

ASSIGNMENT 4

Substitution, Trig Integrals, Integration by Parts, Partial Fractions

Show all necessary calculations and relevant explanations. Numerical answers with no supporting explanations will receive no credit.

1. Find the following indefinite integrals (anti-derivatives) using an appropriate substitution.

(a)

$$\int (10t + 9)^{1.5} dt.$$

(b)

$$\int \frac{dx}{9 - 5x} \quad (\text{Remember to use } \log |u| \text{ where appropriate}).$$

(c)

$$\int \frac{x}{(x^2 + 3)^2} dx.$$

2. Find the following definite integrals using an appropriate substitution.

(a)

$$\int_1^2 \frac{e^{1/x^5}}{x^6} dx.$$

(b)

$$\int_6^7 x\sqrt{x-6} dx.$$

(c)

$$\int_e^{e^2} \frac{dx}{x\sqrt{\log x}}.$$

3. If the temperature of a body is given by $T(t) = (1 + 2t)/(1 + t^2)$ over an interval $[0, 2]$, find the average temperature of the body over this time interval.

4. Use integration by parts to find the following indefinite integrals.

(a)

$$\int (x^2 + 2x) \cos x \, dx.$$

(b)

$$\int \log(x + 1) \, dx.$$

(c)

$$I = \int e^x \sin x \, dx. \quad \text{Hint: integrate by parts twice, and solve for } I.$$

5. First make a substitution, and then use integration by parts to find the indefinite integral

$$\int \cos \sqrt{x} \, dx.$$

6. For each of the rational functions below, use polynomial long division (necessary when the degree of the numerator is equal to or greater than the degree of the denominator) and then simplify with partial fractions and finally perform the indefinite integration.

(a)

$$\int \frac{x^3 + x^2 - x + 5}{x^2 + x - 2} dx.$$

(b)

$$\int \frac{1}{x^3 - 2x^2 + x} dx.$$

(c)

$$\int \frac{x^4 + x^2 + 1}{x^3 + x} dx.$$

7. (a) Sketch, on the same axes, the graph of the function $\sin x$ on the interval $[-\pi/2, \pi/2]$ together with the graph of the inverse sine function $\sin^{-1} x$ defined on $[-1, 1]$.
- (b) Find the exact values of $\sin^{-1}(\sqrt{3}/2)$ and $\tan^{-1}(1)$.
- (c) Simplify $\sin(\tan^{-1} x)$.
- (d) Find $f'(x)$ if $f(x) = [\tan^{-1}(x)]^2$.

8. A lighthouse is located on a small island 3 km from the nearest point P on a straight shoreline, and its light makes four revolutions per minute. How fast is the beam of light moving along the shoreline when it is 1 km from P ?

9. Find the following trigonometric integrals.

(a) $\int \sin^3 x \cos^2 x \, dx.$

(b) $\int \sec^3 x \, dx.$

(c) $\int_0^{\pi/2} \sin^2 \theta \cos^2 \theta \, d\theta.$

10. Evaluate the following definite integrals using a trig substitution. Hint: 9(a)(b).

(a) $\int_0^1 x^3 \sqrt{1-x^2} \, dx.$

(b) $\int_0^{\sqrt{3}} \sqrt{1+x^2} \, dx.$