Calculus II  
Final Exam, Fall 2013

You must show all your work and give reasons for your answers. Good luck!

(1) (5 points) Find the angle between the vectors \( <2, -1, 1> \) and \( <0, -1, 2> \). You may express your answer using \( \arccos(\theta) \).

(2) (5 points) Find the equation of the line perpendicular to the plane \( 2x + 3y - z = 5 \) containing the point \( (-1, 2, 1) \).
(3) (5 points) Evaluate $\int x \sin(x) \, dx$.

(4) (5 points) Evaluate $\int \frac{x^3 + x}{\sqrt{x}} \, dx$.

(5) (5 points) Evaluate $\int \ln(x) \, dx$. 
(6) (5 points) Express \( f(x) = \frac{x^2}{3 + x} \) as a power series. **Also state the interval of convergence!**

(7) (5 points) Use series to approximate \( \cos\left(\frac{1}{10}\right) \) with an error less than \( 10^{-6} \). [You do not need to compute and add the terms in the sum.]

(8) (5 points) Evaluate \( \int \sin^3(x) \cos(x) \, dx \).
(9) (5 points) Approximate $\int_{0}^{\pi} e^{-x^2} \, dx$ using a Riemann sum with $n = 3$ terms and the midpoint rule. [You do not need to simplify or add the numbers!]

(10) (5 points) Set up (but do not evaluate) an integral for the volume of the solid of revolution obtained by rotating the region bounded by the curves $x = 1$, $x = 2$, $y = \frac{1}{x^2+1}$ and $y = -x - 1$ around the line $y = -3$.

(11) (5 points) Compute the distance from the point $(1, 1, 1)$ to the plane $x + y + z = 1$. 
(12) (10 points) Find the interval and radius of convergence of the power series
\[ \sum_{n=1}^{\infty} \frac{(x - 2)^n}{\sqrt{n}} \]
(13) (15 points) Evaluate

$$\int \frac{2}{(x + 1)(x - 1)^2} \, dx$$
(14) (10 points) Find the work done in pumping all the water out of a full conical reservoir of height 5 m and radius 3 m? [You may use the approximation $g \approx 10 \text{ m/ sec}^2$ and the density of water $\rho = 1000 \text{ kg/m}^3$.]
(15) (10 points) Use series to approximate $\int_{0}^{10} e^{-x^2} \, dx$ with an error less than $10^{-6}$. (You do not need to compute and add the terms in the sum.)
Bonus Problem 4 points

We can approximate $\int_a^b f(x) \, dx$ in two ways. One by using Riemann sums (like in problem 9) and the other by using series (like problem 15).

(a) What are the advantages and disadvantages in using the Riemann sum method?

(b) What are the advantages and disadvantages in using the series method?