$ and thus $F = 0$ is called an equilibrium point. A region for which $F = 0$ such as the region $x > x_5$ is called a region of **neutral equilibrium**. If we set $E_{mec} = 4$ J the kinetic energy $K = 0$ and any particle moving under the influence of $U$ will be stationary at any point with $x > x_5$.

**Minima** in the $U$ versus $x$ curve are positions of **stable equilibrium**

**Maxima** in the $U$ versus $x$ curve are positions of **unstable equilibrium**
Note: The blue arrows in the figure indicate the direction of the force $F$ as determined from the equation:

$$ F(x) = -\frac{dU(x)}{dx} $$

**Positions of Stable Equilibrium.** An example is point $x_4$ where $U$ has a minimum. If we arrange $E_{mec} = 1$ J then $K = 0$ at point $x_4$. A particle with $E_{mec} = 1$ J is stationary at $x_4$. If we displace slightly the particle either to the right or to the left of $x_4$ the force tends to bring it back to the equilibrium position. This equilibrium is stable.

**Positions of Unstable Equilibrium.** An example is point $x_3$ where $U$ has a maximum. If we arrange $E_{mec} = 3$ J then $K = 0$ at point $x_3$. A particle with $E_{mec} = 3$ J is stationary at $x_3$. If we displace slightly the particle either to the right or to the left of $x_3$ the force tends to take it further away from the equilibrium position. This equilibrium is unstable.
Work done on a System by an External Force
(no friction involved)

Up to this point we have considered only isolated systems in which no external forces were present. We will now consider a system in which there are forces external to the system.

The system under study is a bowling ball being hurled by a player. The system consists of the ball and the earth taken together. The force exerted on the ball by the player is an external force. In this case the mechanical energy $E_{mec}$ of the system is not constant. Instead it changes by an amount equal to the work $W$ done by the external force according to the equation:

$$W = \Delta E_{mec} = \Delta K + \Delta U$$
Work done on a System by an External Force (friction involved)

• 2nd Newton’s Law for x component \( F - f_k = ma \)

• Since \( a = \text{const} \), \( v^2 = v_o^2 + 2ad \), solving for \( a \) and substituting the result in Newton’s Eq.

\[
F_d = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2 + f_kd = \Delta K + f_kd
\]

• Through experiment, we find that the increase in thermal energy by sliding:
  \( \Delta E_{th} = f_kd \)

• \( F_d \) is the work \( W \) done by external force \( F \) on the block floor system

• Hence, \( W = \Delta E_{mec} + \Delta E_{th} \) energy statement for the work done on a system by an external force when friction is involved.
Problem 42. A worker pushed a 27 kg block 9.2 m along a level floor at constant speed with a force directed 32 degree below the horizontal. If the coefficient of kinetic friction between block and floor was 0.20, what were (a) the work done by the worker’s force and (b) the increase in thermal energy of the block-floor system?

Since the velocity is constant, \( \ddot{a} = 0 \) and the horizontal component of the worker's push \( F \cos \theta \) (where \( \theta = 32^\circ \)) must equal the friction force magnitude \( f_k = \mu_k F_N \). Also, the vertical forces must cancel, implying

\[
F_N = F \sin \theta + mg, \quad \text{on the other hand} \quad F \cos \theta = \mu_k F_N
\]

which is solved to find \( F = 71 \) N.

(a) The work done on the block by the worker is, using Eq. 7-7,

\[
W = Fd \cos \theta = (71 \text{ N})(9.2 \text{ m}) \cos 32^\circ = 5.6 \times 10^2 \text{ J}.
\]

(b) Since \( f_k = \mu_k (mg + F \sin \theta) \), we find \( \Delta E_{th} = f_k d = (60 \text{ N})(9.2 \text{ m}) = 5.6 \times 10^2 \text{ J} \).
Conservation of Energy

Energy cannot magically appear or disappear. It can be transferred to or from objects and systems by means of work done on a system.

