MA 125-6C, CALCULUS I
October 14, 2015

Name (Print last name first): .....

Show all your work and justify your answer!

No partial credit will be given for the answer only!

PART I

You must simplify your answer when possible.
All problems in Part I are 10 points each.

1. Find the derivative of the function $y = f(x) = \cos(x^3)$.

2. Find the derivative of $f(x) = (x^2 + x)^8$.

3. Find the absolute maximum and minimum of the function $y = f(x) = (x-2)^2(x+1)^3$ on the interval $[0, 1]$.

4. Find the linearization of the function $f(x) = x\tan(x)$ at the point $a = \pi/4$ and use it to estimate the value $f(\pi/4 + 0.1)$.

5. Find two positive numbers so that their sum is 200 and their product is maximal. [As always you must justify your answer!]

6. Suppose that the **derivative** of a function $y = f(x)$ is given:

   $f'(x) = (x + 2)(3 - x)$.

   (a) Find the x-coordinates of all local max/min of the function $y = f(x)$.

   (b) At which x value is the function $y = f(x)$ most rapidly increasing?

PART II

7. [**15 points**] You work for a soup company. Your boss asks you to design a soup can of volume $1\,dm^3$ and **minimal** surface area. Either specify the radius of top/bottom of such a can or show that such a can does not exist. Then your boss asks you to design a soup can of volume $1\,dm^3$ and **maximal** surface area. Either specify the radius of top/bottom of such a can or show that such a can does not exist.

   You may use that the volume of a can of radius r and height h is $V = \pi r^2 h$ while the surface are of the side is $2\pi rh$ and of the top (and bottom) is πr^2.

8. [**20 points**] **Use calculus** to graph the function $y = f(x) = \dfrac{x}{x^2+1}$. Indicate

   - x and y intercepts,
   - vertical and horizontal asymptotes (if any),
   - in/de-creasing; local/absolute max/min (if any).

   You must show work to justify your graph and conclusions. You can use decimal numbers to plot points (but mark them with exact values).