Computational Homework 1, due Monday, 9th February

Consider the following initial value problems:

$$\begin{aligned} \frac{dy}{dt} &= 2y, \ \ 0 \leq t \leq 2 \\ y(0) &= 1. \end{aligned} \tag{1}$$

$$\begin{aligned} \frac{dy}{dt} &= y - t^2 + 1, \ \ 0 \leq t \leq 2 \\ y(0) &= 0.5. \end{aligned} \tag{2}$$

i. Solve the initial value problem (1) and (2) numerically using the step sizes $h = 0.1, 0.01, 0.001$ using **Euler's method, Runge-Kutta order 2 (midpoint) method, and Runge-Kutta order four method**.

Solving the problem numerically entails coding the algorithms for each methods in your favorite language (like Matlab), generating approximations w_i, and plotting the graph $w_0, w_1, \ldots, w_N$ for each method and step size.

ii. Solve the initial value problem (1) and (2) analytically (paper/pencil way). Then, compute the actual error at each time step, i.e. $e_0, e_1, \ldots, e_N$ where $e_i = |y_i - w_i|$. Plot the graphs of the true solutions as well as the errors.

iii. Discuss the results and compare the three methods. The results should be illustrated in terms of plots of actual solutions, approximated solutions, and errors. Tables are accepted as well but they use up a lot of papers!