

Assignment 2, due Friday, 15th October

Theoretical:

1. Given $\theta = [0, 1]$, find the order of the method

$$(1) \quad y_{n+1} = y_n + hf(t_n + (1 - \theta)h, \theta y_n + (1 - \theta)y_{n+1}).$$

2. Let $\theta = \frac{1}{2}$. Prove the method (1) is convergent.

3. Given $\theta = [0, 1]$, prove convergence for the method

$$y_{n+1} = y_n + h[\theta f(t_n, y_n) + (1 - \theta)f(t_{n+1}, y_{n+1})].$$

Computational:

Approximate the solution to

$$\dot{y} = -2y + t, \quad y(0) = 1$$

using both Euler's method and the Trapezoidal method.

- Calculate and list the errors in the solution at time $t = 2.0$ using timesteps $h = 0.1, 0.05, 0.025$.
- Estimate the rate of convergence.