ID NUMBER:

UAB MATH TALENT SEARCH

This is a ten-problem two hour contest. All answers are either integers ranging between 0 and 999, or one of the multiple choices provided in the problem. Answers to the problems are to be written in the spaces indicated on the question sheet. However, your work and explanation of your answer are to be written on the extra paper provided. In case of a tie, the judging panel will use your work and explanation to choose the winner. Each next problem is harder than the previous one, and is worth the same or greater number of points. The point value of a problem is posted with it. Choosing their strategy, the students must assess the number of points they will get solving problems and the level of difficulty of the problems.

PROBLEM 1 (5 points). Four boys bought a boat for \$60. The first boy paid one half the sum of the amounts paid by the other boys; the second boy paid one third the sum of the amounts paid by the other boys; and the third boy paid one fourth of the sum of the amounts paid by the other boys. How much did the fourth boy pay?

YOUR ANSWER:

PROBLEM 2 (10 points). Ann and Sue bought identical boxes of stationary. Ann used hers to write 1-sheet letters and Sue used hers to write 3-sheet letters. Ann used all her envelopes and had 50 sheets of paper left, while Sue used all her sheets of paper and had 50 envelopes left. What was the number of sheets in the box?

YOUR ANSWER:

PROBLEM 3 (10 points). Given an equation $|x - 4| + |x - 7| = 3$, what is the number of solutions? Choose from the following options:

A) 0; B) 1; C) 2; D) 3; E) > 3.

PROBLEM 4 (15 points). 2 positive numbers a, b are inserted between 12 and 36 so that $12, a, b$ are in a geometric progression while $a, b, 36$ are in arithmetic progression. Find $a + b$.

YOUR ANSWER:

PROBLEM 5 (15 points). The consecutive angles of a quadrilateral form an arithmetic sequence. If the smallest is 75 degrees, then what is the largest?

YOUR ANSWER:

PROBLEM 6 (20 points). Each letter represents uniquely a different digit in base ten: $(MA) \times (HA) = TTT$. Find $M + A + T + H$.

YOUR ANSWER:

PROBLEM 7 (25 points). Which of these triples could not be the lengths of the three altitudes of a triangle:

A) $1/5, 1/12, 1/13$; B) $1/3, 1/6, 1/10$; C) $1/5, 1/8, 1/9$;
D) $1/3, 1/4, 1/5$; E) $1/2, 1/3, 1/4$; F) $1/10, 1/13, 1/21$;
G) $1/4, 1/7, 1/9$; H) $1/4, 1/5, 1/8$.

PROBLEM 8 (40 points). A square ABCD is given. Then the circle of radius AB centered at A is drawn. Inside the square the circle intersects the perpendicular bisector of BC at the point O. Find the value of angle AOC in degrees.

YOUR ANSWER:

PROBLEM 9 (65 points). Let D be a subset of $\{1, 2, \ldots, 700\}$ such that no two numbers from D differ by 2 or 5. What is the largest number of elements that the set D can have?

YOUR ANSWER:

PROBLEM 10 (80 points). Find the largest possible number n such that $n!$ can be expressed as the product of $n - 5$ consecutive integers.

YOUR ANSWER: