Department of Mathematics UAB
Calculus II

ASSIGNMENT 2

Vectors and Integrals

Show all necessary calculations and relevant explanations. Numerical answers with no supporting explanations will receive no credit.

1. (a) Find vector and parametric equations for the line through the point $(6, -7, 4)$ and parallel to the vector $\boldsymbol{v} = \langle 1, 2, 3 \rangle$.

   (b) Find vector and parametric equations of the line through the point $(1, -1, 1)$ and parallel to the line

   $$x = -1 + 4t, \quad y = 6 - 3t, \quad z = 3 + 9t.$$

   (c) Find parametric equations for the line through the points $(1, 1, 1)$ and $(3, -4, 4)$.

2. Determine whether the lines L_1 and L_2 with parametric equations

   $$L_1 : x = 12 + 8t, \quad y = 16 - 4t, \quad z = 4 + 12t$$
   $$L_2 : x = 4 + 16s, \quad y = 12 - 8s, \quad z = 16 + 20s,$$

   are parallel, intersecting, or skew (neither parallel nor intersecting).

3. Find parametric equations for the line through $P(1, 1, 1)$ and perpendicular to the plane $x + 4y + z = 5$. At which point Q does this line meet the plane? Use Q to find the perpendicular distance from the point P to the plane.

4. (a) Find an equation for the plane through the point $(6, -3, , 2)$ and parallel to the plane $5x - y - z = 3$.

   (b) Find an equation for the plane through the three points $P(0, 6, 6)$, $Q(6, 0, 6)$, and $R(6, 6, 0)$.

5. Find an equation for the plane that passes through the point $(9, 0, -1)$ and contains the line

   $$x = 7 - 2t, \quad y = 3 + 5t, \quad z = 8 + 4t.$$

6. Find an equation of the plane that passes through the point $(-1, 3, 2)$ and contains the line of intersection of the planes $x + y - z = 4$ and $x - y + 5z = 4$.

7. At which point does the space curve

   $$\boldsymbol{r}(t) = \langle x(t), y(t), z(t) \rangle = \langle t, 0, 4t - t^2 \rangle$$

   intersect the paraboloid surface $z = x^2 + y^2$?

8. Find a vector function that represents the curve of intersection of the circular cylinder $x^2 + y^2 = 16$ and the surface $z = xy$.

9. Find parametric equations for the tangent line to the space curve

$$x = 1 + 4\sqrt{t}, \quad y = t^5 - t, \quad z = t^5 + t,$$

at the point $(5, 0, 2)$.

10. Estimate the area under the graph of $f(x) = 10\cos x$ from $x = 0$ to $x = \pi/2$ using four approximating rectangles together with right end-points. Include in your answer a sketch of the graph of $f(x)$ and the four approximating rectangles.