The total energy $E$ of a system can change only by amounts of energy that are transferred to or from the system.

$$W=\Delta E=\Delta E_{\text{mec}}+\Delta E_{\text{th}}+\Delta E_{\text{int}}$$

The law of conservation of energy is not something we have derived from basic physics principles. Rather it is a law based on countless experiments. We have never found an exception to it.

Isolated System

If a system is isolated from its environment, there can be no energy transfers to or from it. The total energy $E$ of an isolated system cannot change.
Problem 61. A stone with a weight of 5.29 N is launched vertically from ground level with an initial speed of 20.0 m/s, and the air drag on it is 0.265 N throughout the flight. What are (a) the maximum height reached by the stone and (b) its speed just before it hits the ground?

(a) The maximum height reached is \( h \). The thermal energy generated by air resistance as the stone rises to this height is \( \Delta E_{th} = fh \) by Eq. 8-31. We use energy conservation in the form of Eq. 8-33 (with \( W = 0 \)):

\[
K_f + U_f + \Delta E_{th} = K_i + U_i
\]

and we take the potential energy to be zero at the throwing point (ground level). The initial kinetic energy is \( K_i = \frac{1}{2}mv_0^2 \), the initial potential energy is \( U_i = 0 \), the final kinetic energy is \( K_f = 0 \), and the final potential energy is \( U_f = wh \), where \( w = mg \) is the weight of the stone. Thus, \( wh + fh = \frac{1}{2}mv_0^2 \), and we solve for the height:

\[
h = \frac{mv_0^2}{2(w+f)} = \frac{v_0^2}{2g(1 + f/w)}.
\]

Numerically, we have, with \( m = (5.29 \text{ N})/(9.80 \text{ m/s}^2)=0.54 \text{ kg},\)

\[
h = \frac{(20.0 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)(1+0.265/5.29)} = 19.4 \text{ m}.
\]
(b) We notice that the force of the air is downward on the trip up and upward on the trip down, since it is opposite to the direction of motion. Over the entire trip the increase in thermal energy is $\Delta E_{th} = 2fh$. The final kinetic energy is $K_f = \frac{1}{2}mv^2$, where $v$ is the speed of the stone just before it hits the ground. The final potential energy is $U_f = 0$. Thus, using Eq. 8-31 (with $W = 0$), we find

$$\frac{1}{2}mv^2 + 2fh = \frac{1}{2}mv_0^2.$$  

We substitute the expression found for $h$ to obtain

$$\frac{2f v_0^2}{2g(1 + f/w)} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

which leads to

$$v^2 = v_0^2 - \frac{2f v_0^2}{mg(1 + f/w)} = v_0^2 - \frac{2f v_0^2}{w(1 + f/w)} = v_0^2 \left(1 - \frac{2f}{w+f}\right) = v_0^2 \frac{w-f}{w+f}$$

where $w$ was substituted for $mg$ and some algebraic manipulations were carried out. Therefore,

$$v = v_0 \sqrt{\frac{w-f}{w+f}} = (20.0 \text{ m/s}) \sqrt{\frac{5.29 \text{ N} - 0.265 \text{ N}}{5.29 \text{ N} + 0.265 \text{ N}}} = 19.0 \text{ m/s}.$$
ALL QUESTIONS ARE WORTH 20 POINTS. WORK OUT FIVE PROBLEMS.

NOTE: Clearly write out solutions and answers (circle the answers) by section for each part (a., b., c., etc.)

1. **Motion along a straight line with a constant acceleration**

\[ v_{\text{average speed}} = \frac{\text{[dist. taken]}}{\text{[time trav.]}}, \]

\[ v_{\text{average vel.}} = \frac{\Delta x}{\Delta t}; \]

\[ v_{\text{ins}} = \frac{dx}{dt}; \]

\[ a_{\text{aver.}} = \frac{\Delta v}{\Delta t}; \]

\[ v = v_0 + at; \quad x = \frac{1}{2}(v_0 + v)t; \quad v^2 = v_0^2 + 2ax \text{ (if } x_0 = 0 \text{ at } t_0 = 0) \]

