Department of Mathematics UAB
Calculus II

ASSIGNMENT 1: Solutions.

Calculus I Review and Vectors §10.1,10.2,10.3

1. We assume that throughout the day the salesperson (SP) is located on a single straight road on which the home/office is also located.

   (a) At 8am the salesperson is located at the office/home, as the distance is zero.

   (b) From 8am to 9am SP travels away (distance is increasing) from the office with the velocity (slope) increasing, so SP is accelerating; SP suddenly stops at 9am.

   (c) From noon to 1pm SP is stationary, possible having lunch.

2. $$\lim_{x\to 8}\frac{x^2+x-72}{x-8}=\lim_{x\to 8}\frac{(x+9)(x-8)}{x-8}=\lim_{x\to 8}(x+9)=17.$$

3. (a) From the sum and power function rules for differentiation $f'(x)=8x-3$; so $f'(2)=13$.

   (b) The slope of the tangent line at $(2,10)$ is 13; an equation for the tangent line is

   $$y-10=13(x-2),\ \text{or } y=13x-16.$$

4. The graph c is position, the graph b represents velocity, and a is acceleration. The slopes of each graph run the show here. Notice that the slopes in c increase until about half way, and then decrease to 0. At the same time the *values* for the b graph (these are velocity values) increase until the same halfway point and then decrease. So b is consistent with the look of the velocity here. The c graph is also convex up (positive acceleration) until halfway, and then convex down (concave) after that, consistent with negative acceleration; the a graph matches this behaviour.

5. Anti-differentiating $v(t)=s'(t)=\sqrt{t}$ using the power function rule, we get

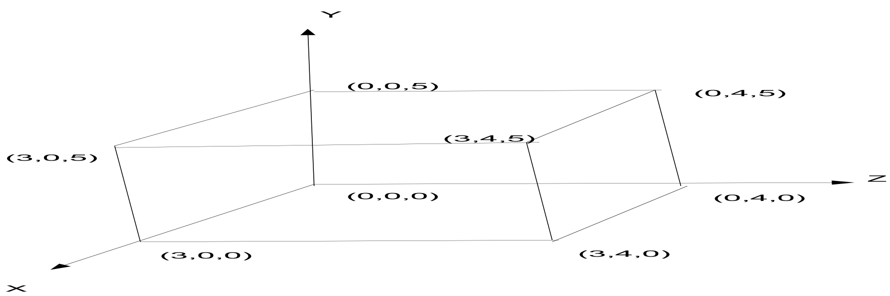
   $$s(t)=\frac{t^{3/2}}{3/2}+C$$

   for some constant C. In this set $t=1$ to get

   $$2/3=s(1)=2/3+C,$$

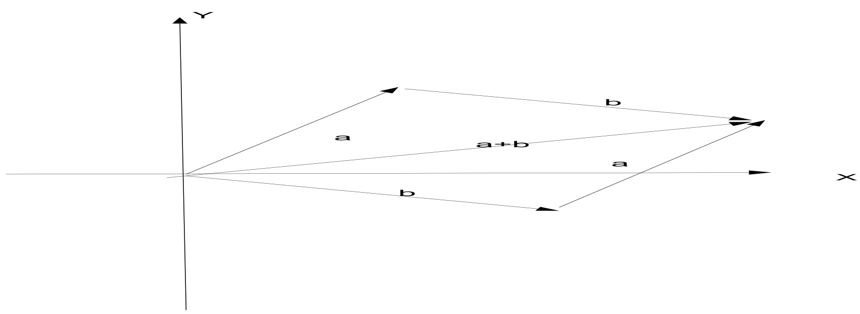
   so $C=0$ and $s(t)=t^{3/2}$.

6.



7.

$\boldsymbol{a} + \boldsymbol{b} = \langle 1, 2\rangle + \langle 2, -1\rangle = \langle 3, 1\rangle.$



8. The force $\boldsymbol{a}$ has magnitude 5 pounds and angle $\pi/4$ radians. So $\boldsymbol{a}$ has components

$$\boldsymbol{a} = 5\langle \cos\pi/4, \sin\pi/4\rangle = \langle 5/\sqrt{2}, 5/\sqrt{2}\rangle.$$

The force $\boldsymbol{b}$ has magnitude 4 pounds and angle $-\pi/6$ radians, so

$$\boldsymbol{b} = 4\langle \cos\pi/6, -\sin\pi/6\rangle = \langle 2\sqrt{3}, -2\rangle.$$

The resulting force is then

$$\boldsymbol{a} + \boldsymbol{b} = \langle 5/\sqrt{2} + 2\sqrt{3}, 5/\sqrt{2} - 2\rangle.$$

9. Let $0 \le \theta \le \pi$ be the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$. Then

$$\cos\theta = \frac{\boldsymbol{a}\cdot\boldsymbol{b}}{|\boldsymbol{a}|\,|\boldsymbol{b}|} = \frac{1}{\sqrt{3}},$$

so $\theta = \cos^{-1}(1/\sqrt{3})$ radians.

10. The scalar and vector projections of $\boldsymbol{b}$ on to $\boldsymbol{a}$ are

$$\mathrm{comp}_{\boldsymbol{a}}\boldsymbol{b} = \frac{\boldsymbol{a}\cdot\boldsymbol{b}}{|\boldsymbol{a}|} = 1/\sqrt{3}, \text{ and } \mathrm{proj}_{\boldsymbol{a}}\boldsymbol{b} = \frac{\boldsymbol{a}\cdot\boldsymbol{b}}{|\boldsymbol{a}|^2}\boldsymbol{a} = \frac{1}{3}\langle 1, 1, 1\rangle.$$