EXAM II

MA 125-CW, CALCULUS I

March 3, 2016

Name (Print last name first): .....

Show all your work!

No partial credit will be given for the answer only!

PART I

You must simplify your answer when possible.
All problems in Part I are 10 points each.

1. Find the derivative of the function $f(x) = \left(x^3 + x\right)^5$. State which rules you used.

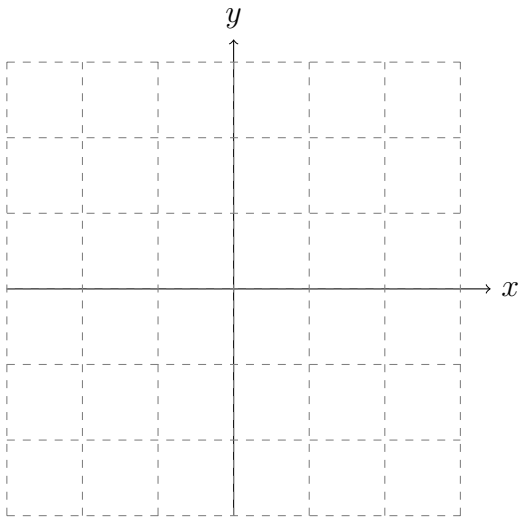
2. Find all critical numbers of the function $f(x) = (2x + 1)^3(1 - x)^7$. Name each step you make.

3. Find the absolute maximum and minimum of the function $y = f(x) = x^4 - 2x^2$ on the interval $[-2, 2]$. Again, name steps you make.

4. State the Mean Value Theorem. Next find the number c which satisfies the conclusion of the Mean Value Theorem for the function $f(x) = x^5$ on the interval $[0, 1]$.

5. Show that the equation $f(x) = x^3 + x + 5 = 0$ has exactly one solution.
   You may want to first show that there is at least one solution using the Intermediate Value Theorem. Next show that two distinct solutions is impossible.

6. Suppose that the **derivative** of a function $y = f(x)$ is given by the graph below.



(a) Find the x-coordinates of all local max/min of the function $y = f(x)$.

(b) At which x is the function $y = f(x)$ most rapidly increasing?

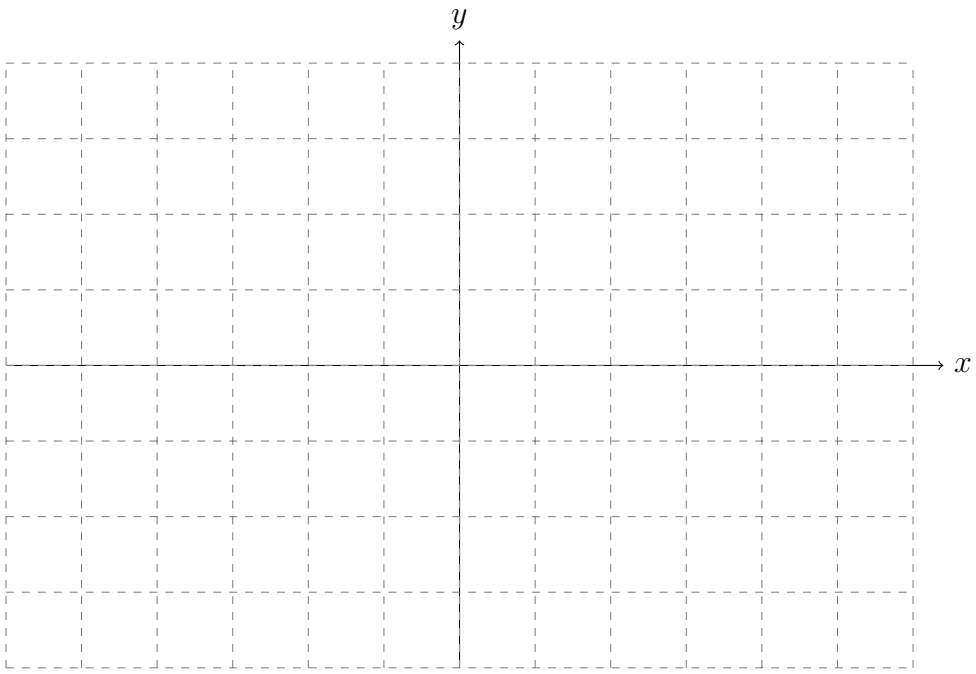
PART II

7. [**10 points**] Find the dimensions of a box such that the volume is 1 m^3, one side of the base is twice the length of the other side, and the surface area is minimum. Recall that the volume of a box is given by volume = (area of base) $*$ height. You may want to draw a box with dimensions of the base x and $2x$, and height y so that its volume $V = 2x^2y$, then compute the area of all sides and add these.

8. [**20 points**] **Use calculus** to graph the function $y = f(x) = \dfrac{1+x^2}{1-x^2}$. Indicate

   - x and y intercepts,
   - vertical and horizontal asymptotes (if any),
   - in/de-creasing; local/absolute max/min (if any).

   You must show work to justify your graph and conclusions. You can use decimal numbers to plot points (but mark them with exact values).



9. This question has two parts.

   (a) [**6 points**] Find the linearization of $f(x) = \sqrt[3]{x}$ at $a = 8$

   (b) [**4 points**] Use this linearization to find the approximate value of $\sqrt[3]{8.01}$.