ALL QUESTIONS ARE WORTH 20 POINTS. WORK OUT FIVE PROBLEMS.

NOTE: Clearly write out solutions and answers (circle the answers) by section for each part (a., b., c., etc.)

Important Formulas:

1. Motion along a straight line with a constant acceleration
   \[
   v_{\text{aver. speed}} = \frac{\text{dist. taken}}{\text{time trav.}} = S/t;
   \]
   \[
   v_{\text{aver. vel.}} = \frac{\Delta x}{\Delta t};
   \]
   \[
   v_{\text{ins.}} = \frac{dx}{\Delta t};
   \]
   \[
   a_{\text{aver.}} = \frac{\Delta v}{\Delta t};
   \]
   \[
   a = \frac{dv}{dt};
   \]
   \[
   v = v_0 + at; \quad x = \frac{1}{2}(v_0 + v)t; \quad v^2 = v_0^2 + 2ax \quad \text{(if } x_0 = 0 \text{ at } t_0 = 0)\]

2. Free fall motion (with positive direction $\uparrow$)
   \[
   g = 9.80 \text{ m/s}^2;
   \]
   \[
   y = v_{\text{aver.}} t;
   \]
   \[
   v_{\text{aver.}} = \frac{(v + v_0)}{2};
   \]
   \[
   v = v_0 - gt; \quad y = v_0 t - \frac{1}{2} g t^2; \quad v^2 = v_0^2 - 2gy \quad \text{(if } y_0 = 0 \text{ at } t_0 = 0)\]

3. Motion in a plane
   \[
   v_x = v_0 \cos \theta;
   \]
   \[
   v_y = v_0 \sin \theta;
   \]
   \[
   x = v_{ox} t + \frac{1}{2} a_x t^2; \quad y = v_{oy} t + \frac{1}{2} a_y t^2; \quad v_x = v_{ox} + at; \quad v_y = v_{oy} + at;
   \]

4. Projectile motion (with positive direction $\uparrow$)
   \[
   v_x = v_{ox} = v_0 \cos \theta;
   \]
   \[
   x = v_{ox} t;
   \]
   \[
   x_{\text{max}} = \frac{(2 v_0^2 \sin \theta \cos \theta)}{g} = \frac{(v_0^2 \sin 2 \theta)}{g} \quad \text{for } y_{\text{fin}} = y_{\text{fin}};
   \]
   \[
   v_y = v_{oy} - gt = v_0 \sin \theta - gt;
   \]
   \[
   y = v_{oy} t - \frac{1}{2} gt^2;
   \]

5. Uniform circular Motion
   \[
   a = \frac{v^2}{r};
   \]
   \[
   T = 2\pi \sqrt{\frac{r}{v}}
   \]

6. Relative motion
   \[
   \ddot{v}_{PA} = \ddot{v}_{PB} + \ddot{v}_{BA}
   \]
   \[
   \ddot{a}_{PA} = \ddot{a}_{PB}
   \]

7. Component method of vector addition
\[ A = A_1 + A_2; \ A_x = A_{x1} + A_{x2} \text{ and } A_y = A_{y1} + A_{y2}; \ A = \sqrt{A_x^2 + A_y^2}; \ \theta = \tan^{-1} \left| \frac{A_y}{A_x} \right|; \]

The scalar product \[ \vec{a} \cdot \vec{b} = ab \cos \phi \]

\[ \vec{a} \cdot \vec{b} = (a_x \hat{i} + a_y \hat{j} + a_z \hat{k}) \cdot (b_x \hat{i} + b_y \hat{j} + b_z \hat{k}) \]

\[ \vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z \]

The vector product

\[ \vec{a} \times \vec{b} = (a_x \hat{i} + a_y \hat{j} + a_z \hat{k}) \times (b_x \hat{i} + b_y \hat{j} + b_z \hat{k}) \]

\[ \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \hat{i} \begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix} - \hat{j} \begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix} + \hat{k} \begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix} =
\]

\[ = (a_y b_z - b_y a_z) \hat{i} + (a_z b_x - b_z a_x) \hat{j} + (a_x b_y - b_x a_y) \hat{k} \]