2. **Free fall motion (with positive direction ↑)**

\[ g = 9.80 \text{ m/s}^2; \]

\[ y = v_{\text{average vel.}} t; \]

\[ v_{\text{average.}} = \frac{(v + v_0)}{2}; \]

\[ v = v_0 - gt; \quad y = v_0 t - \frac{1}{2} g t^2; \quad v^2 = v_0^2 - 2gy \text{ (if } y_0 = 0 \text{ at } t_0 = 0) \]

3. **Motion in a plane**

\[ v_x = v_0 \cos \theta; \]

\[ v_y = v_0 \sin \theta; \]

\[ x = v_{ox} t + \frac{1}{2} a_x t^2; \quad y = v_{oy} t + \frac{1}{2} a_y t^2; \quad v_x = v_{ox} + at; \quad v_y = v_{oy} + at; \]

4. **Projectile motion (with positive direction ↑)**

\[ v_x = v_{ox} = v_0 \cos \theta; \]

\[ x = v_{ox} t; \]

\[ x_{max} = \left(\frac{2 v_o^2 \sin \theta \cos \theta}{g}\right) = \left(\frac{v_0^2}{2} \sin 2\theta\right)/g \text{ for } y_{in} = y_{fin}; \]

\[ v_y = v_{oy} - gt = v_0 \sin \theta - gt; \]

\[ y = v_{oy} t - \frac{1}{2} gt^2; \]

5. **Uniform circular Motion**

\[ a = v^2/r; \]

\[ T = 2\pi r/v \]

6. **Relative motion**

\[ \vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA} \]

\[ \vec{a}_{PA} = \vec{a}_{PB} \]

7. **Component method of vector addition**
\[ \mathbf{A} = \mathbf{A}_1 + \mathbf{A}_2; \quad \mathbf{A}_x = \mathbf{A}_{x1} + \mathbf{A}_{x2} \quad \text{and} \quad \mathbf{A}_y = \mathbf{A}_{y1} + \mathbf{A}_{y2}; \quad A = \sqrt{A_x^2 + A_y^2}; \quad \theta = \tan^{-1} | \frac{A_y}{A_x} |; \]

The scalar product \( \mathbf{A} \cdot \mathbf{B} = ab \cos \phi \)

\[ \mathbf{A} \cdot \mathbf{B} = (a_x \hat{i} + a_y \hat{j} + a_z \hat{k}) \cdot (b_x \hat{i} + b_y \hat{j} + b_z \hat{k}) \]

\[ \mathbf{A} \cdot \mathbf{B} = a_x b_x + a_y b_y + a_z b_z \]

The vector product \( \mathbf{A} \times \mathbf{B} = (a_x \hat{i} + a_y \hat{j} + a_z \hat{k}) \times (b_x \hat{i} + b_y \hat{j} + b_z \hat{k}) \)

\[ \mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \hat{i} \left| \begin{array}{cc} a_y & a_z \\ b_y & b_z \end{array} \right| - \hat{j} \left| \begin{array}{cc} a_x & a_z \\ b_x & b_z \end{array} \right| + \hat{k} \left| \begin{array}{cc} a_x & a_y \\ b_x & b_y \end{array} \right| =
\]

\[ = (a_y b_z - b_y a_z) \hat{i} + (a_z b_x - b_z a_x) \hat{j} + (a_x b_y - b_x a_y) \hat{k} \]

1. **Second Newton's Law**
   \[ ma = F_{net} \]

2. **Kinetic Friction**
   \[ f_k = \mu_k N \]

3. **Static Friction**
   \[ f_s = \mu_s N \]

4. **Universal Law of Gravitation**
   \[ F = GMm/r^2; \quad G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2; \]

5. **Drag Coefficient**
   \[ D = \frac{1}{2} C \rho A v^2 \]

6. **Terminal Speed**
   \[ v_t = \sqrt{\frac{2 m g}{c \rho A}} \]

7. **Centripetal Force**
   \[ F_c = \frac{mv^2}{r} \]

