MA 125-CT,   CALCULUS I
September 26, 2016

Name (Print last name first): ....

$\boxed{\text{TEST I}}$

$\boxed{\textbf{Show all your work! No partial credit will be given for the answer only!}}$

$\boxed{\textbf{PART I}}$

Part I consists of questions. Clearly write your answer in the space provided after each question. Show all of your your work!

All 8 problems in Part I are 7 points each.

Question 1

Use the **definition** of the derivative to show that the derivative of $y = f(x) = x^2$ is $f'(x) = 2x$.

Question 2

Find the derivative of $f(x) = x(x^2 + \sqrt{x})$

Answer: ....

Question 3

Find the derivative of $y = f(x) = x \sin(x)$.

Answer: ....

Question 4

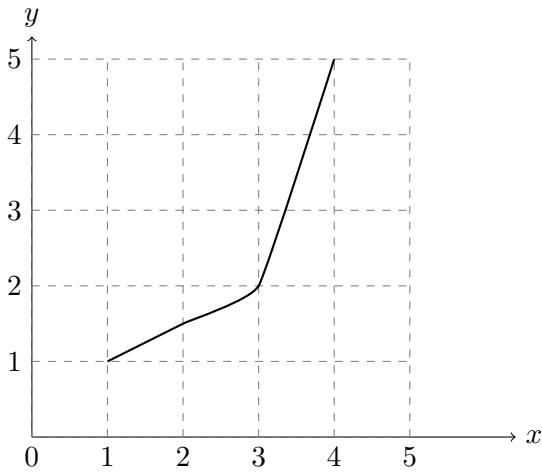Find the derivative of $y = f(x) = \dfrac{x^2}{x+1}$.

Answer: ....

Question 5

Find the equation of the tangent line to the graph of $y = f(x) = 2x^3 + x$ at the point $x = 1$.

Answer: ....

Question 6

Using the graph of the function $y = f(x)$ on the next page to estimate: $f(2)$ and the derivative $f'(2)$.

Answer: ....

Question 7

Find the following limit:

$$\lim_{x \to 3} \frac{x^2 - 4x + 3}{x - 3}$$

Question 8

Evaluate the following limit:

$$\lim_{x \to \infty} \frac{\cos^4(x)}{\sqrt{x}}$$

PART II

Part II consists of 4 problems. You must show correct reasons to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit.
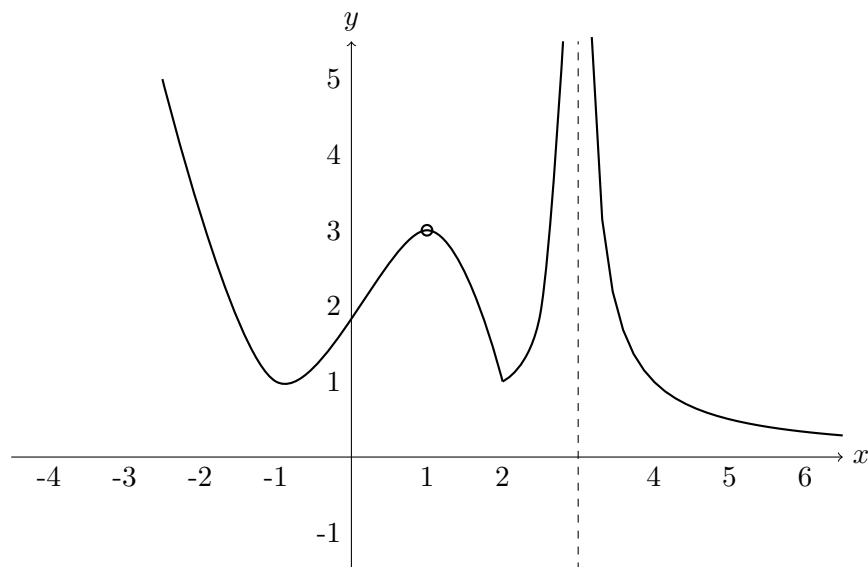
Problem 1 (12 points)

Recall that the displacement of a particle is the difference between its two positions associated with two moments of time. Suppose that $S(t) = 2t^3 + 3t^2 - 12t$ m is the position of a particle at time t (in seconds) on a line. Find:
(a) the velocity at time $t = 1$;
(b) the displacement from $t = 0$ to $t = 3$.

Problem 2 (10 points)

Given the graph of the function $y = f(x)$ below find:

1. $\lim\limits_{x \to 1^-} f(x) =$

2. $\lim\limits_{x \to 1^+} f(x) =$

3. $\lim\limits_{x \to 1} f(x) =$

4. $\lim\limits_{x \to 3^-} f(x) =$

5. $\lim\limits_{x \to 3^+} f(x) =$

6. $\lim\limits_{x \to 3} f(x) =$

7. $\lim\limits_{x \to \infty} f(x) =$

8. $\lim\limits_{x \to -\infty} f(x) =$

9. State all intervals on which $f(x)$ is continuous.

10. State all intervals where $f(x)$ is differentiable.

Problem 3 (12 points)

Evaluate the following limits:

a) $\lim_{x \to \infty} x^3 + x$

b) $\lim_{x \to \infty} x^3 - x$

c) $\lim_{x \to \infty} \dfrac{\tan(x)}{x}$

Problem 4 (10 points)

Define a function

$$f(x) = \begin{cases} \frac{-x^3 + 5x^2}{x-5}, & x \neq 5, \\ k, & x = 5, \end{cases}$$

a) (8 points) Find a value of k so that $f(x)$ is a continuous function for all real numbers x.

b) (2 points) For the value of k you found, is $f(x)$ differentiable at $x = 5$? Briefly explain.