1. Second Newton’s Law
   \[ ma = F_{net}; \]

2. Kinetic friction \[ f_k = \mu_k N; \]

3. Static friction \[ f_s = \mu_s N; \]

4. Universal Law of Gravitation: \[ F = GMm/r^2; \ G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2; \]

5. Drag coefficient \[ D = \frac{1}{2} \rho A v^2 \]

6. Terminal speed \[ v_t = \sqrt{\frac{2mg}{\rho A}} \]

7. Centripetal force \[ F_c = mv^2/r \]

8. Speed of the satellite in a circular orbit: \[ v^2 = GM/r \]

9. The work done by a constant force acting on an object: \[ W = Fd \cos \phi = \vec{F} \cdot \vec{d} \]

10. Kinetic energy: \[ K = \frac{1}{2} mv^2 \]

11. Total mechanical energy: \[ E = K + U \]

12. The work-energy theorem: \[ W = K_f - K_o; \ W_{nc} = \Delta K + \Delta U = E_f - E_o \]

13. The principle of conservation of mechanical energy: when \[ W_{nc} = 0, \ E_f = E_o \]

14. Work done by the gravitational force: \[ W_g = mgd \cos \phi \]
1. Work done in Lifting and Lowering the object:
   \[ \Delta K = K_f - K_i = W_a + W_g; \text{ if } K_f = K_i; W_a = -W_g \]

2. Spring Force: \( F_x = -kx \) (Hook's law)

3. Work done by a spring force: \( W_s = \frac{1}{2} kx_i^2 - \frac{1}{2} kx_f^2; \text{ if } x_i = 0 \text{ and } x_f = x; W_s = -\frac{1}{2} kx^2 \)

4. Work done by a variable force: \( W = \int_{x_i}^{x_f} F(x)dx \)

5. Power: \( \overline{P} = \frac{W}{\Delta t}; \overline{P} = \frac{dW}{dt}; \overline{P} = FV \cos \phi = \vec{F} \cdot \vec{v} \)

6. Potential energy: \( \Delta U = -W; \Delta U = -\int_{x_i}^{x_f} F(x)dx \)

7. Gravitational Potential Energy:
   \[ \Delta U = mg(y_f - y_i) = mg\Delta y; \text{ if } y_i = 0 \text{ and } U_i = 0; U(y) = mgy \]

8. Elastic potential Energy: \( U(x) = \frac{1}{2} kx^2 \)

9. Potential energy curves: \( F(x) = -\frac{dU(x)}{dx}; K(x) = E_{mec} - U(x) \)

10. Work done on a system by an external force:
    - Friction is not involved \( W = \Delta E_{mec} = \Delta K + \Delta U \)
    - When kinetic friction force acts within the system \( W = \Delta E_{mec} + \Delta E_{th} \)
    \[ \Delta E_{th} = f_k d \]

11. Conservation of energy: \( W = \Delta E = \Delta E_{mec} + \Delta E_{th} + \Delta E_{int} \)
    - for isolated system (\( W = 0 \)) \( \Delta E_{mec} + \Delta E_{th} + \Delta E_{int} = 0 \)

12. Power: \( \overline{P} = \frac{\Delta E}{\Delta t}; \overline{P} = \frac{dE}{dt} \)
1. In order to keep a platform stationary a man with a mass \( m \) must pull with a force \( T = 150 \) N as shown in the figure. If the platform, pulleys and strings have negligible weight, what is his mass?

