Assignment 8, due Friday, 3rd December

Theoretical:

1. The two-step method

$$w_{n+2} - w_n = 2hf(t_{n+1}, w_{n+1}), \quad n = 0, 1, \ldots$$

is called the *explicit midpoint rule.*

a: Let $w_1(z)$ and $w_2(z)$ be the zeros of the underlying function $\eta(z)$. Prove that $w_1(z)w_2(z) = -1$ for all $z \in \mathbb{C}$.

b: Show that $\mathcal{D} = \emptyset$.

2. Determine the order of the two-step method

$$w_{n+2} - w_n = \frac{2}{3}h\left[f(t_{n+2}, w_{n+2}) + f(t_{n+1}, w_{n+1}) + f(t_n, w_n)\right], \quad n = 0, 1, \ldots$$

Is it A-stable?

3. Suppose the following multistep method

$$x_{n+1} - x_n = \frac{1}{2}h\left[3f(t_n, x_n) - f(t_{n-1}, x_{n-1})\right] \tag{1}$$

is used to increase accuracy (instead of estimating the error) in the multistep method

$$y_{n+1} - y_n = \frac{1}{2}h\left[f(t_{n+1}, y_{n+1}) + f(t_n, y_n)\right]. \tag{2}$$

a: Prove that following:

$$y(t_{n+1}) - y_{n+1} = -\frac{1}{12}h^3 y^{'''}(t_n) + \mathcal{O}(h^4) \tag{3}$$

$$y(t_{n+1}) - x_{n+1} = \frac{5}{12}h^3 y^{'''}(t_n) + \mathcal{O}(h^4) \tag{4}$$

b: Neglecting the $\mathcal{O}(h^4)$ terms, solve the two equations (3-4) for the unknown $y^{'''}(t_n)$.

c: Substituting the approximate expression back into

$$y(t_{n+1}) - y_{n+1} = ch^3 y^{'''}(t_{n+1}) + \mathcal{O}(h^4)$$

results in a two-step implicit multistep method. Derive the method explicitly and determine the order. Is it convergent? Can you identify it?

4. Prove that the embedded RK pair

$$\begin{array}{c|ccc} 0 & & & \\ \frac{1}{2} & \frac{1}{2} & & \\ 1 & -1 & 2 & \\ \hline & 0 & 1 & \\ \hline & \frac{1}{6} & \frac{2}{3} & \frac{1}{6} \end{array}$$

combines a second-order and a third-order method.

Computational:

No computation!