Assignment 3, due Friday, 22nd October

Theoretical:

1. Find a solution of the consistency conditions for which $c_2 = c_3$ and $b_2 = b_3$ in the explicit Runge-Kutta method of order 3. The resulting explicit method is known as Nyström's third order method.

2. Consider the autonomous equation $y' = f(y)$. Prove that the ERK method with the Butcher array

$$\begin{array}{c|cccc} 0 & & & & \\ \frac{1}{2} & \frac{1}{2} & & & \\ \frac{1}{2} & & \frac{1}{2} & & \\ 1 & & & 1 & \\ \hline & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} \end{array}$$

is of order four.

3. Suppose that an $s-$stage ERK method of order s is applied to the linear scalar equation $y' = \lambda y$. Prove that

$$w_n = \left[\sum_{k=0}^{s} \frac{1}{k!} (h\lambda)^k \right]^n w_0, \quad n = 0, 1, \ldots$$

Computational:

Consider the initial value problem

$$\frac{dy}{dt} = \frac{3t^2 + 4t + 2}{2(y-1)}, \quad y(0) = -1. \tag{1}$$

Implement the classical fourth order Runge-Kutta method and approximate the solution to the problem (1).

- Give the approximate solution at time $t = 2.0$ for timesteps $h = 0.1, 0.05, 0.025$, and 0.0125.
- Estimate the rate of convergence for each pair of successive approximate solutions.
- Find experimentally the timestep h_E for which Euler's method applied to (1) gives the same accuracy as the fourth order Runge-Kutta with $h_{RK} = .5$. Compare the efficiency of both algorithms applied to (1).