Assignment 6, due Friday, 12th November

Theoretical:

1. Determine the region of absolute stability for the method

$$w_{n+1} = w_n + hf(t_n + \theta h, (1-\theta)w_n + \theta w_{n+1}) \tag{1}$$

for several $\theta \in [0,1]$ to make the general picture. Notice that the method above includes explicit Euler, implicit Euler, and implicit midpoint rule as special cases.

2. Determine all the values of $\theta \in [0,1]$ such that the method (1) is A-stable.

Computational:

Consider the following initial value problem

$$\begin{aligned} \frac{d\vec{y}}{dt} &= \Lambda\vec{y}, \quad 0 \le t \le 1 \\ \vec{y}(0) &= (1,1)'. \end{aligned} \tag{2}$$

where

$$\Lambda = \begin{bmatrix} -50 & 1 \\ 0 & -\frac{1}{10} \end{bmatrix}$$

Apply both the forward and backward Euler methods to the ivp (2).

- Find a stepsize $h > 0$ for which the explicit Euler's method is unstable. Plot $ln\|w_n\|$ vs n.
- Use the same stepsize h for the implicit Euler method to solve the problem (2). Plot $ln\|w_n\|$ vs n.
- Explain your results.