8. **Speed of the Satellite in a Circular Orbit**
   \[ v^2 = GM/r \]

9. **The Work Done by a Constant Force Acting on an Object**
   \[ W = F d \cos \phi = \mathbf{F} \cdot \mathbf{d} \]

10. **Kinetic Energy**
    \[ K = \frac{1}{2} m v^2 \]

11. **Total Mechanical Energy**
    \[ E = K + U \]

12. **The Work-Energy Theorem**
    \[ W = K_f - K_i; \quad W_{nc} = \Delta K + \Delta U = E_f - E_o \]

    when \( W_{nc} = 0; \quad E_f = E_o \)

14. **Work Done by the Gravitational Force**
    \[ W_g = m g d \cos \phi \]
1. Work done in Lifting and Lowering the object:

\[ \Delta K = K_f - K_i = W_a + W_g; \text{ if } K_f = K_i; W_a = -W_g \]

2. Spring Force: \( F_s = -kx \) (Hooke's law)

3. Work done by a spring force: \( W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 \); if \( x_i = 0 \) and \( x_f = x \); \( W_s = -\frac{1}{2}kx^2 \)

4. Work done by a variable force: \( W = \int_{x_i}^{x_f} F(x) \, dx \)

5. Power: \( P_{avg} = \frac{W}{\Delta t}; P = \frac{dW}{dt} \); \( P = F \cos \phi = F \cdot \vec{v} \)

6. Potential energy: \( \Delta U = -W \); \( \Delta U = -\int_{x_i}^{x_f} F(x) \, dx \)

7. Gravitational Potential Energy:

\[ \Delta U = mg(y_f - y_i) = mg\Delta y; \text{ if } y_i = 0 \text{ and } U_i = 0; \text{ } U(y) = mg \cdot y \]

8. Elastic potential Energy: \( U(x) = \frac{1}{2}kx^2 \)

9. Potential energy curves: \( F(x) = -\frac{dU(x)}{dx} \); \( K(x) = E_{m ec} - U(x) \)

10. Work done on a system by an external force:

Friction is not involved \( W = \Delta E_{m ec} = \Delta K + \Delta U \)

When kinetic friction force acts within the system \( W = \Delta E_{m ec} + \Delta E_{th} \)

\( \Delta E_{th} = f_kd \)

11. Conservation of energy:

\( W = \Delta E = \Delta E_{m ec} + \Delta E_{th} + \Delta E_{int} \)

for isolated system (\( W = 0 \)) \( \Delta E_{m ec} + \Delta E_{th} + \Delta E_{int} = 0 \)

12. Power: \( P_{avg} = \frac{\Delta E}{\Delta t} \); \( P = \frac{dE}{dt} \)
1. In order to keep a platform stationary a man with a mass \( m \) must pull with a force \( T=150 \) N as shown in the figure. If the platform, pulleys and strings have negligible weight, what is his mass?

\[ \begin{align*}
T + N - mg &= 0 \\
N &= mg - T \\
T + 2T - N &= 0 \\
T + 2T &= mg + T = 0
\end{align*} \]

\[ mg = 4T \]

\[ m = \frac{4T}{g} = 61.2 \text{ kg} \]
If the blocks on the double frictionless incline in figure are at rest or move with a constant speed.

a.) Which block is more massive \( m_1 \) or \( m_2 \)?
b.) How many times more massive?

**Object 1:**
\[
x: T - m_1 g \sin 30^\circ = 0
\]

\[
\Rightarrow m_1 g \sin 30^\circ = m_2 g \sin 37^\circ
\]

\[
\Rightarrow m_1 = \frac{\sin 37^\circ}{\sin 30^\circ} m_2 = \frac{0.602}{0.5} = 1.2 m_2
\]

\[
\Rightarrow m_1 \text{ is 1.2 times more massive than } m_2
\]

