Review Questions for Midterm

The midterm on Oct. 31st will cover materials from §1.1, 1.2, 2.1, 2.3, 2.4, 3.1, 3.2, and 8.3

1. Write an essay on the Secant method.

2. a. Which of these sequences converges quadratically? Please justify your answer.

i. $\frac{1}{n^2}$
ii. $\frac{1}{2^{-2^n}}$
iii. $\frac{1}{\sqrt{n}}$
iv. $\frac{1}{e^n}$

b. Find the conditions on α to ensure that the iteration

$$x_{n+1} = x_n - \alpha f(x_n)$$

will converge linearly to a zero of f if started near the root.
(Hint: use Taylor's Theorem about p: $f(p) = f(p_n) - f'(p_n)e_n + O(e_n^2)$)

3. What is the Intermediate Value Theorem? Mean Value Theorem? Taylor's Theorem? Rolle's Theorem? For each one give an example of where it's been used so far in the course.

4. For each of the Bisection Method, Newton's Method, and the Secant Method answer the following questions.

a. What are sufficient conditions for convergence?
b. What's the order of convergence (and under what conditions)?

5. Given the table of x versus $\sin x$ do the following:

x	$\sin x$
$-\frac{\pi}{6}$	$-\frac{1}{2}$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$

a. Find the Lagrange interpolating polynomial of degree 2 using the data on the table on interval $[-\frac{\pi}{6}, \frac{\pi}{6}]$ Find the error bound of $|\sin x - P_2(x)|$.
b. Use divided differences to get $P_2(x)$ interpolating polynomials. What's the error bound of $|\sin x - P_2(x)|$?

6. a. Find the zeros of the monic Chebychev polynomial $\tilde{T}_3(x)$.
b. Construct the Lagrange interpolating polynomial $P_2(x)$ using the zeros of $\tilde{T}_3(x)$ on interval $[-1, 1]$.
c. What's the error bound of $|\sin x - P_2(x)|$?