Computational Homework 3, due Friday, 12th March

Consider the following boundary value problems:

$$y'' = -2e^x - y, \quad 0 \le x \le 1 \tag{1}$$
$$y(0) = 2 \text{ and } y(1) = e + cos(1)$$

$$y'' = -\frac{2}{x}y' + \frac{2}{x^2}y + \frac{\sin(\ln x)}{x^2}, \quad 1 \le x \le 2 \tag{2}$$
$$y(1) = 1 \text{ and } y(2) = 2.$$

The true solutions for (1) is $y(x) = e^x + \cos x$ and for (2) is $y = cx + \frac{d}{x^2} - \frac{3}{10}\sin(\ln x) - \frac{1}{10}\cos(\ln x)$ where $c = \frac{11}{10} - d$ and $d = \frac{1}{70}(8 - 12\sin(\ln 2) - 4\cos(\ln 2))$.

i. Write a general-purpose code to solve linear two-point boundary value problems by the finite-difference method as described in *Algorithm 11.3* (pp.662) in *Burden and Faires*. You may either download the tridiagonal linear system solver from the class webpage, tweak your old LU decomposition code from last quarter or code up your own solver.

ii. Test the program written in *part i* on problems (1) and (2) with step size $h = 0.1$ and $h = 0.01$. Calculate $\|y - w\|_\infty$ and $\|r\|_\infty$ where $y = (y(x_1), y(x_2), \ldots, y(x_n))'$ (true solution evaluated on the mesh points), $w = (w_1, w_2, \ldots, w_n)'$ (the approximated solution) and $r = (r_1, r_2, \ldots, r_n)'$ (the residual vector associated with w).

iii. Replace the linear system solver with an iterative scheme (Jacobi, Gauss-Seidel or both) in the code from *part i*. Take advantage of the special structure of the matrix when coding the iterative scheme of choice.

iv. Test the program written in *part iii* on problems (1) and (2) with step size $h = 0.1$ and $h = 0.01$. Choose a sensible initial guess $w^{(0)}$, a tolerance for the stopping criteria, TOL, and a maximum number of iterations, N_{max}. Plot $\|y - w^{(k)}\|_\infty$ and $\|r^{(k)}\|_\infty$ at each iteration k.