**Object 2:**
\[
x: m_2 g \sin 37^\circ - T = 0
\]
3. A 4-kg block and 2-kg block can move on the horizontal surface with coefficient of kinetic friction \(\mu_k=0.2\). The blocks are accelerated by a force \(F=12\) N that pushes the larger block against the smaller one. Determine the force that the 2-kg block exerts on the 4-kg block.

\[
\begin{align*}
\text{Picture & object of interest - block 1 & 2 together} \\
\text{F. B. P.} & \quad \text{N} \\
\text{x & y axis} & \\
\end{align*}
\]

\[
\begin{align*}
\text{\(\mathbf{f}_k\)} & \quad \text{\(\mathbf{F}\)} \\
\text{\((m_1+m_2)g\)} & \quad \text{\(\mathbf{F}-\mu m_1(m_1+m_2)g\)} \\
\text{\(\mathbf{a}\)} & \quad \text{\(\mathbf{a}=\frac{\mathbf{F}-\mu m_1(m_1+m_2)g}{(m_1+m_2)g}\)} \\
\text{\(\mathbf{N}\)} & \quad \text{\(\mathbf{N}=(m_1+m_2)g\)} \\
\text{\(\mathbf{f}_k\)} & \quad \text{\(\mathbf{f}_k=\mu m_1\mathbf{N}=(m_1+m_2)g\)} \\
\text{\(\mathbf{F}\)} & \quad \text{\(\mathbf{F}=12\) N} \\
\text{\(\mathbf{a}\)} & \quad \text{\(\mathbf{a}=\frac{12-0.2(4+2)}{4+2}\) \(9.8\) m/s}^2 \\
\end{align*}
\]

\[
\begin{align*}
\text{Object 2} & \quad \text{Object 2} \\
\text{\(\mathbf{f}_{k2}\)} & \quad \text{\(\mathbf{F}_2\)} \\
\text{\(m_2g\)} & \quad \text{\(m_2g+\mu m_2g=2.64 \times 10^{-1} + 0.2 \times 2.98 = 4 \text{ N}\)} \\
\text{\(\mathbf{N}_2\)} & \quad \text{\(\mathbf{N}_2\)} \\
\end{align*}
\]

\[
\begin{align*}
\text{3. N.L.} & \quad \text{3. N.L.} \\
\text{\(|\mathbf{F}_{21}|=|\mathbf{F}_{12}|\)} & \quad \text{\(|\mathbf{F}_{21}|=|\mathbf{F}_{12}|\)} \\
\text{\(4 \text{ N}\)} & \quad \text{\(4 \text{ N}\)} \\
\end{align*}
\]
5. A plane is traveling at constant speed 200 m/s following the arc of a vertical circle of radius \( R \). At the top of its path, the passengers experience "weightlessness". To one significant figure, what is the value of \( R \)?

\[
N + mg = \frac{m v^2}{R} \]

\( N = 0 \) due to "weightlessness"

\[
= mg = \frac{m v^2}{R} \]

\[
R = \frac{v^2}{g} = \frac{200^2}{9.80} = 4081.6 =
\]

\( = 4 \times 10^3 \text{ m} \)
Two boxes are connected to each other as shown. The system is released from rest and the 1.00-kg box falls through a distance of 1.00 m. The surface of the table is frictionless. What is the kinetic energy of box B just before it reaches the floor?

The only nonconservative force acting on the box system (box A - string - box B) is normal reaction of the surface of the table.

but \[ N \text{ I displacement} \rightarrow W_{nc} = 0 \text{ } \Rightarrow \text{from Work-Energy Theorem} \]

\[ W_{nc} = E_f - E_i \]

\[ E_i = mgh_A + mgh_B \]

\[ E_f = mgh_A + m_A \frac{v^2}{2} + m_B \frac{v^2}{2} \]

\[ E_i = E_f \Rightarrow m_A g h_A + m_B g h_B = m_A \frac{v^2}{2} + m_B \frac{v^2}{2} \]

\[ \Rightarrow v^2 = \frac{2 m_B g h_B}{m_A + m_B} \]

\[ \frac{m_B v^2}{2} = \frac{m_B g h_B}{m_A + m_B} = \frac{(1.00 \text{kg})^2 (9.8 \text{ m/s}^2)(1.00 \text{ m})}{4.00 \text{ kg}} = \boxed{2.45 \text{ m/s}^2} \]
7. A 700 g block is released at height $h_0$ above a vertical spring of constant $k=400$ N/m and negligible mass. The block sticks to the spring and momentarily stops after compressing the spring 19.0 cm. How much work is done