Object of interest is a man

\[ T + N - mg = 0 \]
\[ N = mg - T \]

Object of interest is platform

\[ 2T = T_1 \]
\[ T + 2T - N = 0 \]
\[ T + 2T - mg + T = 0 \]

\[ \Rightarrow mg = 4T \]
\[ m = \frac{4T}{g} = 61.2 \text{ kg} \]
2. If the blocks on the double frictionless incline in figure are at rest or move with a constant speed
   a.) Which block is more massive, \( m_1 \) or \( m_2 \)?
   b.) How many times more massive?

\[ \begin{align*}
\text{Object 1:} & \quad x: T - m_1 g \sin 30^\circ = 0 \\
& \quad \Rightarrow m_1 g \sin 30^\circ = m_2 g \sin 37^\circ \\
& \quad \Rightarrow m_1 = \frac{\sin 37^\circ}{\sin 30^\circ} m_2 = \frac{0.602}{0.5} = 1.2 m_2 \\
& \quad \Rightarrow m_1 \text{ is 1.2 times more massive than } m_2
\end{align*} \]
3. A 4-kg block and 2-kg block can move on the horizontal surface with coefficient of kinetic friction $\mu_k = 0.2$. The blocks are accelerated by a force $F = 12$ N that pushes the larger block against the smaller one. Determine the force that the 2-kg block exerts on the 4-kg block.

\begin{align*}
F = m_1 a \\
F - \mu_k(m_1 + m_2)g &= m_1 \frac{F}{m_1 + m_2} \\
N - (m_1 + m_2)g &= 0 \\
f_k &= \mu N \\
N &= (m_1 + m_2)g \\
f_k &= \mu N = \mu (m_1 + m_2)g
\end{align*}

\begin{align*}
a &= \frac{F - \mu (m_1 + m_2)g}{m_1} \\
&= \frac{12 - 0.2(4+2) \cdot 9.8}{4} \\
&= 4 \times 10^{-2} \text{ m/s}^2
\end{align*}

\begin{align*}
F_{12} - f_{k2} &= m_2 a \\
F_{12} &= m_2 a + f_{k2} \\
N_2 - m_2 g &= 0 \\
f_{k2} &= \mu N_2 = \mu m_2 g
\end{align*}

\begin{align*}
F_{12} &= m_2 a + \mu m_2 g \\
&= 2 \times 10^{-2} \cdot 9.8 \\
&= 4 \text{ N}
\end{align*}
5. A plane is traveling at constant speed 200 m/s following the arc of a vertical circle of radius $R$. At the top of its path, the passengers experience “weightlessness”. To one significant figure, what is the value of $R$?

\[ N + mg = \frac{mv^2}{R} \]

$N = 0$ due to “weightlessness”

\[ mg = \frac{mv^2}{R} \]

\[ R = \frac{v^2}{g} = \frac{200^2}{9.80} = 4081.6 = \]

\[ = 4 \times 10^3 \text{ m} \]
6.

Two boxes are connected to each other as shown. The system is released from rest and the 1.00-kg box falls through a distance of 1.00 m. The surface of the table is frictionless. What is the kinetic energy of box B just before it reaches the floor?

\[ \frac{M_B v^2}{2} = ? \]

The only nonconservative force acting on the system is the string force. The normal reaction at the surface of the table is not a displacement.

\[ \Rightarrow W_{nc} = 0 \; \Rightarrow \text{from} \; W-E \; \text{theorem} \]

\[ W_{nc} = E_f - E_i \; \Rightarrow E_f = E_i \]

\[ E_i = M_A g h_A + M_B g h_B \]

\[ E_f = M_A g h_A + M_A \frac{v^2}{2} + M_B \frac{v^2}{2} \]

\[ E_i = E_f \Rightarrow M_A g h_A + M_B g h_B = M_A \frac{v^2}{2} + M_B \frac{v^2}{2} \]

\[ \Rightarrow v^2 = \frac{2 M_B g h_B}{M_A + M_B} \]

\[ \frac{M_B^2 g h_B}{M_A + M_B} = \frac{(1.00 \text{ kg})^2 (6.8 \text{ rad})^2 (1.00 \text{ m})}{4.00 \text{ kg}} = 2.45 \text{ m}^2/\text{s}^2 \]
7. A 700 g block is released at height \( h_0 \) above a vertical spring of constant \( k = 400 \) N/m and negligible mass. The block sticks to the spring and momentarily stops after compressing the spring 19.0 cm. How much work is done (a) by the block on the spring? (b) by the spring on the block? (c) What is the value of \( h_0 \)? (d) If the block were released from height 2.00 \( h_0 \) above the spring, what would be the maximum compression of the spring?

