Sample Questions for the Midterm Exam

Review Session: May 4th, Tuesday at 2-3pm during Discussion Section. The midterm is on **May 5th, 2:00-2:50pm at MS 5117**. It will cover materials from sections: 1-3, 5-8, 10-14, 17-22.

1. Consider a simple spring-mass system with no forces other than the spring force and friction.

a. If mass is 1, the spring constant is 2, and the friction coefficient is -2, what is the governing equation?
b. What are the roots of the characteristic equation corresponding to the system above? Is the system overdamped, underdamped, or critically damped? What is the circular frequencey ω?
c. What is the general solution of the system?
d. Given $x(0) = 1$ and $\frac{dx}{dt}(0) = 0$, what is the solution?

2. Consider a spring-mass system with the following form:

$$\frac{d^2x}{dt} = -f(x).$$

Let $f(x) = (x - 2)$.
a. Derive the energy equation for the given system.
b. What is the potential energy? What are the equilibrium points? Are they stable?
c. Sketch some energy curves on the phase plane.

3. Consider a nonlinear pendulum with damping force:

$$L\frac{d^2\theta}{dt^2} = -g\sin\theta - k\frac{d\theta}{dt}.$$

a. Use the perturbation method to linearize the system around $\theta_E = 0$.
b. Under what condition does the linearized pendulum continually oscillate back and forth with decreasing amplitude around $\theta_E = 0$.

4. Given the nonlinear pendulum,

$$L\frac{d^2\theta}{dt^2} = -g\sin\theta.$$

a. Derive the energy equation.
b. Sketch the energy curves when $0 < E < 2g$.