Sample Questions for the Midterm Exam

Review Session: Feb. 17th, Tuesday at 9am-10am during Discussion Section. The midterm is on **Feb. 20th, 9:00-9:50am at MS 5137**. It will cover materials from sections: 5.1, 5.2, 5.3, 5.4, 5.6, 5.9, 5.11, 11.3, and 7.1 (vector and matrix norms).

1. Show that when Heun's method (second-order Runge-Kutta method) is applied to the problem $y' = \lambda y$, the formula for numerical solution is

$$w_{i+1} = \left[1 + h\lambda + \frac{h^2\lambda^2}{2}\right] w_i.$$

Prove that the local truncation error is $\mathcal{O}(h^2)$.

2. Consider the second-order initial value problem,

$$y'' = y\cos t + e^t y' + 3t^2 + 7$$
$$y(1) = 5 \text{ and } y'(1) = 9$$

Explain how to solve this problem numerically using Euler's method and modified Euler's method.

3. Solve the two-point boundary-value problem

$$y'' = -2y' - 10y$$
$$y(0) = 1 \text{ and } y(2) = 2$$

for $y(\frac{1}{2})$ using the finite-difference method with $h = \frac{1}{2}$.

4. Consider a system of two differential equations:

$$(1) \qquad \begin{aligned} x' &= \alpha x + \beta y, \quad x(0) = 2 \\ y' &= \beta x + \alpha y, \quad y(0) = 0 \end{aligned}$$

a. Verify that the solution to (1) is

$$\begin{aligned} x(t) &= e^{(\alpha+\beta)t} + e^{(\alpha-\beta)t} \\ y(t) &= e^{(\alpha+\beta)t} - e^{(\alpha-\beta)t} \end{aligned}$$

b. If Euler's method is used to compute a numerical solution of (1), then write out the corresponding difference equations.

c. The solutions of these difference equations are

$$x_i = (1 + \alpha h + \beta h)^i + (1 + \alpha h - \beta h)^i$$
$$y_i = (1 + \alpha h + \beta h)^i - (1 + \alpha h - \beta h)^i$$

Assume that $\alpha = -20$ and $\beta = -19$, explain why (1) is a stiff system. Find an interval for the step size h for which the numerical solutions using Euler's method decay to 0.