We place the reference position for evaluating gravitational potential energy at the relaxed position of the spring. We use \( x \) for the spring's compression, measured positively downwards (so \( x > 0 \) means it is compressed).

(a) With \( x = 0.190 \) m,
\[
W_s = -\frac{1}{2} kx^2 = -7.22 \text{ J} \approx -7.2 \text{ J}
\]
for the work done by the spring force. Using Newton's third law, we see that the work done on the spring is 7.2 J.

(b) As noted above, \( W_s = -7.2 \) J.

(c) Energy conservation leads to
\[
K_i + U_i = K_f + U_f
\]
\[
mgh_0 = -mgx + \frac{1}{2} kx^2
\]
which (with \( m = 0.70 \) kg) yields \( h_0 = 0.86 \) m.

(d) With a new value for the height \( h'_0 = 2h_0 = 1.72 \) m, we solve for a new value of \( x \) using the quadratic formula (taking its positive root so that \( x > 0 \)).
\[
mgh'_0 = -mgx + \frac{1}{2} kx^2 \quad \Rightarrow \quad x = \frac{mg + \sqrt{(mg)^2 + 2mgkh'_0}}{k}
\]
which yields \( x = 0.26 \) m.
8. A particle moving along the x axis is acted upon by a single force \( F = F_0 e^{-kx} \), where \( F_0 \) and \( k \) are constants. The particle is released from rest at \( x = 0 \). What is the maximum kinetic energy it will attain?

According to the work energy theorem \( W = K_f - K_i = K_f \) since we started from rest \( (K_i = 0) \)

On the other hand \( W = \int_0^{x_f} F(x)dx \rightarrow K = \int_0^{x_f} F(x)dx = F_o \int_0^{x_f} e^{-kx}dx = F_o \left[ -\frac{e^{-kx}}{k} \right]_0^{x_f} = F_o \left( -\frac{e^{-kx}}{k} + \frac{1}{k} \right) \)

Max \( K_f \) occurs at a point when \( K'(x) = 0, \ K'(x) = F_o (e^{-kx}) = 0 \) only for \( x = \infty \)

\( \rightarrow K_{\text{max}} = \frac{F_o}{k} \text{ Joules} \)
9. The potential energy of a 0.20-kg particle moving along the x axis is given by
\[ U(x) = (8.0 \text{J/m}^2)x^2 + (2.0 \text{J/m}^4)x^4. \]
When the particle is at \( x = 1.0 \text{m} \) it is traveling in the positive x direction with a speed of 5.0 m/s. At what x coordinate it will stop momentarily to turn around.

(a) Calculate total mechanical energy of the particle at coordinate \( x = 1.0 \text{ m} \)

\[ U(1.0m) = 8.0J + 2.0J = 10.0J; \quad K(1.0m) = \frac{mv^2}{2} = \frac{0.2 \times 5.0^2}{2} = 2.5J \]

\[ \rightarrow E_{mec} = 12.5J \]

(b) The particle will stop momentarily to turn around when \( K(x) = E_{mec} - U(x) = 0 \)

\[ 8.0x^2 + 2.0x^4 = 12.5 \]

\[ x^2 = \frac{-8.0 \pm \sqrt{8.0^2 + 4(2.0)(12.5)}}{4.0} \]

\[ x^2 = (-2.0 \pm 3.2)m^2 \]

\[ x^2 = 1.2m^2 \]

\[ x = 1.1m \]