

### Review Questions for the Final Exam

Review Sessions: Dec. 4th, Thursday at 9am-10am during Discussion (MS 5137) and Dec. 5th, Friday at 9am-10am during the regularly scheduled Lecture (MS 6229).

The final is on **Dec. 11th, 11:30am-2:30pm at MS 6229**. It will cover materials after the midterm

§4.1, 4.2 4.3, 4.4, 4.7, 6.1, 6.2, 6.5, 6.6

and before the midterm

§1.1, 1.2, 2.1, 2.3, 2.4, 3.1, 3.2, and 8.3

1. Derive two-point and three-point formulas for numerical differentiation. What are their respective truncation errors? Give examples where you use these formulas to approximate derivative of some functions. Find error bounds and actual errors.
2. How do you accelerate convergence of low-order Newton-Cotes formulas using Richardson's extrapolation.
3. Derive the basic numerical quadrature formulas: Trapezoidal and Simpson's rule. What are the truncation errors for both? Give examples using these formulas and calculate actual errors and error bounds.
4. How is Gaussian quadrature different from the Newton-Cotes formulas? Given the roots, coefficients, and weight functions corresponding to the Chebychev and Legendre polynomials, how do you approximate definite integrals? Give examples.
5. How is LU decomposition equivalent to Gaussian elimination? Give an example. How does one interpret  $PA=LU$  where  $P$  is a permutation matrix in Gaussian elimination with pivot?
6. What is partial pivoting? scaled partial pivoting? When or why are they executed? Comment on the instability of Gaussian elimination (in the case of without pivot and in general). Give some examples of linear systems where Gaussian elimination without pivot fails.
7. Explain why the work needed for Gaussian elimination is  $\mathcal{O}(n^3)$ ? backward substitution is  $\mathcal{O}(n^2)$ ? (You may use geometric argument to get the leading order term.)
8. What special types of matrices can be solved using Gaussian elimination without pivoting? Please give at least 3 examples.
9. What is Cholesky's factorization? Give an example.

*Note: I did not put any questions pertaining to the materials tested on the midterm. This does not mean there aren't going to be questions from those sections. Please review those materials tested on the midterm albeit there will more weight given on the latter sections.*