(a) by the block on the spring?
(b) by the spring on the block?
(c) What is the value of $h_0$?
(d) If the block were released from height 2.00 $h_0$ above the spring, what would be the maximum compression of the spring?

We place the reference position for evaluating gravitational potential energy at the relaxed position of the spring. We use $x$ for the spring's compression, measured positively downwards (so $x > 0$ means it is compressed).

(a) With $x = 0.190$ m,

$$W_s = -\frac{1}{2}kx^2 = -7.2\text{ J} \approx -7.2\text{ J}$$

for the work done by the spring force. Using Newton's third law, we see that the work done on the spring is 7.2 J.

(b) As noted above, $W_s = -7.2\text{ J}$.

(c) Energy conservation leads to

$$K_i + U_i = K_f + U_f$$

$$mg h_0 = -mgx + \frac{1}{2}kx^2$$

which (with $m = 0.70$ kg) yields $h_0 = 0.86$ m.

(d) With a new value for the height $h'_0 = 2h_0 = 1.72$ m, we solve for a new value of $x$ using the quadratic formula (taking its positive root so that $x > 0$).

$$mg h'_0 = -mgx + \frac{1}{2}kx^2 \Rightarrow x = \frac{mg + \sqrt{(mg)^2 + 2mgkh'_0}}{k}$$

which yields $x = 0.26$ m.
8. A particle moving along the x axis is acted upon by a single force $F = F_0 e^{-kx}$, where $F_0$ and $k$ are constants. The particle is released from rest at $x = 0$. What is the maximum kinetic energy it will attain?

According to the work energy theorem $W = K_f - K_i = K_f$ since we started from rest ($K_i = 0$)

On the other hand $W = \int_0^{x_f} F(x) \, dx \rightarrow K = \int_0^{x_f} F(x) \, dx = F_0 \int_0^{x_f} e^{-kx} \, dx = F_0 \left[ -\frac{e^{-kx}}{k} \right]_0^{x_f} = F_0 \left( -\frac{e^{-kx_f}}{k} + \frac{1}{k} \right)$

Max $K_f$ occurs at a point when $K'(x) = 0$, $K'(x) = F_0 (e^{-kx}) = 0$ only for $x = \infty$

$\rightarrow K_{\text{max}} = \frac{F_0}{k} \text{ Joules}$
9. The potential energy of a 0.20-kg particle moving along the x axis is given by
\[ U(x) = (8.0 \text{J/m}^2)x^2 + (2.0 \text{J/m}^4)x^4. \]
When the particle is at \( x = 1.0 \text{m} \) it is traveling in the positive x direction with a speed of 5.0 m/s. At what x coordinate it will stop momentarily to turn around.

(a) Calculate total mechanical energy of the particle at coordinate \( x=1.0 \text{ m} \)

\[
U(1.0\text{m}) = 8.0J + 2.0J = 10.0J; \quad K(1.0\text{m}) = \frac{mv^2}{2} = \frac{0.2 \times 5.0^2}{2} = 2.5J
\]

\[ \rightarrow E_{mec} = 12.5J \]

(b) The particle will stop momentarily to turn around when \( K(x) = E_{mec} - U(x) = 0 \)

\[ \rightarrow 8.0x^2 + 2.0x^4 = 12.5 \]

\[ x^2 = \frac{-8.0 \pm \sqrt{8.0^2 + 4(2.0)(12.5)}}{4.0} \]

\[ x^2 = (-2.0 \pm 3.2)\text{m}^2 \]

\[ x^2 = 1.2\text{m}^2 \]

\[ x = 1.1\text{m} \]